

## OCTOBER TENTH, ELEVENTH, TWELFTH NINETEEN HUNDRED AND TWELVE

VOLUME TWO

# THE BOOK OF THE OPENING OF THE RICE INSTITUTE

BEING AN ACCOUNT IN THREE VOLUMES OF AN ACADEMIC FESTIVAL HELD IN CELEBRATION OF THE FORMAL OPENING OF THE RICE INSTITUTE, A UNIVERSITY OF LIBERAL AND TECHNICAL LEARNING FOUNDED IN THE CITY OF HOUSTON, TEXAS, BY WILLIAM MARSH RICE AND DEDICATED BY HIM TO THE ADVANCEMENT OF LETTERS, SCIENCE, AND ART

#### VOLUME II



HOUSTON, TEXAS



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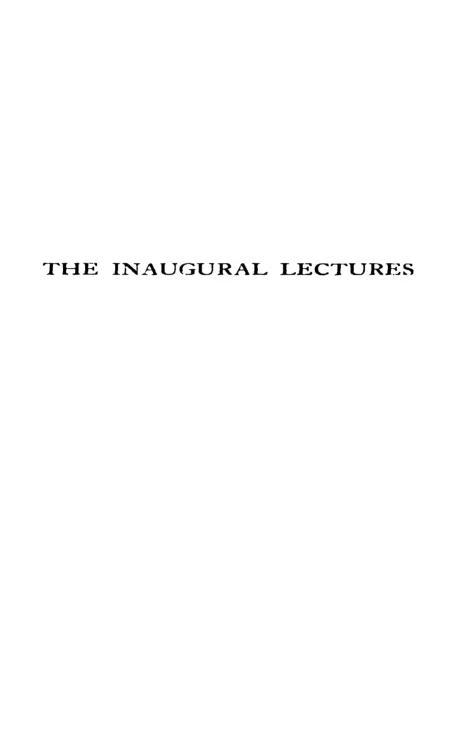
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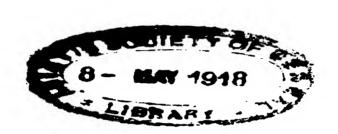
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Rafael Altauria

### THE PROBLEM OF THE PHILOSOPHY OF HISTORY

## THE THEORY OF CIVILIZATION THE METHODS OF EXTENDING CIVILIZATION AMONG THE NATIONS<sup>1</sup>

#### First Lecture

#### THE PROBLEM OF THE PHILOSOPHY OF HISTORY

IN all the dominions of science, and especially in those re-L lating to the human subject and dealing with first principles, there are questions—I will not say of eternal standing and controversy (because to say "eternal" is to anticipate an issue of which, in view of the future's uncertainty, we are not authorized to speak), but indeterminate questions which from the beginning of the known history of scientific thought down to the present have been treated by the different schools of thinkers very differently. Seen thus through the medley of systems and opinions, these questions give the impression of something which is insoluble and by all our processes of knowledge unattainable, something in regard to which it is useless to devote time and energy, since the solution arrived at will not give universal satisfaction, a sign that it is not truly scientific, - and in this, indeed, is explained the position of those individuals (by no means few in number) who, intent on the scientific requirements of precision

<sup>&</sup>lt;sup>1</sup> Three lectures presented at the inauguration of the Rice Institute, by Professor Rafael Altamira, late Professor of the History of Spanish Law in the University of Oviedo, Director of Elementary Education in the Spanish Ministry of Public Instruction.

and exactitude, exclude such problems from the sphere of science and disdain and abandon their investigation.

In spite of such exclusion, the thinking classes of humanity (which are not limited to the professional scientists) persist in stating these problems and in asking questions relating to them or derived from them. These inquiries demonstrate that the problems themselves are a part of an inherent and natural curiosity within us, and are a necessity inseparable from the human spirit—at least as it has been constituted up to the present. We can say no more than this, for it should not be forgotten that all our observations regarding our own nature are based on what has emanated from a period of human life which may seem long, but which is short when considered in comparison with what that life may be prolonged to in the future. Our hypothesis, given the present nature of our intelligence, can never, however fecund the imagination, exceed the finite number of occurrences which embraces the known reality. As this limitation to actual experience is common to all the orders of our reason, it is clear that we are obliged always to work upon the basis of our mind as it now is and has for some time presumably to continue.

The curiosity which belongs to our minds as to-day constituted, then, inevitably causes at one time or another the same questions to be raised, and impels even the professional scientists to formulate them, notwithstanding the futility of previous efforts. But if all this is certain, it is not less so that some of them, although lacking solutions unanimously accepted, begin to show, amid the medley of opinions in regard to them, a certain general orientation or certain points of common acquiescence which signify their advance toward a more scientific basis, a surer and more satisfactory ground than that hitherto occupied. It is this which is occur-

ring with the question of the Philosophy of History, and to signalize and determine in regard to this question that general orientation and those points of acquiescence seems to me a service that would be of indisputable utility.

It will be useful, in the first place, as a basis of future investigation, as a basis of real progress on the road to a solution,—on a road which is, properly speaking, scientific, since progress in the knowledge of things depends on the clarity and security of what has already been established. But it will also be useful for another reason, a consideration of a social character which professionals are in the habit of overlooking. I refer to the influence exerted by their doctrines on the masses among whom these doctrines become translated into lines of opinion and of conduct. For a scientist that which alone is of importance and alone is worthy of attention is the truth or the error of a theory, and from this standpoint he may, and does, neglect all theories which appear to him untrue, discarding them from that which merits his attention. Thus, in the Philosophy of History a providentialist will reject and disqualify the doctrines of a rationalist or those of a positivist, and vice versa, but neither one nor the other will be able to prevent these conflicting doctrines from influencing large numbers of people and guiding them in not a few questions of their lives. With equal reason the contrary positions of those who admit a Philosophy of History and those who deny such a thing collide with and annul one another, but both are powerless before the fact that many people will accept one position or the other; and as, in the long run, that which matters is that which influences the masses, the conflicting theories which claim the solution of these indecisive questions come to possess for the sociologist, for the practical man, and for the historian himself a value which is at best only equally pro-

portioned to the scope of their diffusion and to the force of the conviction they produce. All, then, which may tend to eliminate divergences, discover points of contact, or, better expressing it, to intensify in the public mind the consciousness of common affirmations in what has arisen from distinct starting-points and systems,—affirmations which have not, perhaps, been realized by the majority,—is preparing the way for an ever greater homogeneity in thought and action.

Now, of late years, in the sphere of the Philosophy of History, owing to the discussions which the actual statement and formulation of the question has produced, there has been a fairly concrete determination of factors and a clarification of ideas relating to the subject. Neither movement has descended to the great sphere of those who are nonspecialist but cultured sufficiently to produce in it a favorable change of the same character; but this same lack of correspondence between the scientific position up to date and the sediment of antiquated and already scientifically rectified ideas which have passed down into the masses as accepted knowledge renders all the more necessary that labor of diffusion whose first effect has to be the clear determining and sizing up of fundamental opinions and authorities. The necessity is all the greater in so far as one may consider included in the masses the large number of persons whom, at first sight, we should qualify as cultured, persons who have obtained university degrees and who undoubtedly possess wide information and clear intelligence. Thus, I have heard my book "The History of Spain and the Spanish Civilization" described as a work of historical philosophy, although it is simple and unmistakable narrative, simply because it contains, with the usual chapters on political history, others on what has been called Kulturgeschichte, or internal history.

This very common error signifies not just a vagueness in the conception of the Philosophy of History (vagueness there is as well, and in due course we shall examine it), but an absolute disorientation in which it is impossible to form any argument whatever or even make one's self intelligible to those laboring in the fallacy, for the simple reason that while employing the same name, they imply something wholly different. Let us begin, then, by rectifying this error, that it may once and for all be deleted from the public mind. Every history-book is pure narrative if it limits itself to relating facts. Although it may embrace in entirety every sphere in the whole life of a state, including the history of its thought in the various orders of the sciences and in those treating of human questions, it is not a book of Philosophy of History. It may be the work of an historian who does not believe that science possible or regards it as dissevered from his professional mission: his ideas in this respect will not in the least have been invalidated.

Equally common with this error, and perhaps more so, there is another one more difficult of eradication and of graver consequences for the reason that it comes near, apparently, to the actual field of philosophy itself instead of being plainly and at a glance outside of it. This is the error in which, in the name of philosophy, is inferred every generalization regarding historical facts. To those laboring in this error everything of a general character that may be gleaned from an individual history of concrete facts—the character of an institution in a given epoch, the dominant and central current in a series of events, the distinctive feature of the history of a state, the trajectory and orientation of an order of ideas—is Philosophy of History. But as, apart from such works of erudition as are purely concrete and monographic, every historian must generalize without de-

parting from his own material of facts, it may be deduced, according to this criterion, that there will scarcely be a history-book which is not philosophical. A book which summarizes in a great compendium, a great "synthesis," as it is commonly but erroneously expressed, the facts of a period, of an age, or of a state, and popular lectures which epitomize the great results of detailed investigation, would be Philosophy of History when, in general, they are rigorously limited to the field of what is narrative—that is to say, purely historical. The celebrated lectures, for example, on the "History of Civilization in Europe," by Guizot, do not in any way possess the philosophical character, although their eloquent expression and the reflections and opinions often to be found in them which do not cover a ground that is, properly speaking, historical, added, moreover, to the lax and careless criticism of contemporaries to whom all this justly came as something new, led to the lectures being designated by many as philosophical. Generally speaking, one may affirm, on the contrary, that every generalization about facts, while it remains a generalization, and however abstract be its character, is not philosophical. What always result from it are facts, very general, very comprehensive, but, in the end and in the long run, facts. Laws themselves, or the course they follow in a more or less extended period, are likewise facts, although of an abstract character. They express what is the line and orientation of individual happenings; they do not explain them philosophically or, to be more precise, metaphysically.

I have now just enunciated what, in my opinion, is a basal quality in the Philosophy of History; but, to avoid confusion, it will be necessary to define it. Every explanation of facts is not a philosophic explanation. Naturally it is not so when it treats of causes which are directly or indirectly his-

torical—that is to say, determines temporal origins and precedents, the factors behind an appearance and effect, the necessity of a phenomenon in a given moment. No one will describe as philosophical the explanation of the collapse of the Invincible Armada, an explanation which is entirely confined to the most concrete facts and as historical as any in the world; nevertheless many other analogous explanations of greater or less significance than the above are still described with manifest equivocation as philosophical. The explanation of the Hellenic genius and culture as a consequence of oriental origins, of such and such influences derived from the geographical situation of that people, is equally not of a philosophic character. All such explanation moves entirely amid temporal causes and on a ground which is purely historical, however vast and general its embrace of the concrete facts and data. For the explanation to assume a philosophic character it must treat not of temporal but of permanent causes and must inclose facts in a metaphysical impulsion and causality outside of the field of history. It is not without purpose that the science under consideration is called Philosophy of History (of human history, it is clear), which means that it is a philosophic science and ought to be treated according to its nature and not on historical lines. The antagonism between the Philosophy of History and the History of Philosophy, which has been shown and explained by certain schools of thinkers, defines thoroughly the distinctive character of each of these sciences, notwithstanding that the terms employed in them are identical: the different relative position of both terms in each of the two cases signalizes plainly the opposition in question.

It is necessary, then, to abandon all false conceptions of the science concerned with these reflections in order to place ourselves in the actual field with which it corresponds. Once

settled there, the discussion of the problems belonging to this science becomes disentangled because we know now the value of the words employed and are no longer in the plight of discussing indefinitely and without understanding one another two things which have nothing else in common but the name we give them, a name which is applicable only to one.

With this point settled, it is now possible to propound the first question of the Philosophy of History, which is precisely that now most under discussion in our times—to wit, the possibility of the science in question. In any case this would have to be the first question to be discussed and to be solved; for, what would be the use of fantastically pursuing the principles of a science devoid of all reality—that is to say, impossible? We should be involved in a labor that is not only useless but pernicious, through the false ideas that would be disseminated.

Before examining this question and expressing in regard to it, if necessary, a personal opinion, it is important to separate it from another which is often confounded with it, the one prejudging the other with its own solution. It is one thing to question the possibility of a Philosophy of History, be what it may the field of science in which it is established, and it is another thing to inquire if historians as such are capable of creating it, or even merely if its existence concerns or ought to concern them. The distinction between these two questions is all the more necessary in so far as many treatises have dealt only with the second of the two, and presumed, in the solution of it, to have solved the first and fundamental question. In reality, the second question, as it is commonly propounded, is beside the point. If the Philosophy of History, given that it is possible, is a philosophic and not an historical science, it clearly follows that it devolves not on the historian but on the philosopher to for-

mulate and clarify it. It is legitimate and comprehensible on the part of the historian to declare himself as such incompetent; to refuse to employ his energies in the investigation of an aspect of human history which does not concern him; and to demand the requisite time and energy for what does. For this reason it is a strong position which has been adopted by those who, under the title of historians, refuse to busy themselves with that problem, and even regard it as pernicious that it should be mixed with those peculiar to history; basing their opinion either on the supposition that the character of historical knowledge fundamentally prohibits a philosophical explanation, or on the supposition that the actual position of historical science does not as yet authorize it.1 Observe, however, that the majority of those of this opinion admit that outside the sphere of history, in the field of other science, the problem is legitimate and is one that may be formulated and considered. If he wishes to abide in his own sphere, it is not the professional historian who will study it, but of the results of the investigations which others have accomplished he will be able to take advantage.

It is clear of course that this does not exclude a historian from studying the Philosophy of History, just as he may be interested in astronomy or any other science, nor can it be denied that in the fact of his being a historian his preparation in the study of the problem is the more adequate for a deep penetration into a given one of its aspects.<sup>2</sup> The natu-

<sup>&</sup>lt;sup>1</sup> An exposition of the situation of that question to date is to be found in my book "Questions of Modern History" (Madrid, 1904), Introduction and Chapter III.

<sup>&</sup>lt;sup>2</sup> One of the scientific weaknesses in many authorities on the Philosophy of History who would be styled classical—and even of not a few modern philosophers—consists in their not being or not having been sufficiently historians; that they do not see the problem in its essential historical perspective; and that they have failed to fulfil that exigency which Dilthey ("Emlestung ni de Gentenvissenschaften") formulated, saying: "The thinker who takes as his object the historical world, ought to be intimately acquainted with the immediate material of history and should be entirely the master of his medium."

ral supposition, in fact, is that it will be the historian who will be interested in that problem because the constant vision of the historical material will continually produce in him a desire for an explanation transcending the mere facts themselves; and, in any case, as a man of intelligence he will be brought up against the problem, though he may not embark on the solution of it. Nor, moreover, in the preceding affirmations relative to the independence of position between the scientific sphere and the philosophical is there any denial of the intimate bond which unites them, and in virtue of which not only does the philosopher require, as was said, to be master of historical matter, but the historian will find in philosophy a force which, although it is not his business to create it, will help him in the handling of his data.

Now, it is quite another thing to state the objection in regard to a Philosophy of History to the philosophers themselves, basing one's position on the present status of our knowledge of the history of mankind. Such an objectiondistinct from that embodied in this argument against the possibility merely of the "historians" creating a Philosophy of History-may be based on an affirmation of that strict interdependence in which, we affirm, both terms are to be found. Kohlen has expressed it in a decisive manner with reference to the Philosophy of Law: "Without a universal history of law a true juridic philosophy is as impossible as is a philosophy of humanity without a similar history of mankind and a philosophy of language without linguistics." This, then, denies for all men the possibility of a Philosophy of History, although only so long as it fails to fulfil that fundamental requisite of previous acquaintance with the facts in all the amplitude necessary that it may be possible to philosophize about them; and, to my mind, this is the strongest objection

that can be opposed to the present possibility of a Philosophy of History.

As a matter of fact, it is only by the force of habit and the suggestion exerted by those books (that is to say, the doctrines elaborated in them and the systems formulated, which give the false appearance of something perfect and conclusive) that we say and even believe that we are acquainted with the History of Humanity. Certain it is that considerable in range as is our historical information, and although that information has augmented so vastly in one century in regard to the above branch of history in particular, and become perfected in certitude and thoroughness, there still remains much for us to learn, still many points of obscurity and vagueness, many facts and theories in suspense; and that on a basis so imperfect any philosophic structure will be flimsy, collapsing at the least pressure. For, if we do not possess our facts securely and in entirety, how can we build upon them anything stable or secure? To the immense force embodied in this argument is due the most useful and fruitful of the results which modern criticism has produced in the discussion of the problem now before us. By dint of this argument has been demonstrated the inconsistency between systems relating to the Philosophy of History constructed a priori by writers who, in not a few cases, are ranked among the great. This failure was merited, as merited is the smile with which, to-day, we regard, for example, that infantile endeavor to inwrap the history of mankind in periods or ages of development which limited the future and closed up the eternity of life. In drawing up a clear table of all in these systems which was warrantable and final, the criticism of the professional historians has constituted a service to science of immense value, clearing the road so that it should be unobstructed by pseudo-scientific-though some of them

colossal—structures which would render it difficult to make the labor of the future step by step and in certainty. It is true, however, that it has produced also a pernicious skepticism in many people who, with the precipitancy so natural and difficult to check in human nature when a definite conclusion is arrived at and a judgment passed, have confused the breakdown of the Philosophy of History as interpreted by certain authors with the total collapse of the whole science. To convince the public of the error of assuming the second issue as a consequence of the first is in fact one of the duties of men of science in the social aspect of their labor.

Let us return now to the starting-point of these considerations. To deny the present possibility of a Philosophy of History because we do not as yet know enough of the history of mankind is not to deny its possibility absolutely and forever; agreed, however, on this point, the affirmation which has led us to it reappears and confronts us. We are still at grips with the fundamental problem. In short, if it is proved that it is definitely impossible for us to arrive at that initial historical knowledge which has to be the basis of a scientific philosophy regarding it, or if it is true, as many believe, that historical knowledge is incapable of scientific qualities and even of precision and of certitude, then to philosophize about it will be eternally impossible. The problem, therefore, is transferred to another ground and obliges us to discuss previously all those questions alluded to, and which in our days cover, as is known, an extensive literature. From the discussion as to the degree of generalization which is possible in regard to facts about humanity (a discussion maintained on the extreme wing by Xenopol, who denied that there could be any generalization), to the transference of history wholly and solely into the field of science, the series of minor problems presented in the different opinions upheld by the

specialists to-day require to be tackled and cleared up in order that we may either be free of all incubus in the affirmation of a Philosophy of History or else abandon the dream of its possibility. It would be long and wearisome here and now to enter on this task which I have already elsewhere taccomplished.1 I will refer only to the conclusion I there arrived at, and take my stand upon it under the plea of a personal opinion. The doctrine may be thus epitomized: In the present situation of our knowledge relating to these questions, and of the opinion of men of science respecting them, there is a decided weakness to be observed in the arguments employed to deny the scientific character (the possibility of such) in history, either because the general conception of science renders it possible to-day to state the problem with a different meaning to that of Aristotle, or because it is not so certain as is commonly believed that history is confined purely to the observation of individual facts, forming itself into a narrative without any generalization (of a more or less abstract character, that is, as all generalizations are), in which each fact conserves its unique and differential characteristic and only on the strength of it is mentioned. For myself, personally, however, the crux of the problem is not in whether historical knowledge conforms or not to the Aristotelian definition of science, and whether it is susceptible to abstractions of greater or less amplitude, but in whether it can attain those qualities of truth, clearness and certainty which distinguish scientific from vulgar knowledge. If to the scheme and elaboration of true, evident and certain knowledge which has as its objective the facts about humanity in time and space (and derives from that objective its own internal coherence) is begrudged the denomination

<sup>&</sup>lt;sup>1</sup> In the book mentioned previously, "Questions of Modern History," Chapter III, No. 3.

"scientific," the question at issue is solely the question of a name. What matters is that our knowledge of man and of the manifestations of society in past ages shall arrive, by means of a rigorous employment of the critical methods of investigation, at being as certain as our knowledge about Nature and the facts concerning her, though neither one nor the other, either to the observer or to the experimentalist, delivers the totality of its abundant and (from day to day at least) mysterious contents.

The objection, then, which, if valid, would make it impossible forever, through lack of a foundation, to philosophize about the history of mankind, possesses no scientific authority for opposing an insuperable barrier to this philosophic aspiration; but it does serve most effectively to moderate impatience and to check precipitancy in the task of solving the main problem, showing the connection between this problem and many questions of importance still under discussion, revealing also its complexity and suggesting that even on the strong basis of a personal conviction rooted in the feeling that a right solution is arrived at, we are to preserve the judicious cautiousness which is characteristic of the truly scientific mind, and which safeguards against the possibility of error and makes us respectful toward contrary opinions. All that may avoid that suspicious simplification of a problem in easy terms—only subjectively arrived at while the problem itself is divested of many elements inherent in its complexity and which we fancifully qualify as incidental—and that provides us with the maximum quantity of proofs in support of our opinions by probing them and developing them with every kind of verification and analysis, will become a guarantee in support of our conclusion and of the doctrinal fabric we erect on it. It is for this reason that I have been explaining and examining the principal

objections to a Philosophy of History and the errors and confusions of thought in regard to it which draw into a distinct field—and one conducive to confusions—the interpretation of the name.

Over and above all this cautiousness and reservation, howlever, stands out one fact which even the most decided antagonist of a Philosophy of History has to recognize, not only as a reality but as a thing of importance and significance. This fact is the persistence in the human mind—in every man who thinks at all about the world and about life—of those fundamental interrogatories in regard to the actual problem of the philosophy in question.

It is true that, in view of the potential immensity of future history and the paucity of that at our disposal (as was observed not many months ago by your compatriot Professor Sloane<sup>1</sup>), the persistence in humanity or in great masses of it, of a given idea or preoccupation does not in itself always signify that the notion or ideal in question is consubstantial with our nature, since it may well be a survival, a vibration from primitive stages of thought not vet modified. and to which, in fact (in that relative value of time), we are chronologically very near. For this reason it is not a plausible argument in support of the necessity of an idea or a belief that for many centuries down to the present a more or less considerable number of people have supported it and held it to be something fundamental. The future may wholly disillusion us. But if we ascertain that a definite idea or an ideal exists throughout mankind and is the stronger in a man according to his degree of culture-in an inverse relation to other spiritual phenomena, which exist principally on a sentimental basis and are rooted above all in the uncul-

<sup>&</sup>lt;sup>1</sup> "The Vision and Substance of History," address delivered at Buffalo, New York, December 27, 1911. Published in "The American Historical Review," January, 1912.

tured masses or where culture is incipient—we have a very powerful argument in favor of its essential necessity for us. It is this which occurs with the problem of the Philosophy of History. Be it with a clear understanding of their meaning, their classification in the Encyclopedia of the Sciences, or be it without ever suspecting the relationship they bear to that, great masses of people are to-day, as in the first stages of civilization, formulating questions which correspond to the fundamental problems of our science; and each individual unit in those masses answers these questions from the point of view of a religion, a system of philosophy, or simply that of a common sphere of culture which finds reflection in himself or in which he has been educated.

It is true that many people pass through life without experiencing a moment in which those questions flash before their consciousness, because the material occupations of the daily struggle for existence leave no room for attention to other questions. It is equally true that among those who have broken free from this material incarceration, and even among those who move by custom in an intellectual circle, these questions pass often enough like swiftly flying sparks rapidly extinguished, or do not acquire that standard of importance which is given to a question as the result of deep preoccupation. For a long time, owing to doctrinal considerations arising from the predominance of certain philosophic systems (philosophic although some of them discountenance philosophy), there has existed an indifference and an apathy on the part of many people in regard to those questions. Although there has been a reaction in this respect, it is a fact that the number is still large of those who fail to appreciate their urgency—a fact, however, which depends on general causes traceable to the conditions of our modern life. The feverish activity, the superficiality and

show in which the majority exist, cause our moments of privacy and meditation, of communion of the spirit with itself and of self-examination in regard to life, to become more difficult and rare. Distracted by the outside spectacle, we lose the habit of self-examination and become deaf to the promptings of the soul, and often enough we pass through life in ignorance of the exalted curiosity within us. At times, in moments of brief solitude and thought, these questions suddenly appear to us, but the intellectual effort required in pursuing them, and the time they would demand, make us shy and half afraid of them, with the result that we suppress them and continue as though in ignorance of their presence, until, in another moment of doubt, anguish, discouragement or pessimism in which the mind has nothing to fall back upon or other resources but its own, they reappear before us, without, however, our ever possessing the hope of finding time or opportunity for their consideration and their answer.

Such a state of inattention to the problem is not enough, then, to deny that it exists; this state of mind, on the contrary, continually affirms the problem as a presence. Whenever we wish to hear its voice, it is with the utmost clearness that the voice echoes, and this in itself will be enough to guide us in the circumstances.

The historian derives a knowledge, or what he believes to be a knowledge, of the principal facts concerning the history of mankind; he traces the rise and fall of the great empires; he describes in its separate stages the process of civilization, its oscillating and, at times, contradictory movement, the advantage to one state of the labor of another which it resumes and carries on, the things which have been accomplished in modern times, and the trajectory and law of development of institutions and aspirations regarded as fundamental in importance; and then, over and above all

this remain those same great, disquieting questions which embody the whole program of the Philosophy of History: Where and toward what is mankind traveling? Is there a goal of which, at present, it is ignorant, but toward which is moving the central current of its history? Is it being impelled toward that end by something beyond and transcendental to it? What is its significance and value in the whole, in the general process of the universe? Is it the creature of chance, or has it an orientation and direction? And if it has, can we deduce that movement through such of the facts about humanity as we have knowledge of? Does there exist in the actual conditions of its life some other foundation than the corner-stone of history? And, following from all this, what state is it which marks or is to mark the triumph of that history, the culminating situation most nearly approaching and conforming to the purpose of the universe? Is it possible to define and predict for the future some main path for man, or is the Philosophy of History ever restricted to the limits of the present? Of the utmost clarity for every one engaged in the investigation of those questions which history, deeply contemplated, raises, must be the real and logical hierarchy which exists between them. Not all are on the same level, not all are equally far-reaching, and if I may use a phrase which is unscientific and inexact but which well reflects what would be thought by an uneducated person (that is to say, by the majority of people), they are not all equally philosophical, but some more so and others less. This question of a hierarchy and of a relative importance possesses a greater significance than would at first sight be imagined, because if we regard it as a proper and wellfounded one, it at once brings us to the point as to whether or not the professionals, the writers who have propounded scientifically the problem of the Philosophy of History, have

grasped in fact the whole and entire problem, or whether they have limited themselves merely to the study of some one or several of the questions it embodies, and perhaps to some of them which, compared with those embracing the main object of the science, would be called secondary; and more than this, we are even led to the question whether it may not be the case that, while preoccupied with what they regarded as the real problem, they were not confining themselves, through an error of perspective, to aspects of history quite general and comprehensive in themselves, but above which they have never risen, never attaining a transcendental vision in the true philosophic field to which they were aspiring. I am not far from thinking that it has been thus in the majority of cases, at least with those great systems which have attempted a fundamental revolution in the Philosophy of History. I do not allude by this to the observation, continually reiterated by the critics and some of the most recent exponents in the matter, that the majority of these systems, if not all of them, losing sight of the complex nature of the problem, have given an ingenuous explanation of the History of Mankind to which is owing their failure or insufficiency. I refer to that which, apart from the degree of comprehensiveness in the problem they embrace, it is impossible to ask in regard to whether those systems embark on the true problem of the Philosophy of History, on which problem depends a series of others to be called consequences, or whether, on the contrary, it is not from one of these selfsame "consequences" or minor problems that they have arisen, the minor being mistaken for the greater problem in whose solution rests that of all the others. That this equivocation is clear in Montesquieu, in Rousseau, in Voltaire ever so much more so, and in other authors of an analogous scientific standing in relation to the Philosophy of History,-

# THE RICE INSTITUTE that they failed to get abreast of the question and seriously

tackle its solution, -no one will deny. But even with the great masters of the school, the same doubt is legitimate. and the decision may be actually against them. Will it be said that Herder, notwithstanding the discrimination with which he subordinated to the more general standpoint those secondary questions which were almost the only preoccupation of his predecessors in the century, actually raises in his problem of the factors and issues of the History of Mankind the real and basic question of the Philosophy of History? Was it approached by Kant in his own explanation of human progress—that is, the solution which is offered to the conflict between individual liberty and the general welfare-in the State? After this is there no room, even when the Kantian solution is accepted, for questions regarding the metaphysical problem of the plan of history, questions above and beyond the antagonism of individual liberties among themsclves—that is to say, questions of a more general and comprehensive character, by the side of which the above is subordinate and over concrete? And in spite of the incontestable grandeur of the conception of Hegel, are we not left, perhaps, with the impression that in reality it lowers and depreciates the problem and denies it what should be a higher point of view, in which the development of the moral conscience, of freedom, and of the functions of the State becomes subordinated? The observation of history and its mode of development, and the interpretation of it exclusively from the viewpoint of a standard of ethics, notwithstanding a metaphysical quality, is yet something which too nearly approaches a broad but, in certain respects, very concrete vision of historical development which allows a vaster and remoter problem to float above it. Yet clearer is this in Comte and his disciples, and in Marx and his, the character of whose

philosophies is purely an analysis of the factors behind the phenomena of human history, factors which only explain these phenomena in a secondary manner. Even in the acutest and most comprehensive of these systems the mind is not left satisfied as when one has set hands on the real solution to a problem; it feels (and I say it without wishing to depreciate the value of those investigations and the clear light they have thrown on the movements of mankind) that there is something still wanting, something greater which remains we answered, and which, if answered, would respond more fully to aspirations, properly speaking, philosophical.

I regard as scientifically legitimate this dissatisfaction of the mind even with the profoundest and minutest analysis of human progress. I am also of opinion that the problem of the Philosophy of Human History ought not to be wholly limited to the two questions formulated by Herder,—on the value of that history and the conditions in regard to its development,—since, although, in the consideration of the latter question, there may have been a glimpse of the ultimate and basic problem, the systems soon settle down into a mere analysis of conditions and a generalization about the facts of history which is secondary to the main problem. . . . That which cannot be described as an explanation of human facts by other facts of a like nature (they may be as general and fundamental as you like, but that does not affect their nature) cannot be described as history; and thus, what has by some schools of thinkers been called the "anatomy" and the "physiology" (or the "psychology," from another standpoint) of human action, is not Philosophy of History.1

<sup>&</sup>lt;sup>1</sup> It is in not passing from that narrow standpoint that those claiming to have construed doctrines and systems of a Philosophy of History have been able to introduce and discuss the question of the anticipation of future history. In the concrete conception of this question it has been affirmed: "Humanity, in the future, will act in such and such a way, and attain such and such standards of civilization and development." The question is neither

And now, in conclusion, there remains this culminating question: Does there exist any actual reality and basis corresponding to that aspiration of ours towards a transcendental explanation of what is a greater problem than all those scientifically formulated until now in the so-called Philosophy of History, or is it a pure whim and caprice of the spirit that is never to be satisfied? To this question I do not believe we can provide at present a scientific answer; but I should point out that neither our present nor permanent impotence regarding the solution of what is an idealistic problem can banish that problem from the mind, which continues to formulate it as an aspiration that is ineradicable and to which it is forever hopeful of finding a solution.

And lastly we should remember, in order that the logical statement of the problem may leave no loophole of uncertainty, that the questions in which we embody the main substance of the Philosophy of History do not, in their formulation, prejudge an affirmative answer, nor is such an answer an ineludible necessity for their existence. Although our answer to all these questions were in the negative, they would continue to be problems present in our minds—so long, that is, as the answer is not indisputably a scientific one; and even if it were, it would, none the less, be legitimate material for a Philosophy of History as real and settled as if it answered in the affirmative those same interrogations which for the majority of men correspond to a desire, latent but ineradicable, to see explained in an ordered, rational and scientific method, according to the general plan of the whole universe, the Life of Man.

permissible nor can it be included in the field of the Philosophy of History. Thus, Meyer is right (in his "History of Antiquity") when he judges that such predictions are impossible, since in that which is generally referred to, the individual element predominates, escaping all prognostication; and affirms, always from that standpoint, that history only allows of comprobation, and not of any fixing of the future.

For this reason the essential necessity of a Philosophy of History depends neither on a special solution of its problems nor on the actual possibility of a solution being afforded It arises principally from the presence of the problem in our minds and from the corroborated fact that the highest expression of what, as concerns our history, is called progress, consists in the awakening of humanity to the idealistic quality behind its actions, of the things it is accustomed to perform in ignorance of their value and significance; and in the guidance of his life by man, ever increasingly, through the medium of that consciousness and with an ever clearer vision of the "why and wherefore" of things. To assist, by due attention to this problem, in promoting the study of it, and, some day (whenever that may be), the solution of it. is more reasonable and human than to bang the door upon it with an a priori negative against its possibility, or than to belittle and discard it.

#### Second Lecture

#### THE THEORY OF CIVILIZATION

HAVING tackled the main problem of the Philosophy of History, we should now ascertain what practical issues have arisen from the study of those would-be philosophic problems undertaken by the specialists, and what, in this connection, deserves further attention.

We saw, it will be remembered, that all these so-called systems of a Philosophy of History, all the interpretations of this science to which the above name has been arrogated, have been limited, in reality, to the scope of history, transcending this field only in brief moments of the investigation or in theological conceptions which we are not concerned with. But, although none of the systems in question may have afforded a real basis for the science they proposed, they have served, on the other hand, in no small measure as a means of deepening our conception of history itself and of widening our vision of it, while revealing all that is embraced in what is called historical material, determining the more important and decisive factors which (some of them in distinct periods or epochs, others at all times) are at play in the action of mankind. In spite of the exaggerations which in most of these systems are conspicuous, and in some notorious, it is an undeniable fact, once having discarded the false, unilateral pretension common to nearly all of them and transferred them to their own sphere of history, in which such of their investigations as are of value may be developed, that to the science of historiography they have rendered immense services, at once widening its horizon and

revealing the complexity of human labor which each one of them has studied in an aspect not infrequently as real as it was hitherto unrealized. We can appreciate the positive fruits of these investigations on observing the great difference between our method to-day of conceiving and writing history and that which prevailed some centuries ago; and even, it may be said, between the historians of the seventeenth century and those of the nineteenth. The methodologists, advancing theoretically ahead of the historiographers (the latter exerting themselves to fulfil the exigencies of the former and turning to account the suggestions obtained from the "philosophers" of history, or at times actually raising systems of their own by way of experiment and illustration-e.q., Taine), have paved the way for our modern conception, ever becoming wider and profounder, of human history. And this labor, which has enabled us to elucidate man's past with ever increasing vividness and with a keener penetration of its meaning, is a solid basis on which we may hope to find an answer to several of the questions which are suggested to us in the contemplation of that past. Starting, then, from such a basis, with all due prudence and a rigorous employment of those critical methods of investigation which are essential if one is to avoid wandering into fantasies (fantasies, though, not necessarily philosophic in pretension), we shall be able often enough to arrive at conclusions of real scientific value, while other hypotheses will serve as a scaffolding for subsequent investigation. And as this field embraces what is positive and certain, and all that we are interested in, deriving from a great portion of the moral and political applications of historical knowledge, it is our business to approach and examine it rather than sacrifice it to the lure of a higher and remoter explanation, which, even if possible, in no way excludes the above study nor renders it

useless. Within purely terrestrial aims and limitations of which we ourselves are cognizant—that is to say, human aims, of human interest—while equally, also, in our legitimate anxiety to understand more fully the way in which, from one moment to another, a community conceives its task and function in the world and tackles and solves the problems which are its own, what is of immediate consequence is the investigation of all those historical elements which may afford us the knowledge we require and establish our conclusions; for, in the long run, that in the study of history which descends among the crowd and interests it, is the critical estimation in which, as a result of historiography, each historical epoch and entity is held, and the estimation of the general movement of mankind in regard to the question of social development or "progress" as we define it, though with error, since a meaning is, in this connection, attributed to the word which implies actual betterment, improvement. Clearly such a point of view will be a very subjective and uncertain one, since it entails that each epoch judges past ages according to its own social and moral criterion, and this criterion is not eternally the same; but there is no other standpoint open to us, nor can there ever be another, with the result that our only course is to reconcile ourselves to the manner and circumstances in which these questions must be considered and in which they have attracted us. If we are bent on verifying history ever more widely and more precisely, it is not for the simple esthetic pleasure of knowing things, of reading or hearing narratives as children read and are told stories, but for the object of explaining to ourselves why men have acted in such and such a manner, of apportioning their responsibility and forming our opinions about their conduct. Whether or not we are conscious of this object, it is this which is the initial force behind our curiosity regard-

ing history, our researches either aiming at an explanation and justification of that particular national or political aggregate to which we may belong, or a criticism of the others foreign to it; and the judgments and appreciations which are left over in our minds from these researches are factors which determine the conduct we pursue in our own private sphere of action and in our relationship with other minds. From a broader and more disinterested standpoint, above mere national distinctions, we are desirous, also, of learning the road humanity is taking in what we suppose to be a definite trajectory toward a more perfect state; what actual advances have so far been achieved; and what are the surest means, such as the experience of history has confirmed, for guaranteeing and augmenting this improvement. And here, in this higher sphere, that which in the other province of concrete criticisms and estimations regarding given communities amounts to a conflict between national influences and interests, is now a conflict of general theories about life, of distinct methods and systems of organization, a conflict for priority between such and such factors in the life of man which, on the supposed justification of history, claim, in regard to that life, the right to be made the controller of it.

And this practical issue which men deduce from historical investigation adds a new value to it over and above what it represents in the sphere of pure speculation, and is one of the motives on which its study may be justified against those combating it, in the name of a common utilitarianism which is eternally in doubt but forever reappearing.

The investigation which is proposed here embraces the two points of view referred to, responding to the suggestion of the theme taken by Dr. Edgar Odell Lovett, President of the Rice Institute, for the present inaugural celebrations. We shall discuss first of all, as a general question, the prob-

lem of the history of mankind, following this with a special investigation into the Spanish backgrounds of American history.

The general problem of human history, as we shall interpret it, is the problem of "civilization," or, as it is also expressed, of "progress." Is the process of human civilization something continual and indefinite? Is civilization a thing which is permanent, transmissible, and which grows in successive stages? What is the actual stage of civilization we of this era have arrived at, taking the criterion of humanity in general, or of those we regard as the most highly developed groups of it? These are the first questions which the problem raises. By what means is civilization produced? What, in consequence, is the procedure to be adopted in order to insure and further it? These are the questions which immediately follow.

Now, as regards both series of questions the answer is naturally to be sought in history, since civilization is an historical fact. This historical fact, however, has been translated in our minds into a conception, or, to define better this appellation of "civilized" which we apply to certain ways and customs, certain principles of life and conduct adopted by men in their relations with one another (as distinct from other ways which we should not describe as civilized), into an idealistic criterion—a classification, that is, of the particular conception and ideal we stand for. It is thus that the first question to be considered and settled is the question of the exact categorical meaning we shall agree upon for the word in which are embodied all those different principles and customs—that is to say, the first question to be answered is: What is civilization? As regards the common meaning of the word, the vague acceptance accorded it, such as is usually

accorded words, and which admits of their use in conversation and even in books without the necessity, on each occasion, of explaining them, the answer to the above question would appear simple. Yet, nevertheless, as occurs in so many other cases when one endeavors to fix the meaning of a term, there is not merely a variation in the acceptance of this word among different people,—a variation, let it be noted, singularly conspicuous among professionals and specialists,—but often enough an utter contradiction.

A rapid inquiry into the principal interpretations of the word "civilization" will enable us to become master of this difficulty on which, sooner or later, one inevitably stumbles. . . . We will discard, at the outset, that acceptance of the word, common in modern historiography and prevalent as early as the eighteenth century, according to which the history of civilization (Kulturgeschichte) is held in contraposition to "political history," or which also makes the term "history of civilization" synonymous with the internal history of communities in opposition to what is external history, and comprehensive only of political facts, or rather that section of political facts most superficial and least permanent in character.1 Such a contradistinction is illogical because there is no justification for it in fact. The history of man has not evolved in this fashion, divided into two fundamentally separate branches of equal magnitude; and, moreover, there are no grounds for maintaining that many-or, in fact, any-of the facts of political history are extraneous and immaterial to the sphere of civilization. . . . Rid, however, of this illogical distinction, we are still faced and obstructed by the twofold difficulty that among the definitions of civilization offered under the title of scientific there

<sup>&</sup>lt;sup>1</sup> On this question also reference should be made to the book previously mentioned.

are scarcely two that coincide, and that the criterion by which a community judges its own and other civilizations is not common and the same for all—at least, that is, when it is a question of fixing the basic and essential characteristic of the civilized state.

As a first group of opinions may be mentioned those according to which "civilization" designates, inexclusively, the general situation in any country which has graduated through a certain phase of development in its intellectual and material life,—the requisite development in question being fixed as the invention of the use of iron, or the discovery of the art of writing, or any other analogous event prior to which man would be described as without culture, as "barbarous" or "savage." Dismissing, however, the doubts and uncertainties raised by this artificial limit, all that need be emphasized is the general standpoint shared by all the opinions in this category, and in virtue of which such expressions are used as "the civilization of Egypt" or "the civilization of Greece," terms embracing in totality the life of each, inclusive of all phases, good or bad, concomitant or not with true "civilization" in the modern acceptance of the word. Thus the historian who with this criterion and terminology describes the civilization of Greece will not exclude as a phase and feature of it either the slave system or the Greek religion, though the one appear to him unjust and the other false.

Diametrically opposed to this interpretation of the term is the category of opinions which, starting from a given dogmatic conception of civilization, partly ethical and in part material, excludes from the scope of the word anything which

<sup>&</sup>lt;sup>1</sup> It is unnecessary to formulate here a list of these definitions; any one can find them out from the well known writers on the subject,—to quote, for example, several tendencies: Guizot, Burke, Gumplowicz, Henry George, Kidd, Metchnikoff, Tolstoy, etc.

is not adjustable to this conception; so that out of what is called the civilization of a given people, or of man in general, would be abstracted as uncivilized and barbarous many phases-not invariably the same-which according to the other terminology would be left included. In this group may be included all those authors who hold to be essential, before a people or a person may be called civilized, either a certain development in regard to material conditions or a certain standard of attainment respecting moral relationship and conduct. It is clear, of course, that such a category of opinions becomes divided into an infinity of sub-groups, according as the writer judges that it is impossible to regard as compatible with the ideal of civilization—being typical only of the barbarian or savage—the lack (according to his view) of justice and morality in such and such orders of life, or the need of a given religious faith, or the absence of such and such ideals, or of certain conditions of culture, comfort, hygiene, etc. And this diversity of opinion becomes still further complicated when, as often happens, it is not merely that human manifestations are split up into two categories. but further than this, that one or more of them, in a certain grade of development, are fixed upon and requisitioned as an indispensable necessity without which no historical epoch or community can be said to have been civilized,—the claim being that without this given factor all other phases of life, material and spiritual, advanced as their development may be, are at a discount and insufficient in themselves to warrant for those who represent them the description of "civilized." Most of the interpretations in question refer to cardinal necessities in the moral, juridical or intellectual order; there being others, however, for whom the favored sphere is the material, more or less associated with a certain social and juridical organization.

Now, in the truly scientific mind all these distinct standpoints and suggestions do not at all awaken the alarm usually produced in those who, for lack of a personal opinion, depend upon the opinions of others, fluctuating and distracted amid the variety afforded them. The scientific mind, on the contrary, accepts as its definition nothing other than what is naturally suggested by a clear grasp of fact-to wit, that civilization is a status of human life constituted of several and fundamental and integral elements (embracing alike intellectual, moral, artistic, anthropological and social development, with the development of mind and character), all being necessary in that they respond to conditions and exigencies of human life that are also fundamental: further, that their respective development is not parallel and uniform, either in the general history of humanity or in the individual history of each realm, and that what is properly speaking the conception of civilization is a standard and ideal of life according to which we appreciate every historical actuality and gauge the status and situation of every phase and order of the life of nations. Our basis is the conception of a perfected existence, and it is in relation to this conception that we signalize grades of perfection and development, of approximation to the ideal.

Now, for ourselves, for the nations of America and their offspring in America this ideal is the ideal of European civilization in what it possesses that is common and inherent among all the nations which have collaborated in it through the ages. But now, above and beyond this there exist other communities which it cannot be denied have attained a high level of "progress" in other directions, and which cannot therefore be ostracized from civilization—communities whose standard and ideal differ consciously from ours in many fundamental aspects. Such is the case with China, for

example; and it is the truth, however much one hesitates to recognize it through attachment to our own special manner of regarding things, that in this fact is demonstrated beyond doubt the existence of different historical directions of civilization, or at least of two—namely, European and Asiatic. The greater or lesser probability of the former ultimately absorbing the latter, apart from the fact that it is a moot question whether the probability embraces an absolute absorption or only a partial substitution in given phases of activity, does not invalidate the fact that there have existed, and exist to-day, these two fundamentally distinct directions, and ought to create in us a certain caution in venturing on dogmatic assertions.

Returning again, then, to the question of the integral elements of which civilization is constituted, there are two things we must observe: first, that these elements respond to different manifestations or types of human development: and, secondly, that our researches are not limited to merely ascertaining the existence of such elements, or even their degree of development, but their adaptability, their qualifications for fulfilling the ideal of life aspired to. And, moreover, it should be noted that the importance of the elements in question as inherent properties of the human species is not enough to satisfy us, but that we insist emphatically on the question of their relative importance, their situation in a hierarchy and order of necessity, either in recognition of a factor which is higher than the will of man, or as an operation preparatory to uniting the best efforts of men in developing and perfecting in a self-conscious plan that element which, of all human manifestations, is most highly prized and estimated, and regarded perhaps as the basis of the rest. And it is of course undeniable that, from the distinct standpoints adopted in this problem result distinct social, political

and educational criteria and distinct views of history, past and present, and of the achievements of man in general or given countries in particular.

But observe, now, the difference between the problems and divergences here raised and those resulting from the admission or non-admission of such and such phenomena into the sphere of civilization. In the present case there is nothing of that contradiction and confusion resulting from mutually destructive exigencies of inclusion and exclusion, for in this case we admit the reality and necessity of all. What is proposed is to determine a scale of importance or a hierarchy between the factors of civilization. All we have to do is to compare, for example, the position of Ruskin, who maintained that Art is the most important element in life, with that of Marx, or the position of those who regard intellectual development as the main factor on which everything depends, with that of the advocates of the moral or religious factor in place of the intellectual.

This question of hierarchy is the cardinal question, indeed, which the problem of civilization raises, because it affords at the same time the key for our judgments of both the present and the past and the solution of the question as to what sort of rational influence and guidance is to be exerted by the will and intelligence of man in the directing of his life along a certain route, or the adoption of a given organization and régime. It will be said, without doubt, that this is not, properly speaking, an historical question, but rather a political one (in that it embraces the organization of life), or pedagogical, in the higher and wider acceptance of the word. There

<sup>&</sup>lt;sup>1</sup> Some schools, however, have considered it as, actually and strictly speaking, historical: for example, Marx, who does not affirm his theory of the predominance of the economic factor as a rational necessity which ought to be granted, but as a fact and a reality which has always existed, and which from this historical basis derives its real essentiality.

is no denying, however, that any one who approaches this question is obliged to seek in history many of his data for the solution of it, and that its solution is bound inevitably to react on his outlook upon history. At least no one can be indifferent to this question. The question as to whether it is the egoistical and utilitarian principle, in the material acceptance of these terms, which is to triumph in the world, or the ethical and altruistic; the question as to whether our present life embodies in itself its own aim and culmination, or has to be directed toward a posterior and ultra-terrestrial goal, in relation to which it is merely a transitory and preparatory phase to be regarded as such and nothing else; the question as to whether the world of the future has or ought to be "Greek" in character or "Carthaginian," interpreting these names, for the moment, in the idealistic signification which a tradition, whose reliability it would be out of place to discuss here, has given them across the centuries, is one that ought to be the concern, and in fact is the concern, of everybody, and in the solution of which that experience which is offered by history in the shape of the positive issues which characterize two main directions of civilization is a guide of considerable importance. For this reason, in the theoretical argument conducted between educationalists. politicians, theologians, and philosophers, full and comprehensive knowledge of the civilizations of the various nations as inspired by one or other of the ideals in advocacy, or by a proportionate conjunction of them, is a basis that is indispensable, bringing us away from problems which are in the melting-pot of other sciences to the strict field of history-itself a fresh comprobation, let it be noted, of the organic relationship, close interdependence and essential intrinsic unity in which all departments of human thought are included. A true understanding of man's labor in the world, and of the prac-

tical issues and effects of each of the great human divisions of civilization, without the admixture of prejudice and fiction, without the substitution for corroborated truth of unscientific suppositions, is thus an exigency which is more than merely historical, which transcends the proper limits of history and brings us into the arena of man's highest preoccupations in relation to the future; while it is clear, of course, that if there is much in history which, after an impartial segregation of what is definite and trustworthy, is left over as uncertain, that section of historical knowledge which is a secure and arguable basis can only possess a relative value and a limited application,—this, indeed, being the first point which it is both the right and the duty of the historian to confess and discuss before such as apply to him, in the interests of other sciences, for the material and data which are his monopoly, and in regard to which he alone is qualified to speak. . . . Hence, then, the paramount importance of a comprehensive and scientific history of civilization; for this reason, also, all the investigations of historians, properly described, and of sociologists, economists, pedagogues, psychologists, etc., respecting the factors which, as such, have really actuated and are actuating the life of man,-respecting their manner of operation, their mutual action and reaction, their hierarchy and, finally, their issues and effects, -are indispensable in the attainment of a real and thorough understanding of human history, and demand, therefore, the most rigorous exactitude as regards scientific proof. So long as they lack the security of corroborated truth, there can of course be no deductions regarding them—a fact which should be remembered by such as are impatient for categorical conclusions.

The other question which stands out with the above as of cardinal importance is that of the persistence and continu-

ance of civilization. We know, as a fruit of modern criticism and research, that the theory of continuous progress is, at any rate, a false one; that history offers repeated instances of reaction and decadence now on the part of one particular community, now in a whole group of such communities (those, even, of an entire continent). We know also that there have been highly advanced civilizations that have disappeared from the world without any transmission or absorption into other communities distinct from those that embodied them, civilizations whose thread has broken and whose labor has remained for centuries and centuries buried and abortive; and in the contemplation of these facts it is only natural that uneasiness should gain possession of us with respect to the future. Is it not possible that the future may witness regressions such as that of the Middle Ages—a reaction which embraced all the most civilized races of the world? Is there not a possibility that the entire labor that man, up to the present, has accomplished, may one day be annihilated, swept from the face of the earth and lost as a heritage for future ages? Ought we not take into account the intervention of geological upheavals such as those which fiction-writers have depicted in stories-without, of course, any scientific value? Moreover, in the background (it is useless to deny it) there is always this same awful specter, the possible annihilation of the whole human race itself, some sudden uprooting of its entire records, a possibility which chills the spirit of those who contemplate it, and engenders a skeptical feeling of futility—the futility of a struggle upward toward a better life which is ultimately to better no one, which is doomed to be abortive. It is enough, indeed, to recall the possibility that, apart altogether from climatic aberrations or the destruction of large parts of the earth's crust, this discontinuance may, none the less, occur,

as has happened in times past, without the factor of geographical changes.

Against these potentialities of the future we cannot thoroughly tranquilize ourselves or remove misapprehensions without a thorough investigation of the following historical questions: the conditions which are normally favorable to the diffusion and transmission of the distinct civilizations represented in the different communities, and the difference or resemblance to be noted between present conditions and past; the object being to ascertain whether, in the existing situation, there are not certain new conditions which render less possible, and perhaps impossible, a repetition of those reactions and recessions in the progress of great masses of humanity (masses embracing, apparently, the most important branches of the race) which have imperiled or delayed during immense intervals the general labor of mankind, and entailed endless recommencement and repetition. ward as a practical issue of this, we ought to determine the actual safeguards necessary in order that this function of transmission may be better guaranteed for future generations.

In regard to the first question, modern science already possesses certain positive knowledge resulting from the concrete investigation of given historical instances of the transmission process, as also from the criticism and speculation which has been accorded the phenomenon in connection with the comparative method of investigation, especially in regard to the legitimacy of deducing and presupposing the fact of a transmission (without previous knowledge or detailed investigation of the case) from the simple fact of a coincidence of institutions.<sup>1</sup> It should not be forgotten, however, that

<sup>&</sup>lt;sup>1</sup> The same may be said of the Theory of Imitation of Tarde, which can only be applied with great caution. Imitation is a phenomenon of diffusion.

for historians there are still many doubts and uncertainties in the verification of this phenomenon with relation to events that are of great historical importance, the study of which cannot yet be considered as exhausted or reduced to definite conclusions. A definite though general theory, of wide application apart from the specific differences of each particular case, cannot strictly be established except after a series of monographic studies of other data in connection with this process, extending over as wide a field as possible and necessitating what has still to be a long and complicated labor before generalizations will be permissible.

While fully appreciating the great importance and interest of these investigations, we must observe, however, that for our own particular purpose—in connection, that is, with the problem we are here considering—they lose much of that interest when we come to the second of our questionsnamely, the question of the difference or resemblance between past conditions and present in regard to the facility with which the issues and achievements of civilization may be transmitted and secured as permanent; for, if we could be certain that existing conditions, over and above being more favorable to the process of transmission, actually guarantee and safeguard for humanity in general all the labors realized in its service, then our conclusions in regard to the first question are, for all practical purposes, at a discount. In effect, without further parley, it is actually the case that, from what we know of the past and from our observations of the present, there are enough grounds for affirming as a definite conclusion that existing conditions are, indeed, far more propitious than at any other period of history; on this there is no longer any serious doubt. And with the reassurance the fact brings us, we may satisfy our qualms, confident that what we are accomplishing to-day will not be wasted in

the future, and that the fruit of present labors will be reaped by our successors. We are aware, also, that this security is due chiefly to the development of material civilization, which, indeed, possesses here one of its foremost vindications and highest claims to attention and furtherance; for, in augmenting and facilitating the means of communication between communities, it is not only approximating but at the same time solidifying them in a bond of mutual interests forever widening and forever becoming more closely associated and interlaced, rendering thus more feasible and rapid the diffusion of that culture which, from being self-centered and destructible as in the old days, is evolving now into the universal and the permanent. The fact of life's present uniformity, of the expansion and domination of a common type, and even of the same forms and details in many branches of activity, is sufficient evidence for this contention; and although it may be resented and deplored from another—an idealistic -standpoint, in so far as it threatens us with a monotonous sameness throughout life, destructive of the personal character of each given people, it stands out among the facts of history as one of the most important and significant circumstances in the question at issue. Concomitantly with this immense attainment, modern times have witnessed also a wide and fruitful labor of assimilation which applies both to the modern world and the ancient. For, in regard to the former, the modern aspect, material civilization, while it spreads and implants a fixed form of life and a series of common industrial appliances disseminated from their point of origin over the face of the globe, at the same time, and as an inevitable issue of this centrifugal movement, gathers in and abstracts from each individual person or community of persons the fruits of the original genius of the individual, further developing thereby the whole—that is to say, making

it forever richer and more complex, and facilitating the reciprocal action and reaction of the one upon the other. regard to the former aspect, the amazing renovations in historical knowledge and the resurrection of so many peoples buried for centuries from human ken, and for this reason useless in the advancement of man's labor, have enriched quite suddenly, or in a space of time so short that it is almost negligible as such, the heritage of modern civilization, and enabled us to reap the richest fruits of defunct civilizations of the past, which we have incorporated in our own-to the extent, that is, of all that is of use to us, whether in the shape of some practical element of utilitarian service or some educational contribution toward our imagination, taste or ideality. We have only to compare what at the end of the eighteenth century was known of Greece, Egypt, the oriental civilizations, and even of Rome itself, as regards the art, industry, literature, science and jurisprudence of these countries, with the information now at our disposal, to appreciate the immense advantage which in many matters we possess over our predecessors. The classical restoration movement initiated in the Renaissance has, in these days, developed and augmented in a manner unhoped for and amazing; and if to this we add the deeper and more extensive penetration we have realized into so many other epochs of the past, and from which so much, until now buried and forgotten, has returned to enrich our civilization,-medieval literature, primitive art, the pre-Renaissance philosophies, etc.,—we realize in how great a measure, in the past unparalleled, modern civilization embodies the civilization of all history and is truly universal, truly the civilization of man. And this stupendous achievement, let it be added, is due to the historians, the disciples of a school of study whose practical value is so often superciliously denied.

But now, when all this is said, with all the hope and reassurance for the future which it brings, we cannot deny, after an analysis of our feelings on the subject, that we are not equally at ease on all the issues it embraces. Although it is true there is no longer any doubt as to the persistence of everything which signifies material progress and of that common heritage, scientific and literary, which now seems definitely embodied in our life, we are not equally certain as to the continuance of other elements of our civilization more closely dependent on changes of thought and conduct. The material progress we have realized is so intimately associated with primordial necessities of human life and with appetites—or, if you prefer, aspirations—inseparable from human nature, such as the competitive stimulus and the commercial factor, the craving for economic profit and material comfort, that renunciation of these things seems to us impossible outside the hypothesis of some general mental aberration in mankind. Nor is the retainment of all the learning and culture elaborated through the centuries, and of the beauties of literature and art, a cause for anxiety in so far as such a process of retainment is purely passive in character, while the inspiration of these beauties and that culture is practically inextinguishable in the human species. But in everything that is influenced by opinion such as is not secured on the bedrock of the experimental sciences, or in which other factors are at work in the form of speculative conceptions whose foundation is rational and not empirical, or of feelings of another order from the appetites and aspirations previously referred to, a good deal of misgiving, despite the optimistic outbursts persistently indulged in, has to be confessed after considering matters impartially and scientifically. Who, for example, has not felt the possibility that the unmistakable advances we have realized as regards social

and political organization, in the general province of law and the moral conception of life, may not, after all, be doomed to immolation before some sudden metamorphosis of human thought and opinion, as illogical, according to our present judgment, as you like, but not without precedent in the history of many countries,—embracing, moreover, widely extended areas? What meditative mind has not experienced, at one time or another, uneasiness over the possibility of the general orientation of modern thought being finally supplanted by another, to the entire subversion of our basic conception of the world; or of our literature and art sinking into a decadence in which they will be rendered extravagant and impotent?

With these considerations we are brought to another question that is associated with this theory of civilization namely, that as to whether all the orders of our life are following a necessarily ascendent path—that is to say, a course of indefinite improvement, considering their history as a whole and discarding mere temporary setbacks; or whether there are not certain orders which are exceptions to this rule, different and distinct in character from those subject to a continuous progress; whether, moreover, there are not others whose point of culmination (in man) has now already been attained and will not be exceeded, perhaps not even equaled, in the future. And, as a natural consequence of the comparisons and contrasts necessitated by this study, there follows yet another question which is repeatedly occupving thinkers-namely, the question of a proportion or relative development between the distinct reaches of human activity, or, broadly speaking, between these two (to be taken as embodying the two main divisions of the facts of civilization): the moral order and the material.

Coming to a closer consideration of the first of the two

questions raised, we shall see that while historical investigation has enabled us to determine the existence, across the ages, of a fundamental current which, in spite of temporary deflections, has always, in the long run, triumphed, mounting now higher and higher in the conquest of Nature and the applications to human necessities of her elements and forces, expanding in the sphere of social organization and in the direction of popular liberties, as also in artistic manifestation of a certain order,—yet, on the other hand, we cannot say the same of all the provinces of, for example, art, nor of all the orders of scientific research, and still less so of the problem of moral conduct, especially as regards certain of its most important branches. How many times has it been asseverated that Greek art, in certain branches, is insuperable, and that none of man's subsequent creations are to be compared with it,—not excepting those of this modern era, despite the higher reaches of modern culture and its boundless sources of nutrition from the past? Who is not aware. that, in spite of the great progress of philosophy since the Renaissance, its present situation is still fundamentally inseparable from the doctrines of the Greek philosophers, whose thought we have not, in many things, so much as widened? How often have we not been told that music in the great German classics was carried to its apex, both technical and ideal? Who can deny that modern literature is far from monopolizing all the greatest productions of literary art, and that many of the great masterpieces in this line have been the work of the ancients—a fact implying that the line of development which this departure is following is not subject to the same law which is guiding other orders and unmistakably urging them still forward? And finally, who can escape the bitter confession that moral development is still exiguous, that customs are not improving all around, and

that the higher ethical doctrines remain untranslated into action in the practical life of the majority?

Let it be observed, however, at the outset, that there is a strong possibility of error in these affirmations and comparisons, owing to the influence of a traditional tendency, still prevalent, in which the "classical" is seen as a type and standard handed down to us from the past as something perfect and insuperable, by which we have unduly limited the future, with all its hidden possibilities—possibilities in the way of new departures in the sphere of art, thought, originality and culture. In face of this doubt and uncertainty arising from indefinite and what are for us mysterious possibilities of new departures and new doctrines, a past status of perfection loses the importance it would otherwise possess could it be definitely stated that never in the future will this standard either be superseded or equaled. It would be sufficient, as regards art and literature, that the future should produce things of equal supernal beauty to the great masterpieces of the past, although the ideal which inspires them and the means and medium of their expression may be different.

Furthermore, it should be remembered that the only conclusion of any practical value which is to be drawn from the fact—supposing it to be a fact—that in certain human departments of thought the goal of achievement has been arrived at in past ages—i.e., Greek sculpture—would be that certain branches of progress are more easy of development than others, and have thus been exploited and exhausted, while others are still in the process of development. The immediate consequence of this conclusion in its influence on our conduct, as one of the educative results of knowledge, would be that we should dedicate the greater part of our energies henceforward to developing all that is relatively backward,

withdrawing such energies to a great extent from the fully exploited branches whose pursuit, it would seem, can only be attended now with lesser results. Perhaps, indeed, in certain modern propensities, in certain orientations of the main body of humanity to-day, which seems to be cultivating by choice precisely those branches which are only imperfectly developed, there is a vague but effective consciousness of this necessity.

What is of real and actual gravity, however, is the fact of the enormous disproportion between the highest results which have been achieved in the ethical department and those of the other orders. This is an historical fact which is evident, even without any special study of the matter, to anybody, and on the strength of which we may divide the manifestations of human life into two groups: one in which are embodied all those branches which, it may be said, have on the whole expanded and developed and are continuing to develop in a conspicuous manner, or else have already in the past attained their apex of perfection, though to-day in a state of collapse and effeteness,—manifestations belonging to the artistic and intellectual sphere, or representing the material civilization which has resulted from man's dominion over nature and from the applications of science, and also to certain aspects of social organization; while on the other hand is the group which embraces the element of moral conduct and certain other directions of social and juridical organization, phenomena which either have not developed in any perceptible degree or are obviously behindhand compared to the phenomena included in the first group.

It would be superfluous to reopen here the discussion which years ago, when the literature of the Philosophy of History was flourishing (that literature which dazzled and misled so many people, while it offered little that was of real

scientific value), raised such impassioned argument owing, perhaps, to the radical form in which it was planted and the rash manner, disregardful of requisite historical data, in which it was approached,—the discussion of the question: Is there or is there not such a thing as moral progress? Such absolute questions it will be a matter of common agreement to discard as fruitless because no one doubts the fact today that, in certain aspects of his moral conduct, social and individual man has actually advanced, and that the practical ideal which is being realized in the higher circle of society is superior to that which prevailed in such circles some centuries ago. And simultaneously, in the juridical sphere, in the strict meaning of the term, accepting the common distinction between morality and law, -a distinction which is not necessarily exact,—it is equally beyond doubt that justice is, on the whole, becoming more and more actual in many of the human relations it affects.

But by the side of this twofold conviction which we possess it is equally unmistakable that the moral and juridical order still, in many of its phases and even in the most advanced communities, embraces what is immoral and unjust, and that the majority of individuals are likewise immoral and unjust in many features of their lives. The discouraging impression which these facts produce in us is not so much suggested by the evils they infer as by their exposure of the inefficacy of doctrines and ideals proclaimed and effusively embraced by millions of human beings many centuries ago. It is comprehensible that there are certain sciences which have not at all times realized the perfection and development they have now attained, because the advance of these sciences has followed from the grasp of certain truths which have only latterly been realized; but the ethical and juridical ideal, in its application to social and individual life, has been realized in

many of its fundamental aspects since immemorial time,—yet, nevertheless, it has produced only the most exiguous effects relatively to the situation which preceded its adoption or to its exigencies as an ideal. This inefficacy or extremely limited efficacy of the moral ideal is what disheartens the sincere observer and at times causes him to despair of the province of morality, even theoretically admitting the development attained in the other provinces of life, or at least to demand why it is that this element is to be found in what is perhaps an immense inferiority to others, and is, at all events, held in less importance among the problems of life.

This situation is explained, according to modern theories. on the hypothesis that moral advancement is not solely dependent on the advancement of ethical ideas, but also on other factors belonging to other orders-factors which in most cases have made their appearance long after the actual ethical ideal. A good illustration of this doctrine is Buckle's instance, in connection with war, of the decline of the warlike spirit in humanity. For Buckle, as is known, the three great causes of this change have been: the invention of gunpowder, Adam Smith's book on the "Wealth of Nations," and the use of steam in land and maritime communication; that is to say, three factors wholly distinct in origin and character from the moral sentiments which, at first sight, would have seemed to be the principal causes of this momentous change. In like manner, other authors, of philosophic affiliations very different from those of Buckle, have shown that in the abolition of slavery in Europe and in the betterment of the juridical situation of the land-laboring classes, moral motives represented only an exiguous influence, while economic motives, on the contrary, were paramount. These

<sup>&</sup>lt;sup>1</sup> For all that is to be learned from Spain in this matter, reference should be had to the standard work of Eduardo de Hinojosa: "The Feudal System and the Agrarian Question in Cataluña," Madrid, 1905.

and many other historical examples appear to establish the theory of the school in question, according to which moral progress is made dependent on scientific development, or on the changes at work in other very distinct orders of life,—a theory according to which the relatively backward situation of the moral order is explained by this observation of two facts—the fact, primarily, of this same dependence of position and the fact of the personal and intransferable quality of moral actions. "Whereas intellectual acquisitions," says an exponent of the theory, "are transmitted scrupulously from one generation to another and the attainments of the moral faculties are not transmissible, in that every one must practise goodness for its own sake, by the nature of it goodness is essentially personal and private, and even the good which is realized by the purest and most diligent philanthropy is of limited duration and can only benefit a comparatively small number of people. The actions of the bad produce a transient evil; those of the good, a good which is equally unenduring: it is only the discoveries of the great thinkers which subsist eternally, survive the ruin of empires and the fluctuation of beliefs, follow and are added to each other in succession, and stand alone immutable amidst the ephemeral and fugitive, serving as landmarks in the progress of humanity."

There is of course obvious exaggeration in some of the above affirmations, for neither is the moral element so changeable as is suggested,—a certain sediment always having persisted and affirmed itself through history,—nor can it be said that nothing of what is attained in this order can be added to previous attainments in the way that intellectual advancements are recorded and accumulated; nor even is there entire justification for the theory that the effect of a moral effort can only be passing in duration, for such an

effort, when it becomes crystallized in a social labor or social institution or in a reform of customs, may be prolonged through great periods of time and become incorporated in the general conduct of a people almost finally and unalterably. descending and extending to an immense number of human beings. These discrepancies, however, do not invalidate the general truth of the theory as regards the intervention of non-moral factors—factors, that is, of a different physical and spiritual order-in the achievement of advances in the actual domain of morality, nor the force of the theory as an explanation of this same disproportion in development which we are concerned with—this albeit that it is not a matter of such certainty that the inevitable action of the intellectual over the moral implies an absolute subordination of the latter to the former, in so far as the influence exerted by the human intelligence over human conduct does not invariably signify the actual suggestion of new lines of conduct, but represents in many cases merely the thought and reflection granted certain principles of life defended by the moralists. -reflections that have resulted in a conviction of the essential necessity of the principles in question; -- intellectual progress, in the strict meaning of the term, thus, apart from all it represents in its own sphere, being converted through this relationship into a means for serving and furthering the end of most importance—the object, that is, of moral progress. The fallacy in the argument that because intellectual advancement, as is contended in this theory and in fact admitted by us, is the impulse of civilization, it has for this reason to be considered the measure and criterion of it, is evident when we consider that progress does not consist merely in the declaration of principles or in the act of mentally appreciating them, but in their practice and realization -assuming, that is, that the first and basic necessity in life is

goodness; the contradiction, moreover, between belief and conduct, between thought and action, is sufficiently glaring in our lives to save us from the error of deducing the purity of the latter as an inevitable issue of the truth and beauty of the former.

But now, so far as our main question is concerned, -the actual question under discussion, -- the fact remains, whether we hold this theory to be valid or regard the two spheres in question—the scientific and the moral—as independent, or at least independent in many of their aspects, that we are still left with the same doubt as we started with, though embodied in two forms. On the first hypothesis—that of our accepting the theory-it is necessary to ask: Up to what limit will scientific development be able to influence the moral conduct for whose growth it is responsible? In the second case we are faced always with this question: Is the present disproportion between the development and evolution of both spheres to be permanent; will it, in time, become diminished, or is it to be augmented still further in the future? And in either case, what is the impression, optimistic or pessimistic, that we are left with after the study of all, in this connection, that history up to the present has afforded us?

But now again, it is not impossible—in fact, it is very probable—that the question is still imperfectly stated owing to the need of a further discrimination. In short, are we so very certain that all the actions usually comprehended in the sphere of moral conduct belong to the same order and destiny? Does not historical observation, on the contrary, suggest that there are two distinct classes of manifestations in this order whose difference may be said to have found expression in the distinct directions they have taken across history? This very obvious distinction, already noted in a preceding argument, that exists between certain features, on

the one hand, of social morality, embracing determined aspects of human relationship—orders that have developed in moral status, and become purified, possessing what is perhaps an inexhaustible capacity for continued purification and development-moral attainments such as honor, tolerance, veracity, impartiality, etc., -between these and other features of social and individual morality, as far as the distinction is possible, which are plainly making no headway and in which the element of evil is as prevalent to-day as centuries ago, is surely a powerful argument in favor of the theory that there is one branch of our moral life which is capable of development and another in which all progress seems impossible, or at least has seemed so up to the present. That this is the case is, in my opinion, beyond doubt: I believe that the experience of history demonstrates with the utmost clarity that there are moral inclinations in our nature which can actually be checked—which have, indeed, been suppressed among certain communities, with a resulting transformation in popular customs; while, on the other hand, there are others, always precisely the same, which, subsisting as they do in passions apparently ineradicable, dominated and subdued by only a limited number of people, not in each case the same elements, have not been subject to this rectification and continue as sources of evil. Such is the case with envy, anger, cupidity, ambition and the craving for luxury, and a whole series of other tendencies elemental in our nature whose products in the form of misery and privation are utterly horrifying as represented to us by modern sociologists, psychologists and criminologists, such abominations in our days scarcely being considered possible.

These, then, are the actual facts of the case, the results of historical investigation, and beyond the field of these facts, on any scientific basis, we cannot venture; for every predic-

tion is merely a hypothesis, a problematical supposition with relation to an uncertain future. Human aspiration, however, does not resignedly surrender to a simple recognition of the facts as they now are and have been in the past—in a recognition, that is to say, of history. Hope ventures into the belief that it will also be possible to rectify, finally, that which has seemed incorrigible, to subdue those forces which up to the present have been irrepressible, and so to subdue them that the change shall constitute a social triumph, wicorporated as a definite conquest in the civilizations, first of the most advanced communities, and finally of all. Such a labor, in fact, if we come to think of it, embodies the cardinal problem of education, and it is on the appreciation of this problem in the alternative attitude of optimism or pessimism that depends an important difference in the prevailing scholastic system. "Education will do everything!" or, "Education is subject to impassable limits in human nature generally and in each individual case in particular!" Such are the two conflicting statements. The second bases itself on the concrete data of experience, the first on a generous confidence in the perfectibility of human nature and the efficacy of method; and so inspiring is the conception it awakens in us of the future that it has won the powerful support of great men like Goethe and Guyau. Although the main course of pedagogy is to-day following another direction, refusing to admit the omnipotence of education, it is certain, for the moment, that any absolute and categorical answer to the question will be problematical. This question the advances of psychology, social and individual, may enable to be answered in the future. At present the most we can do is to formulate the problem.

But this same uncertainty and doubt which arise, on the one hand, from the weakness of our hypothesis respecting

the future, and, on the other, from the results of our study of the past, serves at any rate to bring us to grips with the urgent and dominating question: What is it that is of most importance in life? If mankind is not improving morally, what value is there in the other branches of his progress? For what do they serve but as a merely superficial satisfaction and a delusive mask to the virtual wretchedness in which the immense majority of individuals live?

Let us now fearlessly approach this question, which, although, like others we have been dealing with, is apparently disassociated from an investigation properly speaking historical, is as a matter of fact essentially allied to such a study. The question is inevitably associated with the ideal of life which ranks the ethical factor (and quite rightly so, no doubt) at the head of all, maintaining that, as compared with this, material or purely intellectual advantages are of little value; while, for another thing, it presupposes that all the elements, both material and spiritual, of human life have necessarily to be equally perfectible. As a result of this double supposition every deficiency in the moral order fosters, it is clear, discouragement, pessimism or censure, with all the perplexities that historical data awaken with regard to the disproportion between the march of the two orders. But the question to be considered is whether, while admitting the first supposition (for me it is beyond doubt, and in fact I believe most firmly that the main value and significance of our advances in the intellectual sphere and the material consists in such assistance as they provide for the juridical and moral element in its task of facilitating a real understanding of the world and the subdual of natural impulse), there is not a great error in the second. Would it not appear certain that, distinguishing as we do between two spheres or groups of actions and relationships in that province of civilization

whose backwardness we are discussing, we should confine ourselves, without embarking on the impossible, to the perfecting of those elements which are perfectible, according to our evidence from history, while on the other hand recognizing, and resigning ourselves to the admission, that there are other elements which lack this capacity of growth, and in respect of which the only feasible course, with human nature as it is, is to limit their scope for evil, redeeming the maximum number of individual cases, and, in short, diminishing the deplorable influence they exercise (it being impossible to suppress them), as is being done to-day with many of them by means of legislation, police, prisons and reformatories such as are worth the name, and even medical treatment in its particular province?

If we were to take this course and bow to the inevitable. we should be relieved once and for all of the warring preoccupation over an impossible ideal, over the incompatibility between a belief in this ideal and our utter failure to accomplish it; and this relief, freeing us from the despair which is born of failure, would enable us to direct the best of our energies toward what is feasible, discarding from the field of historical investigation problems which have ceased to be problems. And then, indeed, our whole theory of civilization, springing from a recognition of the facts of history and the undoubted progress realized in the majority of our activities, as also of the fundamental orientation which the whole of human history seems to contain below the surface of its racial differences, -- an orientation which is not prejudicial to the original genius, necessary as long as harmless, of each social entity and group,—would have as a practical result for the present and the future the ever intenser application of those means and processes by which, up to the present, progress has been realized, especially with the object of

accelerating the march of those phases of progress which are behindhand, and of maintaining the equilibrium in which the development of one order will not be sacrificed to that of another, either in dragging humanity into a life of egoism for a more or less considerable number of people merely voluptuous and sybaritical, or in depreciating intellectual and material evolution in favor of an esthetic ideal and moral standard, to which mankind is to be converted, incompatible for society with all the other achievements it has realized.

Well, now, if we reflect on the aspirations of contemporary civilization as they are manifested and expressed, we shall see, as was mentioned before, that all these manifestations affirm the resolve to secure and conserve the material civilization now flourishing, to augment and at the same time disseminate it, embracing the widest number of people and thus converting it from the monopoly of the few into the heritage of the majority, and, if possible, of every one; also, that this same centrifugal tendency is to be observed in the sphere of intellectual culture, forever seeking to penetrate more widely the masses at the same time that it is perfecting the conditions of the higher investigation which is reserved for the chosen few, but open to humanity in general in the glory of its issues and conclusions. And concomitantly we shall observe that, alike in the flower of humanity and in the surging masses, there is a cry and clamor for the ethical basis to life, a demand for the reign of justice in the sphere of jurisprudence, of the good in the sphere of morality, these being the things which are our only guarantee against the tragedy of a life of hatred, tears and curses, -in search of these things, however, always in the consciousness, given an impartial recognition of experience, that there is a surplus of evil still undominated, which is probably indomitable, and which embodies the unavoidable lot of human imperfections. human limitations, which are defiant of human will.

#### Third Lecture

# THE METHODS OF EXTENDING CIVILIZATION AMONG THE NATIONS

WE were saying in the preceding lecture that the general problem of human history—or, in other words, of civilization—embraces two classes of questions. The first of these we have endeavored to answer in the before-mentioned lecture. The second, although it has been the subject of many previous allusions, we shall now answer more directly, in order to arrive at the treatment of the concrete question in reference to Spain.

We must bear in mind that our object is to ascertain by what methods civilization is evolved, and what is, in consequence, the best course to adopt in order to strengthen and advance it.

Passing over the beginnings of history, when each family or human group (if we admit the polygenetic theory) or the family nucleus (if we accept the monogenetic theory) either must have been self-taught and have had to select for itself the most important lessons which nature offered, or must have arrived at the principles involved through the inventive power of human intelligence, there is no doubt that the instances of autodidacticism, collective and individual, are the exception, and that when they do appear they have but a limited field of development and leave no lasting impression if they remain in the isolation in which they were conceived.

The general law of civilization, as in education (and, strictly speaking, are they not the same?), is reciprocal influence and mutual teaching. Those who teach others are at

the same time taught. There is a continual ebb and flow of suggestions, corrections, imitations and reflected experiences, by which each individual profits more or less according to his power of assimilation and reaction. This law fulfils itself in each group, acting between individual and individual, between individual and group, and vice versa. The same process takes place between group and group, although it may be possible that during the centuries one group, or a combination of groups, has become isolated and has continued to develop an acquired impulse by virtue of the continuous growth of human powers and the more than geometrical progression of their advance. The latter seems to have been the case in primitive America.

This law takes effect without the knowledge of those it influences, and even against their will, as happens, for instance, between hostile peoples separated by mutual hatred and respective interests,1 or as occurs with those peoples who attempt to isolate themselves from their neighbors (as though this could be accomplished even should all the laws of the world seem not only to sanction but to command it under a thousand penalties). Aside from the fact that this law invariably works itself out naturally, man applies it reflectively. He civilizes individuals through education (schools, academies, etc.). Nations he civilizes sometimes by imposing upon them a régime which influences the great majority (e.q., the process of Romanization of the provinces in so far as this result was intended and sought after by the Romans themselves), sometimes through individuals, these individuals being chosen, as in the modern method of awarding scholarships for study and travel, to learn at first hand the history and customs of peoples who are considered more

<sup>&</sup>lt;sup>1</sup> For example, in the case of Mussulmans and Christians in medieval Spain, who, notwithstanding their constant warfare, influenced each other to a great extent.

advanced, in order that the knowledge thus acquired may be diffused throughout the student's own country.

In this way the civilization of each group continues to progress, impelled by that which each group receives from the other groups and by that which originated within the group itself. The absence of either of these two factors would disturb the equilibrium of the civilizing process, since to influence and to teach, a people must have created something, and even that people which has created nothing equal to the productions of others, must have in its mental composition an original element on which to base and mold into characteristic form those qualities borrowed from its fellow beings. A people lacking this original element (which in its turn will convert a people into an active factor in the common work of civilization) becomes weakened and atrophied as does a disused organ.

Since civilization and education are essential factors in every case, this question immediately arises: Is it right to impose civilization by force? In education this question is presented in the discussion concerning "obligatory learning" imposed upon the child, although he may not desire it, because his resistance to it (if he does resist) is the result of his ignorance of the fundamental importance of education in his life. Had the child as clear a conception of its value as the adult man usually possesses, he himself would ask that he be educated and would demand this as a right, in the same way that he would demand the fulfilment of his right to be provided with the necessities of his material life, for which, in his earliest years, he could only ask by signs and cries (at times he even refused them), but which, nevertheless, were not denied him because of this.

Let us now consider the problem in its bearing upon the relations among peoples. Probably ever since humanity has

existed and groups of men have fought among themselves for a thousand causes more or less clear, in the discussion of the motives which led to aggression men have resorted, whenever the circumstances offered a semblance of justification, to the argument that this aggression was entered upon in the interest of culture and education. In some cases this interest manifested itself in connection with religion (e.q., in recognizing as a duty the conversion of infidel nations. pagans, etc., and their introduction to the true faith); in others, the argument had to do with the general welfare of humanity, which was being jeopardized by the existence of peoples ignorant, backward, fanatic, opposed to all innovation, etc., incapable of developing with intelligent effort the resources offered by their own soil,—peoples, in short, whose continued unproductivity justified the interference of the rest of mankind; others alleged that humanity was imperiled by the existence of peoples stubbornly opposed to the recognition of those fundamental rights of man without which community life and social relations are impossible. This latter argument is of recent origin; indeed, it is the child of our own epoch, and has come to replace almost entirely the argument of religion, just as that of religion replaced to a certain extent the argument of the superiority or inferiority of peoples and individuals which was used to explain slavery in classic times, and which was even advanced by certain philosophers of the Renaissance when referring to the American aborigines.

Apparently we have before us a theory analogous to that on which obligatory education is based. Nations, like children, must be taught to realize the importance of their mission; if they fail to educate themselves voluntarily, others must intervene in their affairs in order to raise them to the level of culture they are capable of attaining. Thus, the

most civilized discharge a tutelary function, aiding and cooperating toward the common good. Of the two forces working in humanity, one to advance all civilization, the other to bring about the sovereignty and independence of individual states, the former is, in the theory, the stronger the usefulness of the latter being destroyed when it serves, as it does here, merely to maintain a group of men outside the established order and conditions of civilized life.

If this theory were correct, we should have an example of a method of civilization distinct from the two common to humanity: viz., individual effort and the normal and pacific influence of others (if this influence is not rejected or deliberately sought after). It would be, simply, the employment of the coercive method when the voluntary method was not spontaneously followed, and all that would remain for us to discuss would be whether this method may rightfully be employed, or whether, on the contrary, there is included among the prerogatives of a people's liberty the right to remain indefinitely barbarous, uncivilized, or backward and markedly inferior to the majority who feel the impulse toward civilization, -the right, in short, to be an obstacle preventing the growth of this civilization in strength, its acquirement of new methods and its extension over the entire world.

But even if we accept the theory simply as such and without raising any difficulties, history provides us with this extremely powerful argument against it: If obligatory education presupposes a compulsion, this compulsion is not used to abuse the child, to diminish his rights, to take possession of what is his,—in other words, to do him harm,—but to portion out to him a benefit in a form equally good. The theory referred to, as has already been noted in pointing out its origin, is only applied to peoples in the form of conquest.

And, even supposing that it is not a disguise for the mere desire for mastery, the form through which it manifests itself usually bears in its train conditions which render the theory worthless. In fact, those who have recourse to it as an excuse to interfere in the life of a nation, to seize its territory and to direct its affairs, are not in the habit of deciding upon this course for the good of that nation (this is the fact, no matter what name may be given to the intervention), but egotistically for their own benefit (to take advantage of the natural and industrial wealth of the vanquished nation, to provide room for expansion, or through pure delight in domination, etc.); or at least these considerations take first place, while the task of education is left very much in the background, or is confined to mere contact with that in which the conqueror is superior; that is to say, the tutelary mission of cooperation and of the regeneration of the less developed neighbor is subordinated to the acquisition of those things which contribute peculiarly to the advantage of the conqueror, or at least it does not occupy the preëminent position which befits it; and instead of a work of love, of concord, of mutual effort, it becomes a work of hatred, of violence, and of plunder more or less dissimulated.

If it should be objected that in such a case the end justifies the means, since in the end the less advanced, conquered people,—the Roman provinces, for example,—assimilating the advantages of the new civilization, will rise to the level of its conqueror,—if this objection is presented, we may answer that neither is this always the case (for there are many inferior peoples who have never risen to the level of their conquerors, but have been absorbed by them and so lost their own identity), nor is violence, ordinarily carried to bloody limits, the proper road to education. This deplorable result is brought about sometimes through lack of tact on

the part of the "educator," sometimes through resistance on the part of those whom he is attempting to educate. It will suffice to recall in this connection the thousands of victims of the Roman conquest in the Iberian peninsula,—victims who cannot be forgotten even in the light of the superior culture which was finally forced upon the descendants of those sacrificed. And as it was effected then, so it has continued to be effected through all history, and so it is still effected in our own times.

The question, then, immediately arises: Is it possible to accomplish this by another method? Is it possible to bring into the field of what is considered the more advanced civilization any nation whatever, without stirring up a conflict animated by that very resistance to improvement which is the result of their ignorance, and without this conflict degenerating into bloody disputes and plunderings? Or, in other terms, is it possible to educate in the same way (that is, through the action of love and kindness) as one would a child who fails to lend himself willingly to education, a people which does not desire progress? In my opinion this question cannot be answered in the abstract. We lack sufficient historical data to give a well-founded answer, for all the material which we do possess is based on contrary proceedings: the conqueror has always commenced by troubling and molesting, and has thus given a motive for the resistance. Some exceptions which we might recall, but which came to nothing (I have in mind the attempt of the Padre Las Casas in Cumaná), have usually followed bloody conflicts, and it is impossible to say what they might have accomplished by themselves if they had been employed from the start. That very division of mankind into peoples stubborn and warlike and peoples docile and submissive in the case of intervention, which the conquerors have been accus-

tomed to make, is in itself suspicious. We cannot be certain that the first classification was not often an excuse for the violent proceedings which the invaders themselves initiated.

There is, moreover, a factor in the problem when dealing with nations which greatly complicates the question and forces it into the field of violence, although this may not be the intention of the one who intervenes. This factor is the total or partial loss of independence which the intervention of a foreign power always presupposes, and which, no matter how slight it is alleged to be, bears down upon and hampers its victims, the more severely the nearer they find themselves to that state of civilization in which liberty is fastidious and does not even recognize the ideal restrictions which separate and distinguish it from free will and the most absolute personal autocracy. In the case of the child forced to attend school there is a loss of independence as he understands it; but his protests may be overruled and his struggles are so insignificant and ephemeral that they leave no traces. The protest of a people, on the contrary, is not so easily overcome, and is strong enough to bring about the violent conflict whose suppression serves to accentuate the hatred and increase the tyranny. Since even the slightest interference, actuated by the most generous purpose, brings with it some limitation of a people's sovereignty,—if this limitation is felt keenly enough by the people interfered with, will all the advantages that accompany it be strong enough to smother the desire to reconquer their former complete freedom? Moreover, the self-esteem, the national pride of a people is far stronger than that of an individual; it reasons less and often fails to recognize the superiority of a neighbor; consequently, as soon as a people whose affairs are under the direction of another begins to comprehend its own powers and is admitted to the same rank of civilization as

that of the nation which is intervening with the intention of teaching, it will oppose this design with all that feeling of repulsion to which the self-respect of a nation is susceptible when it is troubled by the mere suggestion that it needs the guidance of another and is incapable of working out its own salvation.

And it is to be remarked that this fact, natural in the psychology of the group and repeated in history, has been dignified in a theory which, idealizing it, has strengthened and raised it from the rank of an almost instinctive movement of reaction to the category of a recognized necessity, some of whose principles admit of no discussion. This is the position of Fichte when he names independence as a fundamental and essential condition of all culture, since civilization truly serviceable to a people must be the outgrowth of their own effort and not something borrowed or taken over ready-made from others.<sup>1</sup>

Except for a very few and limited examples of missions and governors in the history of our own civilization, we lack, I repeat, such data concerning loving guardianship over a people as we possess concerning the affectionate teaching of a child; but this deficiency does not authorize the statement that, generally speaking, the humanitarian proceeding would not be possible.

That of which we may be certain is, that humanity, taken as a whole, does not know how to use it. It has seen the wisdom of dealing gently with the child, but it has not yet arrived at this method of dealing with the people of another country when that country is open to domination. This historical law, true in ancient times, true in the Middle Ages,

<sup>&</sup>lt;sup>1</sup> History, however, sometimes argues with examples contrary to this statement—e.g., the Romanization of a great part of Europe, which produced extremely beneficial results, notwithstanding the fact that it was accompanied by domination. The truth is that Fichte theorizes concerning peoples already civilized.

true in the epochs of great discoveries and of colonial expansion, still reigns in the world to-day. And furthermore, notwithstanding certain advances in the laws of war usually more theoretical than practical, illusory promises in the reports of the international conventions and frequently contradicted by reality, we note a retrocession in the ideas relative to this point, or a new and unsympathetic assertion (dissembled in form and not very explicit in its outward manifestations, but very clear and definite as a rule of conduct) concerning the incorrigibility of certain human groups, of their unfitness for civilization, and of the advantage of making them disappear as one would an obstacle which stands in the way of progress. At least there is a general indifference to the fact of their disappearance, even in the case when this is brought about by violence and has exceeded the limits of a natural movement for self-defense on the part of the superior group. These sentiments, I repeat, are the dominant ones which in the end direct the decisive acts of statesmen, and those which triumph beneath racial romanticisms which, in some places, have wished to bind the present life with native atavisms open to much question when considered historically, but worthy of respect from the humanitarian point of view.

The question, then, in its practical aspect is answered day by day; and it will be some time at least before any one will be able to change its trend, however fervid and however reasonable may be the propaganda against it. Precisely here lies the problem—in the fitness of one or the other line of conduct. Which of them has reason on its side? Which should prevail in the system of relations between people and people, state and state? Do there exist, in truth, peoples incapable of advancing civilization, refractory to the demands of modern life; peoples whose mere existence in or

out of a country is at least a dead weight upon the progress of that country, if not actually an active factor of disturbance and degradation, the suppression of which is a necessity?

It, must be observed that the judgment of incorrigibility or inadaptability is rendered by the very group which is promoting or predicting the annihilation of that which it considers a disturbing element. This judgment, always open to suspicion, since the giver is at one and the same time judge and party to the suit, is perhaps hasty as well, when we consider that it is applied to those who have as yet experienced no attempted education. If the condemnatory sentence should come as the consequence of a systematic series of efforts sufficiently extensive and intensive to educate the people or the race qualified as a disturbing factor, there would still be room for discussion concerning the logical exactness of the conclusion, but one could never deny the fact that this conclusion had some foundation, and that before arriving at it other methods had been tried. But, as we have previously asked, which one of those peoples who have planted colonies among inferior races can lay claim to having actually made an attempt at such an education, instead of offering a "civilization" produced through alcohol, deception, abuses, and through that contempt which bars from communion with the superior race those men considered as lower in the scale of humanity?

The above consideration, just as it stands, would be sufficient to make us suspend judgment respecting the justice of that policy of domination in the relations among peoples; but we could strengthen it still further by observing that in history this judgment of inferiority has not only been applied to barbarous and savage human groups, but also to those who enjoyed a well-developed civilization; not infrequent

are the cases in which a warlike chauvinism, the smoldering hatred of nation for nation, also applies this judgment to a nation which is almost upon the same plane of development as the one which condemns it and passes this opinion only because the latter nation does not consider the other as belonging to the same "race," or because a gulf of century-long wars separates them and provokes their ill-will, or simply because exciting contempt for any foreign accomplishment was considered a good method for assuring patriotism.

Even laying aside these cases of actual injustice, of judgment blinded by passion, and also those other cases in which the condemnatory sentence is notoriously hasty and is not based on positive facts, there will still remain a few concerning which the question reappears in all its vigor. Around it the two opposed criteria of humanity will continue to contend-the sentimental and optimistic, which abhors all violent suppressions, and the utilitarian and pessimistic. which believes that such suppression is justified in the service of civilization and on the grounds of the positive inability to advance in culture which it presupposes in certain human groups. That is to say, that even on the firm ground of sociology and law, laying aside all the selfishness, all the deceits and tricks of justice which are produced by special interests ever against our wills, and all the illogical precipitancy of judgment, this question may safely be formulated, or rather, in fact, we do formulate it to-day and answer it at each step without scruples, and hence we must consider it as not to be set aside in our minds,—the question as to whether there actually exist people who, because they are refractory under any attempt to guide and educate them, should be eliminated from modern social life, if not by a quick, violent method, then by neglect of their cultural necessities and the absorption of their revenues. This recognition of our pres-

ent attitude of mind toward a question of such importance should serve us as a touchstone for investigation and judgment of past conditions. If humanity to-day, with all its progress and culture, is still doubtful on this particular point, and, what is worse, in actual practice still continues to apply the system of domination and fails to recognize tutelary education, or else does not apply it when it should, how can we be surprised that in other centuries humanity less cultured, harsher, and more implacable toward man, less infloenced by the principles of fraternity and solidarity, should usually have proceeded in the same manner and fulfilled its duty of transmitting civilization either by subordinating it to its own interests, or imposing it by force, or judging that not the conquered people were worthy of it, but rather the conqueror in the dissemination of colonies which conquest itself brings about? Undoubtedly the fundamental work for a knowledge of actual human history is a thorough investigation as to how each people, on coming into contact with an inferior race, has understood its relations with that race in the light of its duty toward civilization, and how it has effectually realized them (favoring now one system, now the other). This investigation up to the present time has been undertaken only in a fragmentary manner (that is, with reference only to certain peoples, and, strangely, to certain definite classes of culture and of social life), and often in a spirit of partiality which sought only faults, not facts. The Kulturgeschichte, aspiration of the theorists of the Renaissance, cultivated in the learned manner by many historians of the eighteenth century and reduced to a system by those of the nineteenth, is still in the main a collection of general laws whose ideal interrogatory lacks many of the questions which might explain its processes and give significance to the material on which it is based. One of these questions—and

one of the most important—is that which we formulated a moment ago. While this question remains unanswered with that fullness which its conception demands and with the scientific accuracy which would exclude passion and injustice, we have no right, even from the most rigorously sentimental and humanistic point of view, to judge any people upon this phase of their conduct, because we would lack the exact and complete knowledge of what they had accomplished, and, consequently, the ability justly to compare this with what the rest of mankind had achieved.

This is the case of Spain considered as a colonizing country. Since Las Casas published his "Destruccion de las Indias" (1552), Spaniards and foreigners have discussed not only the problems proposed by Las Casas—as, for instance, the right of conquest in America (the justice or injustice of the war), the personal liberty of the aborigines, and especially those acts of violence, unauthorized even by war itself and which more than anything else aroused the pity and the just spirit of the famous friar,—but also our entire colonial policy and even our ability as a colonizing people, in so far as colonization is to be regarded as an aid to the progress of the colonizers, which is the consideration that preoccupies those who regard the problem from this point of view. Let us put this question aside since it has no immediate relation to the problem of civilization which is now occupying our attention. Although this is interesting to economists and to

¹ The defense of Spain's colonial policy in America has been very incomplete. Neither Vargas Machuca nor Solorzano nor Nuix, etc., has dealt for the most part with more than one aspect—i.e., the slaughter of the Indians, their slavery through the abuse of agents, and other matters connected with the accusations of Las Casas; and even this they have usually done with arguments which, judged by our modern standards, at times rather make things worse, although such arguments carried great weight at the time they were advanced, because they were in accordance with the legal opinion of the age, a circumstance which we must never fail to take into account. As an example of this type of argument we may take that of evangelization and that of the power of the Pope, which Vargas Machuca employs, etc.

those who with scientific reasoning deduce from every manifestation of a people's character the salient points of their psychology and their fitness for social life, it lacks interest for those who, like us, are putting a very different question—one referring not to the effect of colonization upon the colonizing country, but on the country colonized.

In this respect it is not particularly interesting to note those cases in which the Spaniards of the sixteenth and seventeenth centuries, as sons of their epoch and educators in its ideas, acted as did the world at that time (and as is done even to-day quite frequently) toward the persons and possessions of the natives, their political independence and peculiar civilization, more or less advanced. That which is both interesting and necessary is to note and weigh, after a detailed and calm investigation, the true extent of this proceeding, or, in other words, of this contempt for the Indians and the abuse of their lives and possessions, in order that we may be able to say whether the cases in which this occurred were such, in number and consequence, as to warrant our considering the Spanish conquest and colonization as a unique and extraordinary example of a cruelty and arbitrariness unequaled in history, or, on the contrary, an example of the manner in which human groups which consider themselves more advanced have always treated those inferior to them. And while we are considering those charges unfavorable to Spain, it is equally interesting and necessary to ascertain and scrupulously to judge those actions, laws, sentiments and ideas which counteracted to a certain degree, or attempted to counteract, the usual method of formulating and carrying into effect a system of treatment for peoples of different rank in the scale of culture and civilization, peoples of different religion, etc., etc. The accurate and complete verification and comparison of these two opposed points of

view will enable us to form a just and impartial judgment upon Spain's early proceedings with regard to the countries which she conquered or colonized. This verification of data, however, has not yet been carried out, although it has been suggested and even initiated in certain historical and polemic works, modern and ancient.<sup>1</sup>

The same reaction which is visible to-day in the works of so many authors, not Spaniards, against that exaggeration, admitted and encouraged for centuries, concerning Spanish cruelty as an essential part of our methods of colonization, proves that the matter is not yet fully understood nor the final judgment upon it rendered. The thousands of comments dealing with American history which have not been read and, consequently, not been used in historical investigations are sufficient argument in favor of a just and prudent hesitation in pronouncing this judgment.

There is to be considered, however, a second division of this purely historical problem which is occupying our attention at present. This division deals with the actual benefits conferred by Spain upon the countries she colonized. Mistaken or not, from the point of view of politics, the comparison of the Indies (Spanish possessions in the New World) to Spanish territory, the consideration of their inhabitants as Spanish subjects, which influenced the laws given to them in the same manner as it influenced those given the people of the Spanish peninsula, the frequent transplantation of Spanish institutions to America, the participation in public duties allowed these very natives, etc., etc., are facts which merit consideration as evidence that Spain gave to the new countries she had conquered the same political and administrative system by which she herself was governed,

<sup>&</sup>lt;sup>1</sup> A résumé of all that is known on this subject to-day may be found in the author's "Historia de España y de la Civilización española," Vol. II, secs. 574, 575, 588; Vol. III, secs. 676, 677, 678, 695, 696, 697, 698; Vol. IV, sec. 811.

and not a distinct and inferior system.1 She also followed this identical policy with regard to her culture, establishing in her colonies the same system of education which the mother country possessed and which experienced the same fortune and vicissitudes as did the latter. In this respect there never existed a system of exceptions (we refer to the classical period of colonization), but rather one of perfect equality. For the native races and the half-breeds Spain even went to the extent of founding special centers of education and means of obtaining it (as, for example, in Cuba, Mexico and Chile). If she did no more, and if she did not always succeed in that which she attempted, this failure was due either to the fact that the problem of popular education, as far as the native was concerned, did not at that time present itself with the same clearness and urgency as it does to-day, since culture was then the patrimony of a select class.<sup>2</sup> or because in the mother country herself they either knew no better how to deal with the subject, or if they had at one time known, the decadence of education had greatly reduced this knowledge. Failure was never due, however, to lack of interest in offering to the colonies all that Spain herself possessed of culture and of education.3

When the Spanish governor failed to observe the general rules of the original policy in reference to government and instruction in the colonies, curtailing the rights of the creoles to hold public offices and reducing their opportunities of seeking prosperity through the liberal professions<sup>4</sup> because

<sup>&</sup>lt;sup>1</sup> For references on this subject, see the references quoted in the preceding foot-note.

<sup>&</sup>lt;sup>2</sup> Concerning the aristocratic and narrow field of education, one may consult the author's "Historia de España y de la Civilización española," Vol. III, sec. 745.

<sup>&</sup>lt;sup>8</sup> "Historia de España y de la Civilización española," Vol. III, sec. 774; Vol. IV, sec. 837.

<sup>&</sup>lt;sup>4</sup> In this respect one recalls the typical case of Don José Perfecto Salas (eighteenth century). "Historia de España y de la Civilización española," Vol. IV.

he distrusted the use to which they might turn those advantages, the situation changed and the conflict with these descendants of the Spaniards themselves, not with the native Americans, declared itself. This conflict, for the causes indicated above and for many others extremely complex, was at its bitterest during the nineteenth century with respect to those colonies which remained in the possession of Spain until the close of that century. This change, which was so late in appearing, has, nevertheless, not been thoroughly studied either in its scope or causes, and consequently it is impossible ever to estimate, with any degree of exactness, its historical importance and bearing upon the problem of this paper.

Finally, the study of Spain as a colonizing power would be incomplete, from the point of view from which we are now considering our question, without a realization of the discoveries and contributions drawn from the opportunities afforded by her colonies and added by Spain to the general fund of the world's culture. The services rendered in this respect by her geographers, cosmographers, naturalists, philologists, navigators, etc., make a considerable item which justice demands that we place to the credit of Spain in the general work of civilization -- that is, in the list of contributions which each people owes this work in proportion to the resources with which its history shows it to have been endowed. The just consideration of this point must wait, as does all that precedes it, until historical investigation has ascertained the number, quality, and significance of the facts relating to it.

Let us now return to the general question from which this digression, or rather this practical application, has led us and which most concerns us since it relates to the fundamental structure and scientific purpose of these lessons; in other

words, let us return to our study of the ways in which civilization is communicated or initiated or encouraged among peoples which either fail to possess it at all or possess it in a tentative and elementary state. Without discussing again all the points which we have examined, let us accept the law, just as history past and present shows it, that the peoples superior in culture, wealth and power, and animated by the desire to extend their influence over the world, always intervene in the affairs of other nations which they consider inferior. This interference, however, is undertaken under the pretext or with the sincere intention of aiding a more backward people toward progress through the infusion and transplantation of all the means of culture and of comfort, of the methods and standards of conduct which had aided the intervening nation in becoming a principal factor in all the history of the world during the epoch of its greatest And let us imagine the most favorable case namely, that in which compulsion is limited to the indispensable (a case in which force is used simply to bring the nation under tutelage to submit patiently to the educative action in all its branches), and where this compulsion is actuated solely by purposes of kindness, cooperation and aid. Even then a new problem of unquestionable importance would arise because it concerns the future civilization of the world. This problem is that of the relation which the distinctive characteristics of the educating and educated nations should bear to each other, not so much in the field of politics as in the more fundamental and important field of the culture and philosophy which each nation represents.

The problem is neither useless nor purely hypothetical. On the contrary, it deals with a very common reality which repeats in ethical relations that which constantly appears in the relations of individuals, especially where these relations

enter the field of education. In all grades of instruction there are educators who understand their function as simply one of causing absorption. This interpretation of their duty is sometimes due to a sincere pedagogical opinion, sometimes to a vanity which considers its own culture ultimate perfection and for that reason worth imposing upon others and repeating without the slightest variation or amendment. Such instructors consider that they have faithfully performed their task if they have reduced to the same pattern the minds and characters of their pupils, giving them a single model and smothering in them all manifestations of originality and individuality in order that no one shall either mar or improve the picture. In this same way there exist "absorbing" peoples who understand their duty toward civilization not in the sense of an obligation to arouse and stimulate the free spirit of others, so that through original and unhampered impulse they may attain, in their own way, the highest ends of human endeavor, but in the sense of imposing upon others their particular conception of life and manner of complying with its demands; thus replacing with their own spirit that of the nation they desire to advance—that is, practically crushing this nation out of existence by destroying its national spirit and replacing it with that of the educator. Historical accuracy compels us to admit that not merely some but the majority of colonizing and civilizing nations proceed in exactly this manner. We must also admit that those who have entered foreign territory with the frank desire for conquest have been more justified in so proceeding. This impulse of absorption, this lack of consideration for the mentality and character of other human groups, sometimes results from the instinctive and irrepressible force of the civilizing spirit, which, endowed with overabundance of strength, wherever it appears destroys everything less power-

ful, even without the deliberate purpose of so doing; at other times it emanates from the excessive and inflated estimation which a nation holds concerning its own accomplishment. and from the corresponding contempt which it entertains for the accomplishment of others, in which, indeed, it perceives only those things which call for reform or abolition. In any case, however, the spirit of absorption springs from a lack of sociological and educative orientation caused by ignorance, or at least by the lack of a realization, so complete that it is formulated and applied in a line of conduct, that education produces nothing of worth while it is limited to transferring from one mind to another formulæ and bits of second-hand knowledge, as one pours water from one vessel into another, but is only productive of results when the pupil's own intelligence is stimulated by examples, by suggestions, and by the assistance of his own judgment, which has been encouraged to attain a higher degree of ability to comprehend life and the manners of satisfying humanity's needs, both material and spiritual.

It is interesting to note that this neglect or faulty comprehension of the educational duties of one people toward another has been increasing and growing more prevalent as civilization has advanced. The enormous difference between the civilization of the Greeks and Romans and the primitive, barbarous state of the other European nations which they colonized and ruled explains, on the one hand, the contempt of the former for their colonies, and, on the other, the admiration which the inferior nations felt for the superior, and their eagerness to assimilate the higher culture of the latter. But we must also notice that the Greeks and Romans (we restrict ourselves to the history of European civilization) deliberately refrained from attempting to surpass or restrain any characteristic manifestation on the part of the nations

which they colonized and dominated, except, of course, as these manifestations might relate to politics and government. because this would have concerned their sovereignty. the rest (religion, mode of living, private and even, in part, public law-all those things in which the distinctive characteristics of a people are most clearly shown) they had the greatest respect, or, one might say, since "respect" does not exactly convey my meaning, the greatest indifference. virtue of this indifference each people was enabled to preserve and perpetuate these important institutions in their original form and purpose. Rome had to attain the height of her power in order that Romanization as an absorbing force (certainly not repugnant to those subjected to it) might extend to matters originally left untouched, but in which, as a matter of fact, the dominated peoples possessed little that was definitely opposed to the innovation of the conquerors. Only religion was exempt from this uniformity (and perhaps also a part of customary law), although this freedom was without great advantage to those nations whose religion was really less advanced than the Roman paganism, and, more particularly, than the philosophy which was gradually replacing this paganism.

Christianity changed the aspect of affairs by transferring the process of absorption to the religious side of the question. The Germanic peoples, Romanized more or less thoroughly and rapidly and upholding in the field of law the principle that each nation should possess a code suited to its own peculiar conditions and demands, represent only as regards religion the uncompromising uniformist attitude of mind which, notwithstanding the indifference of the Mussulmans in the majority of cases and the spirit of practical compromise which some Christian nations maintained toward them and toward the Jews for many centuries, was

imposed from the twelfth century, and which grew constantly more bitter and severe until the early part of the present age. In other things, however, the conquerors and the colonizers returned to the practice of the Greeks and Romans, and did not insist upon the suppression of the customs and manners peculiar to the inferior peoples as long as these did not infringe upon the question of religion and, as goes without saying, upon the matter of their own sovereignty. Either they left their subjects in freedom upon all other subjects (without this neglect in any way preventing the realization in history of that spontaneous assimilation of superior culture which penetrated everywhere, and which, through imitation, communicated to the inferior race that part of itself which they were capable of adopting), or they made them their legal equals, placing within their reach, as they did in Spain, all the means of culture and progress which the mother country possessed. It must be observed, too, that all this was worked out with peoples in a very primitive state of civilization both socially and intellectually. or even in a state of manifest barbarity.

But to-day the doctrine has taken a new turn, and it is applied in dealing with all classes of peoples. The endeavor of those who uphold it would be to eradicate from within the limits of their political dominion every type of civilization and manner of living which differs from their own, and to replace them with a new expression of their own doctrine of intransigency, which, if it spares religion, affects other phases of life as essential and characteristic, and which is, after all, no more than an expression either of colossal vanity or of inconceivable short-sightedness with regard to the way humanity has progressed and can still continue to progress. The effective mode of progress which, in obedience to a psychological law stronger than human will, the

peoples of all ages have followed, working together for the perfection of civilization as a whole, in spite of humanity's tendency toward jealous anger and the formation of distinct and self-sufficient groups, is not one in which a single philosophy of life and manner of giving expression to mental and spiritual qualities forces into one mold, with deplorable monotony and unjustifiable tyranny, the various activities of peoples; rather is it one in which éach people develops its own culture to the highest point, extracting from each mental trait and quality all that it offers of essential and valuable in order thus to enrich the complex whole of life with customs varied and distinctive (in so much as they are unique and represent the peculiar aptitudes of each people). proceed in any other way-that is, against this principle of consideration of complete and unhampered cultivation of the individuality of each people—is to impoverish civilization. There exist, without doubt, examples of the above-mentioned mode of progress, notably in industrial applications of the great scientific principles—that is to say, applications of our knowledge of natural forces and their laws which, through their very generality, are applicable to all and which all are equally free to use. This also is the case with universal, humanity-wide principles of education and moral conduct. But, on the other hand, there are many qualities of the spirit, or appertaining to it, which fail to develop in all peoples or in all individuals. Each one has been or is master or master artisan in one or various lines of progress, and his accomplishment is offered, in the course of centuries, as a model and spur to others who would not know how to surpass it, and who need, from time to time, to stimulate their energies by contact with an achievement which through its very nature has attained the highest degree of perfection of which humanity is capable. Each particular "civilization" of those

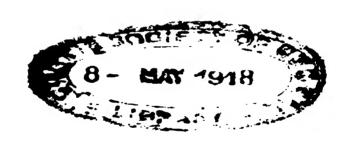
which arrive at productive maturity has contributed its characteristic item. This contribution is the outcome of the coalescence of the most fundamental, most distinctive qualities of the people or the peoples which produced it, and it will always endure as a model for the later civilizations, which, influenced by their own idiosyncrasies, may advance along quite different lines. In this manner civilization has continued to progress through the assimilation of the multitudinous factors which compose it to-day. Each of these factors has had, or has, its most perfect and characteristic form in the achievement of a single people, for the qualities and aptitudes which are called forth by human needs, from the most elementary to the highest, have not been and never will be united in one spirit, national or individual, but distributed among many.

This being the case, what would civilization gain if even one of these contributing factors were destroyed? And what more would she gain if all but one were destroyed in order that this one might dominate the world, subjecting it to a uniformity which would carry with it unforeseen limitations? No one is great or perfect in everything, but only in a very small portion of those things which his life demands of him if he is to be worthy and humane. What will he do, then, without the collaboration of those who can supply the notes which the lyre of his own spirit lacks, or from which, even if they are beneath his fingers, he is unable to produce as deep and full a vibration as is he for whom these same notes sound as spontaneously as the laughter of a child or the song of a happy man? Our human egoism lies in the very fact that we do not lack any collaboration in the task of bringing together the richest variety of essential notes. to accomplish this we must realize, in the first place, the value of them all. We must make each nation, each people,

understand its unavoidable duty and grave responsibility toward the cultivation and perfection of its own distinctive note in the great harmony of civilization. In other words, each people must learn not to flee from the task set before it, nor to fail in that assistance which other people expect from it. It is also necessary to establish a continuous and systematic spiritual communion among nations in order that they may understand and mutually aid each other, that each one may learn from the rest the lessons they are best fitted to teach, and that in this way the work of national civilization may be converted into a truly human work in which all groups and all individuals may coöperate, each contributing the best and most valuable part of its culture, and each bearing always in mind the way in which his contribution will most benefit others.

Only in this manner should civilization spread, perfecting and enriching itself,—civilization, with the present and future of which we are rightly concerned, and the laws of which historians and sociologists do not investigate from mere curiosity alone, but rather in order that their knowledge of these laws may enlighten and guide mankind in all its present and its future actions.

RAFAEL ALTAMIRA.





- Emice Bores

# MOLECULAR THEORIES AND MATHEMATICS

# AGGREGATES OF ZERO MEASURE

MONOGENIC UNIFORM NON-ANALYTIC FUNCTIONS—THE THEORIES OF CAUCHY, WEIERSTRASS, AND RIEMANN<sup>1</sup>

#### First Lecture

#### MOLECULAR THEORIES AND MATHEMATICS 2

I OW could I fail to call up the memory of the illustrious scientist for whose death, so cruelly premature, France and the whole world are mourning? When Henri Poincaré was invited by President Edgar Odell Lovett to deliver an address at this scientific celebration, his acceptance was conditional on the state of his health. A few months later, he finally declined the invitation, promising, however, to send his lecture in writing. I cannot remember without emotion the last conversation I had with him on that subject. I was still hoping that his decision was not final; but, after giving me some friendly advice about my lectures and the journey, he told me with what deep regret he had to give up the thought of ever visiting the United States again, and I felt, for the first time, how serious was the condition which justified his refusal. A few weeks afterward he was gone. In spite of the difficulties of such a task, I should have considered it a pious duty to devote this address to an appreciation of his work; no subject could be more suitable, in this Institute consecrated to science, than the life and works of this noble champion of disinterested research; but my eminent friend Mr. Vito Volterra had, as you know, formed

<sup>&</sup>lt;sup>1</sup> Three lectures presented at the inauguration of the Rice Institute, by Emile Borel, Professor of the Theory of Functions in the University of Paris.

<sup>2</sup> Translated from the French by Professor Albert Léon Guérard of the Rice Institute.

the same plan; and no one among you will regret that I resigned to him the privilege of carrying it out.

THE relations between the mathematical sciences and the physical sciences are as old as these sciences themselves; it is the study of natural phenomena which led man to set for himself the first problems, out of which, by means of abstraction and generalization, the sciences of numbers and of space have grown in all their splendid complexity. Conversely, through a sort of preëstablished harmony, certain mathematical theories, after being developed apparently far from the real, were often found to provide the key to phenomena which the creators of these theories did not have in mind. The most famous instance in point is the theory of conic sections, an object of pure speculation among the Greek geometers, but whose researches enabled Kepler, twenty centuries later, to formulate with precision the laws of the motions of the planets. In the same way, in the first half of the nineteenth century, it was the theory of imaginary exponentials which made it possible to go deeper into the study of vibratory motions, which was found to be of such commanding importance in physics and even in the field of industry; it is to this study that we owe wireless telegraphy and the transmission of energy by polyphase currents. More recently still, we know how useful the abstract theory of groups proved to be for the study of the ideas, so profound and so new, which have been put forward to explain the results of the capital experiments on relativity made by your illustrious compatriot Michelson.

But these illustrations, however important they may be, are special and relate to particular theories. How much

more striking is the universal adoption of the forms imposed on scientific thought by the genius of Descartes, Newton, Leibnitz! The use of rectangular coördinates and of the elements of differential and integral calculus has become so familiar to us that we might be tempted at times to forget that these admirable instruments date only from the seventeenth century, and in the same way the theory of partial differential equations dates only from the eighteenth century: it was in 1767 that d'Alembert obtained the general integral of the equation of vibrating chords. It was the study of physical phenomena which suggested the notions of continuity, derivative, integral, differential equation, vector, and the calculus of vectors, and these notions, by a just return, have become part of the scientific equipment necessary to every physicist: it is through them that he interprets the results of his experiments. There is evidently nothing mysterious in the fact that mathematical theories constructed on the model of certain phenomena should have been capable of being developed and of providing a model for other phenomena; this fact, however, deserves to hold our attention, for it implies an important practical consequence; if new physical phenomena suggest new mathematical models, mathematicians will have to study these new models and their generalizations, with the legitimate hope that the new mathematical theories thus evolved will prove fruitful in their turn in providing the physicists with useful forms of thought. In other words, to the evolution of physics there should correspond an evolution of mathematics which, without giving up the study of classical and well established theories, should develop in taking into account the results of experience. It is in this order of ideas that I should like to examine to-day the influence which molecular theories may have on the development of mathematics.

H

IT was in the hypothesis of the continuity of matter that, at the end of the eighteenth century and in the first half of the nineteenth, what may be termed classical mathematical physics was created; one may take as types of the theories thus constructed hydrodynamics and elasticity. In hydrodynamics every liquid was considered by definition as homogeneous and isotropic; it was not quite the same in the study of the elasticity of solid bodies: the theory of crystalline forms had led physicists to admit the existence of a periodic network—that is to say, of a discontinuous structure; but the period of the network was supposed to be extremely small compared with the elements of matter physically considered as differential elements: the crystalline structure therefore led only to anisotropy, but not to discontinuity; the partial differential equations of elasticity as well as those of hydrodynamics imply that the medium studied is continuous.

Yet the atomic hypothesis, the tradition of which goes back to the Greek philosophers, was not abandoned; apart from the confirmation which it found in the properties of gases and in the laws of chemistry, it was by means of that hypothesis that certain phenomena, such as the compressibility of liquids or the permeability of solids, had to be explained, in spite of the apparent continuity of these two states of matter; but this hypothesis was placed in juxtaposition with the physical theories based on continuity: it did not affect them. The rapid advances in thermodynamics and in the theories of energy contributed to maintain this sort of impenetrable partition between the physical theories and the hypothesis of the existence of atoms, however fruitful this might prove to be in chemistry. For most of the physicists of half a century ago the problem of the reality of atoms

was a metaphysical question, in the original acceptance of the term, a question beyond the domain of physics; it mattered little to science whether atoms existed or were simple fictions, and one might even doubt whether there was any sense in affirming or denying their existence. However, thanks especially to the labors of Maxwell and of Boltzmann, the explicit introduction of molecules into the theory of gases and solutions was proving its fruitfulness; and Gibbs created the new study to which he gave the name Statistical Machanics. But it is only within the last twenty years that all physicists have been compelled, by the study of new radiations on the one hand, and by the study of the Brownian movement on the other, to consider the molecular hypothesis as indispensable to natural philosophy. And more recently a more thorough study of the laws of radiation has led to the unexpected hypothesis of the discontinuity of energy, or of motion. It does not come within my subject to expound the experimental proofs which make these hypotheses seem more and more probable every day; the most striking experiments are perhaps those which have made it possible to observe the individual emissions of the a particles, so that we are actually able to apprehend one of the concrete units with which the physicist builds up the sensible universe, just as the abstract universe of mathematics can be built up by means of an abstract unit.

In order definitely to formulate their hypotheses and to deduce therefrom consequences that can be experimentally verified, the theorists of modern physics make use of mathematical symbols; these symbols are those which were created on the basis of the notion of continuity; no wonder, therefore, if difficulties sometimes appear, the most recent of which is the contradiction, at least in appearance, between the hypothesis of the quanta and the older hypothesis that

phenomena are governed by differential equations. But these difficulties of principle do not prevent the success of what may be called partial theories, by which a certain number of experimental results, in spite of their apparent diversity, can be deduced from a small number of formulæ which are coherent among themselves; thus, for many of the phenomena of physical optics, the formulæ are the same in the mechanical theory of Fresnel and in the electromagnetic theory of Maxwell; in the same way, the formulæ used by electrical engineers are independent of the diversity of theories concerning the nature of the current.

If I have made it a point to call your attention to this use of the mathematical instrument as an auxiliary to the partial physical theories, although it does not lie within my subject, it is in order to prevent any misunderstanding: it seems to me beyond doubt that for a long time to come-perhaps as long as human science itself shall endure—it will be under this comparatively modest form that mathematics will prove of greatest use to physicists. This is no reason why we should take no interest in the general mathematical theories for which physics has provided the models, whether we have to deal with speculations on partial differential equations suggested by the physics of the continuum, or with statistical speculations pertaining to the physics of the discontinuum; but it should be clearly understood that the new mathematical theories which may be suggested by the discontinuity of physical phenomena cannot have the pretension of entirely replacing classical mathematics; these are only new aspects, for which it is proper to make room by the side of the older views, so as to increase as much as possible the richness of the abstract world, wherein we seek for models which will make us understand concrete phenomena better and foresee them more accurately.

Ш

It is frequently a simplification in mathematics to replace a very large finite number by infinity. Thus the calculus of definite integrals is frequently more simple than that of summation formulæ, and the differential calculus is usually simpler than that of finite differences. In the same way, we have been led to replace the simultaneous study of a very large number of functions of one variable by the study of a continuous infinitude of functions of one variable; that is to say, by the study of a function of two variables. By a bolder generalization, Professor Vito Volterra has been led to define functions which depend on other functions—that is to say, in the simplest case, functions of lines, in considering them as the limiting cases of functions which would depend on a great number of variables, or, if one prefers, on a very great number of points of the line.

These various generalizations have rapidly secured permanent recognition in mathematical physics; the use of integral equations, the classical types of which are the equation of Volterra and the equation of Fredholm, has become current. Although these theories are well known to all, it may not be idle to recall their origin by means of a particularly simple example; we shall thus better understand their significance from our present point of view.

Let us consider a system composed of a finite number of material points, each of which can deviate only by a small amount from a certain position of stable equilibrium. The differential equations which determine the variations of these deviations from their position of equilibrium may be considered, under certain hypotheses and to a first approximation, as linear in respect to these deviations. If, moreover,

we introduce the hypothesis that the system conforms to the law of the conservation of energy, the differential equations assume a very simple and classical form, from which the fact can easily be deduced that the motion may be considered as the superposition of a certain number of periodic motions. The number of these elementary periodic motions is equal to the number of degrees of freedom; it is three times the number of the material points, if each of these points can be arbitrarily displaced in the neighborhood of its position of equilibrium. The periods of the simple periodic motions are the specific constants of the system, which depend only on its configuration and the hypotheses made concerning the forces brought into action by its deformation, but which do not depend on the initial conditions: positions and velocities. These initial conditions determine the arbitrary constants which figure in the general integral and which are two in number for each period: the intensity and the phase.

Now let us suppose that the number of material points becomes very large, and let us identify each of them with a molecule of a solid body—a bar of steel, for instance; if our hypotheses are still verified—and this is admitted in the theory of elasticity—their consequences also will remain true; we shall then have a very large number of characteristic constants, each of these constants defining a proper period of the system. Let us increase to infinity the number of molecules; the system of differential equations, infinitely great in number, is then replaced by a finite number of partial differential equations, whose fundamental properties are obtained by passing to the limit. In particular, the proper periods can be determined, and this remarkable result is established—that these periods can be calculated with precision and without ambiguity if we take the precaution of defining them by commencing with the longest; there is only

a finite number of periods superior to a given interval, but this number increases indefinitely when the interval tends toward zero.

The reasoning which has just been outlined is the type of those to which the substitution of continuity for discontinuity leads; in reality, the considerations based on the existence of molecules play but an auxiliary part in them; they put us on the track of the solution, but this solution, once arrived at, satisfies rigorously the partial differential equations of Lame, equations which can be deduced just as well from theories of energy as from molecular hypotheses. The molecular theory has therefore been a valuable guide for the analyst in suggesting to him the course to be followed in studying the equations of the problem, but it is climinated from the final solution. On the other hand, we know that this solution is but an imperfect representation of reality; we obtain an infinitude of proper periods, instead of a very great number of them; the actual number, however, is so great that we ought not, perhaps, to feel any scruple in passing to the limit and considering it as practically infinite. If, however, one bears in mind that the difficulties of the theory of black radiation arise precisely from the very short periods, and that these difficulties are not yet solved in an entirely satisfactory manner, one will perhaps come to the conclusion that one could not be too careful about anything which relates to these very short periods. This is probably the reason why such a physicist as Lorentz has thought that the considerable analytical efforts required by the study of the propagation of waves, when molecules are explicitly introduced into it, were not superfluous. However this may be, even if the substitution of the infinite for the finite is entirely legitimate in certain problems, it may be interesting to propose to one's self, from a purely mathematical point of view, the direct

study of functions or equations depending upon a great but finite number of variables.

ΙV

THE first difficulty which presents itself, when one wishes to study functions of a very great number of variables, is the exact definition of such a function—I mean its individual definition-making it possible to distinguish the function thus defined from the infinitude of other analogous functions. It is true that there exist general properties common to all the mathematical entities of a certain category, independent of the numerical value of the coefficients; for instance, every definite quadratic form (that is to say, one always positive) is equal to the sum of the squares of as many independent linear functions as the number of the variables which it contains. One has at times attempted to deduce physical consequences from mathematical facts of that kind: I must confess that I cannot help being skeptical about this sort of reasoning; it may seem rather strange that one should be able to deduce anything exact from such a general notion as that of a surface of the second degree (let us say, for fixing ideas, a generalized ellipsoid) in a space having a very great number of dimensions. Let us insist a little on the difficulty there is in knowing such an ellipsoid individually: its equation may be supposed to be reduced to a sum of squares by an orthogonal substitution—that is to say, the axes remaining rectangular. Such an ellipsoid then requires, for its complete definition, the knowledge of what we may call the squares of the lengths of its axes—that is to say, the knowledge of as many positive numbers as the space considered has dimensions. The question of knowing whether one can consider as given so many numbers, when a man's lifetime would not suffice to enumerate a small part of them, is

a question which is not without analogy with that of the legitimacy of certain reasonings of the theory of ensembles, such as the one by which Professor Zermelo pretends to prove that the continuum can be well ordered, and which supposes to be realized an infinitude of choices independent of any law, and yet uniquely determined. Opinions may differ on the theoretical solution of these difficulties, and this is not the moment to reopen this controversy; but from the practical point of view, the answer is not doubtful: it is not possible effectively to write the numerical equation of an ellipsoid whose axes are as numerous as the molecules constituting a gram of hydrogen.

In what sense then is it possible to speak of a numerically determined ellipsoid possessing a very large number of dimensions? From an abstract point of view, the simplest method for defining such an ellipsoid consists in supposing that the lengths of the axes are equal to the values of a certain function which is simple for the integral values of the variable; one may suppose them to be all equal (in which case one will say that the ellipsoid is reduced to a sphere); one may also suppose that their values are the successive integral numbers in their natural sequence, either starting from number one or from any other given number; or that they are equal to the inverses of the squares of these integers, etc. In other words, we suppose that the lengths of the axes are all determined by the knowledge of a formula simple enough to be actually written, whereas it is not possible actually to write as many distinct numbers as there are axes.

Another method, to which we are naturally led by the analogies with the kinetic theory of gases, consists in supposing that the values of a function of the axes, such as the square of the lengths of the axes, or of their inverses, etc.,

are not individually given, but that we know only the mean value of this function, and the law of the distribution of the other values around this mean. We propose, under these conditions, not to study the property of a unique and well defined ellipsoid, but only the most probable properties of the ellipsoid, knowing only that it satisfies the required conditions; we can also say that we study the mean properties of the ensemble of the ellipsoids defined by these conditions. Here again we may observe that the probable ellipsoid or the mean ellipsoid is completely defined by the knowledge of the mean value of the law of deviations. If this law is the classic law of probabilities, it includes only two constants; if we were led to introduce a more complicated law, this law might in all cases be explicitly written. The two processes that we have indicated are therefore equivalent from the analytical point of view; it would evidently be the same with all other processes that could be imagined, and in particular with the combinations of these two.

In a word, a figure which depends on an extremely great number of parameters can be considered as numerically determinate only if these parameters are defined by means of numerical data sufficiently few in number to be accessible to us. It is for this reason that the study of the geometrical figures in a space possessing an extremely great number of dimensions can lead to general laws if we can exclude from this study such of these figures as, humanly speaking, cannot possibly be defined individually.

Here are, for example, some of the results to which the study of ellipsoids leads us. In working the equation in the form of a sum of squares, the second member being reduced to unity, the coefficients are equal to the reciprocals of the squares of the axes. If the mean of the squares of these coefficients is of the same order of magnitude as the square

of their mean, one will say that the ellipsoid is not very irregular. The modes of definition concerning which we have just spoken lead to ellipsoids which are not very irregular, since one does not systematically introduce into those definitions functions purposely chosen in a complicated manner. On the contrary, one gets a very irregular ellipsoid in equating to a constant the vis viva of a deformable system composed of a very great number of molecules, this vis viva being written under the classic form of the sum of the zis viva of translation of the total mass concentrated at the center of gravity, increased by the sum of the vires vivæ of the molecules in their motion relative to this center of gravity. The great irregularity comes from the fact that the products of the total mass by the three components of the velocity of the center of gravity are extremely great in comparison with the other terms. When an ellipsoid is not very irregular, several of its properties make it possible to assimilate it to a sphere, which may be called the median sphere; the surface of the ellipsoid is almost wholly comprised between the surfaces of two spheres very close to the median sphere; on the other hand, if a point is arbitrarily chosen on the ellipsoid, it is infinitely probable that the normal at this point passes extremely close to the center.

This geometrical study of figures with a very large number of dimensions deserves, I believe, to be thoroughly investigated; it brings out the abstract basis of the theories of statistical mechanics and physics—that is to say, it enables us to distinguish, among the propositions to which physicists are led, those which are a consequence of physical hypotheses from those which are derived only from statistical hypotheses. But, apart from its physical usefulness, this geometrical study of spaces having a very great number of dimensions offers an interest of its own; it is to the

molecular theories that we are indebted for this new branch of mathematics.

V

We can, however, ask ourselves whether it is legitimate to consider as bound up with the molecular hypothesis a theory which, after all, should depend exclusively on a small number of constants. To say that an ellipsoid with a great number of dimensions is entirely defined by five or six constants, amounts to saying that all the consequences which we shall deduce from its study can be expressed by means of these five or six constants. Can we not suppose, then, that an analytical mechanism could be devised, enabling us to arrive at these same consequences, expressed by means of the five or six constants, without its being necessary to bring in the equation with a very great number of terms—that is to say, without its being necessary to make use of the molecular hypothesis.

This objection deserves careful consideration, although it reminds us of the controversy between the energetists and the atomists, a controversy in which the advantage seems decidedly to have been on the side of the atomists. It may be answered, in the first place, with an argument of fact: it matters little that we might conceive the possibility, without making use of molecular hypotheses, of combining among themselves the consequences of these hypotheses; the important point is to know whether this possibility is realized at present, or if, on the contrary, the calculations based upon molecular hypotheses are the simplest, if not the only, mode of deduction. If the latter alternative be correct, and it seems difficult to deny that it is, molecular hypotheses are therefore at present very necessary indeed, and that alone ought to be of consequence to us.

Under this modest form, which leaves room for future contingencies, this reply seems peremptory; but I believe that many physicists would think it is not categorical enough. It must be noted, however, that the question is independent of the experimental proofs of the reality of molecules. Even if we should succeed in seeing, by means of an instrument more powerful than a microscope, the molecules of a solid body, it would not follow, however valuable this knowledge might be, that one should have to use it in order to account. in the simplest possible manner, for the properties of that body; in a similar way, the possibility of seeing an isolated microbe under the microscope is not an indispensable condition for the attenuation of the viruses and the use of vaccines; or again, in the reproduction of a masterpiece by photogravure, it is not the individual knowledge of the points constituting the negative that interests us.1

From an abstract point of view, if we admit that any human theory must be expressed, in last analysis, by means of a finite and relatively small number of data, it seems difficult to deny the possibility of entirely constituting the theory, without introducing hypotheses which imply the existence of elements more numerous than human imagination can conceive. But the recognition of this abstract possibility cannot prevail against the importance of the services rendered by molecular theories in linking together apparently unrelated phenomena; so it is permissible to consider these reserves on future possibilities as purely theoretical.

This individual knowledge of points has a part in the processes for transmitting the negative to a distance; but in this case these points, however numerous, are none the less finite in number and accessible to our observation. If we transmit by telephone an orchestral selection, we know that all the æsthetic beauties of the piece are, in last analysis, the results of certain vibrations which would require too much time to be known individually; but in fact these elementary vibrations have nothing to do with musical æsthetics: an excellent composer may be ignorant of their existence, and an excellent physicist may be a wretched musician.

Is it possible to go still further, and to do away even with this kind of reserve? In order to answer this question, we should have to examine in detail all the phenomena which are explained by means of molecular hypotheses, and to try to ascertain whether an extremely large number of parameters is indeed necessary to such explanation. Among the discontinuous phenomena whose experimental laws are well known, the most characteristic are those of spectra in series; we know that the positions of the spectral rays are determined with a very great precision by formulæ, the first and simplest of which, due to Balmer, includes the difference of the reciprocals of the squares of two integers. This is perhaps the most remarkable example of the intervention of the integer in natural law; if laws of this kind were more numerous and better known, one might possibly be led to name arithmetic and the theory of numbers among the branches of mathematics which can be connected with molecular physics. Can one, by induction, admit that the formula of Balmer is exact, not only for small integers concerning which the experimental verification is rigorous, but for many other larger integers concerning which this verification is impossible? And if such be the case, is it not one of those discontinuous phenomena whose explanation requires a very large number of parameters? It does not seem so: on the one hand, the formula with the variable integer contains in fact but a small number of constants; on the other hand, the attempts made for explaining the presence of this integer by hypotheses of physical discontinuity have led to the placing of this discontinuity within the atom itself; there is consequently no need of a very large number of atoms: one alone is sufficient, whose structure depends only on certain parameters, on magnétons in the theory of Ritz, parameters the number of which is far from being of the same order as the number of the atoms.

This remark leads us to consider another category of phenomena, to which we have already alluded, and in which the atoms or corpuscles are observed individually. Does not the explanation of these phenomena require atomic hypotheses? It seems difficult to deny it without being paradoxical. Let us note, however, that such phenomena as the emission of the a particles are susceptible only of a globate explanation; it is not possible to foresee with accuracy any particular emission, but only a mean number; scientifically speaking, therefore, this mean number alone has any existence; the phenomenon which consists in the emission of one a particle does not present the characters which permit of rigorous experimentation: one cannot either foresee it or reproduce it at will; it is only the study of the trajectory after the emission that offers these characters; and in fact this study requires only such a limited number of equations that one can write them all. The atomic hypotheses would enable us to foresce each individual emission, if one could in fact calculate with reference to an extremely great number of equations; but that is not possible, and so far as the globate prevision is concerned the atomic hypothesis is not, at least a priori, necessary.

We touch here upon the borders of science, since we reach phenomena accessible to our observation, and which depend upon causes too numerous for us ever to know them with precision in their full complexity. Science remains possible only for mean values which can be calculated with precision by means of data accessible to observation.

It is well understood, I hope, that I do not dispute the legitimacy and usefulness of molecular theories; my remarks as a mathematician cannot attain physical reality; at the bottom, they do not go farther than this: all the calculations we shall ever be able really to effect will comprise only a rather limited number of equations actually written; if we

write one equation, and if we add that we consider several billions of analogous equations, we do not, in fact, calculate these unwritten equations, but only the written equation, taking into account perhaps the number of these unwritten equations, a number which also has been written. Every mathematical theory, therefore, reduces itself to a relatively small number of equations and calculations, which involve a relatively small number of symbols and numerical constants; it is therefore not absurd a priori to suppose that one might conceive a physical model containing also a relatively small number of parameters and leading to the same equations. As long, however, as this model has not been imagined—and perhaps it will never be—the analytical or geometrical researches on functions of a very large but finite number of variables will offer some interest for the physicists.

VI

We have already observed that it is an ordinary proceeding in mathematics to replace a very large finite by an infinite. What result does this method yield when it is applied to physically discontinuous phenomena, whose complexity seems bound up with the very large number of molecules? Such, for instance, are the phenomena of the Brownian movement, which is observed when very fine particles are in suspension in an apparently quiet liquid. These phenomena fall within the category of those we were mentioning a moment ago, of which none but a statistical foreknowledge is possible.

Is it possible to construct an analytical image of such phenomena? Professor Jean Perrin<sup>1</sup> has already called attention to the fact that the trajectories observed in the Brownian

<sup>&</sup>lt;sup>1</sup> Jean Perrin, "La discontinuité de la matière," Revue du Mois, mars 1906. See also Jean Perrin, "Les Atomes," Alcan 1913.

movement suggest the notion of continuous functions possessing no derivatives; or that of continuous curves possessing no tangent. If one observes these trajectories with optical instruments of increasing perfection, one sees, at each new magnification, new details, the curvilinear arc that we could have traced being replaced by a sort of broken line the sides of which form a finite angle with each other; this remains the case up to the limit of the magnifications at present possible. If we admit that the movement is produced by the impact of molecules against the particle, we must conclude that, with a sufficient magnifying power, we should obtain the exact form of trajectory, which would present itself under the form of a broken line with rounded angles, and which would not be perceptibly modified by a still further magnification.

But the analyst is not forbidden to put off indefinitely in his thought the realization of this final state, and thus to arrive at the conception of a curve in which the sinuosities become finer and finer as one uses a higher magnification, without ever obtaining the final sinuosities: this is indeed the geometrical image of a continuous function not admitting of a derivative.

We obtain also a curve of a similar nature, sufficiently interesting to arrest our attention, when we study the function which Boltzmann designates by H and Gibbs by  $\eta$ , and which represents, in the case of a gas, the logarithm of the probability of a determinate distribution of the velocities of the molecules. Each collision between two molecules gives a sudden variation to this function, which is thus represented by a staircase curve, the horizontal projections of the steps corresponding to the intervals of time which separate two collisions, the number of the collisions undergone by a molecule being some billions per second (that is to say, of the

order of magnitude 10<sup>9</sup>), and the number of molecules of the order of magnitude 10<sup>24</sup> (if we consider a mass of a few grams of gas), the total number of collisions per second is of the order of magnitude 10<sup>33</sup>; such is the number of steps projected on a portion of the axis of the abscissæ equal to unity, if the second is taken as the unit of time.¹ What the physicists consider is the mean behavior of the curve. They replace the serrated curve by a more regular curve having the same mean behavior in the time intervals, which are very small in comparison to the second, but very great in comparison to 10<sup>-33</sup> of a second.

These diverse considerations bring interesting suggestions to the analyst, on which I should like to dwell for a moment.

In the first place, referring to the continuous curves without derivatives of which the Brownian movement has given us the image, should the passage from the finite to the infinite lead to a curve all of whose points are points of discontinuity, or to a curve which admits an infinitude of points of discontinuity, but also an infinitude of points of continuity? For a proper understanding of the question, it is necessary briefly to recall the capital distinction between denumerable infinity and continuous infinity. An infinite ensemble is said to be denumerable if its terms can be numbered by means of integers. Such is the case for the ensemble composed of terms of a simple or multiple series; we can also cite as a denumerable ensemble the ensemble of the rational numbers. On the other hand, the ensemble of all the numbers comprised between o and 1, both commensurable and incommensurable. is not denumerable: we say that this ensemble has the same power as the continuum. If we define a discontinuous func-

<sup>&</sup>lt;sup>1</sup> This discontinuity supposes evidently that we consider the duration of a collision as less than the mean interval of two collisions (in the whole mass), a hypothesis difficult to admit. The schema to which this hypothesis leads is not less interesting from the analytical point of view.

tion by a series each term of which admits a point of discontinuity, the ensemble of these points of discontinuity is denumerable, as are the terms themselves. Can we determine a function which shall be totally discontinuous—that is to say, one whose points of discontinuity shall be all the points of a continuous ensemble, and not merely those of a denumerable ensemble? It would seem to be easy to imagine such a function. Such is the oft-studied function which is equal to 1 if v is commensurable and to x if x is incommensurable; this function is indeed discontinuous, as much so for the commensurable values as for the incommensurable values. we look a little closer, we perceive that the discontinuity is not the same in these points: we must note, in fact, that the commensurable numbers occupy infinitely less space in the axis of the x's than do the incommensurable numbers: the ensemble of these commensurable numbers is of dimension zero—that is to say, it can be confined within intervals whose total extent is less than any number given in advance. Speaking in more concrete terms, if we choose a number at random, the probability that it will be commensurable is equal to zero.1 We therefore conclude that the function equal to x for the incommensurable values of the variable is. on an average, continuous for these incommensurable values, whatever its values may be for the commensurable values—that is to say, if we choose in the neighborhood of an incommensurable value, for which we study the continuity, another value taken at random, it is infinitely probable that this value taken at random will also be incommensurable; it is then infinitely probable that the variation of the function will be infinitely small when the variation of the variable is small.

<sup>&</sup>lt;sup>1</sup> To give one's self a number at random, one may agree to choose at random the successive figures of the decimal fraction which is equal to it; the probability that this decimal fraction will be finite or periodic is evidently equal to zero.

This remark enables us to understand that it has not been found possible to define analytically a function all the points of which are effectively points of total discontinuity; it is only in points determined according to the definition of the function, and playing a particular part in this definition, that the function is actually discontinuous on an average.

The passing from the finite to the infinite, when we are concerned with the discontinuity of functions, is, then, not effected after the manner which is most usual in classical mathematical physics, in which matter is supposed to be continuous, and in which the finite is replaced by the continuous; we are led to conceive a different process, which seems, besides, more in harmony with the molecular conception, and which consists in replacing the very great finite by the denumerable infinite.

This is the way in which the analytical generalization of such curves as the H curves presents itself from this point of view. Let us consider a number written in the form of an interminate decimal fraction, and let us imagine that the figures which follow the decimal point are grouped in successive periods, each period containing many more figures than the preceding period. To each period we shall cause to correspond one term of a series, this term being equal to zero if in the corresponding period the ratio of the number of even figures to the number of odd figures is comprised between 0.4 and 0.6; while if this ratio is not comprised between these limits, the term corresponding to the period is equal to the term of the same order of a certain convergent series with positive terms. It is clear that, if the lengths of the successive periods increase rapidly, it is infinitely probable that a small number of periods only will furnish terms different from zero; consequently, the series which corresponds to the decimal number will be terminate; this termi-

nate series has a certain sum, which remains the same as long as the decimal number varies so little that the last one of the periods which gave a term to the series is not modified; at least in the interval thus defined it is extremely probable that the function corresponding to the decimal number preserves this constant and well determined value—that is to say, is represented by a horizontal line; however, there are in this interval, as in every interval, particular decimal numbers for which certain periods of high order, perhaps even an infinitude of such periods, are irregular from the point of view of the distribution of even and odd figures; there are then intervals which are extremely small, and, on an average, extremely rare, but nevertheless dense everywhere, in which the curve runs up above the horizontal line which in general represents it. In one of these points, which we may call maxima of the curve, it is extremely probable that, if we take a value in the neighborhood of the variable at random, the function will diminish—that is to say, that this point has, on an average, the character of a maximum in a point.

In the preceding example the maxima are represented by intervals narrower and narrower, but finite; in modifying slightly the definition, one can obtain a curve which would coincide everywhere with the axis of x, except in points not filling any interval; it is sufficient to agree that, in the series which we have just defined, we replace by zero every term which is followed by an infinitude of terms equal to zero; the new series can then be different from zero only if the terms of the first series are all, after a certain place, different from zero.

The study of analytical models thus obtained leads us to go deeper into the theory of functions of real variables, and even to conceive new notions such as the notion of average derivative, naturally suggested by the physical example of

the function H.<sup>1</sup> Besides, it is necessary to observe that in the study of these functions the notion of continuous ensemble is often combined with the notion of denumerable ensemble; for example, it is easy to see that the ensemble of decimal numbers whose figures are all odd presents certain characters of the ensemble of all the decimal numbers; it has, as we say, the same power as the continuum,<sup>2</sup> but it is, however, of zero dimension.

We may also connect with these considerations the theory of denumerable probabilities—that is to say, the study of probabilities in the case in which either the infinitude of trials or the infinitude of possible cases is denumerable—a study lying between the study of probabilities in the finite cases and the study of continuous probabilities.

#### VII

In spite of the interest of problems relating to functions of a real variable, it is the theory of functions of a complex variable which, since the immortal discoveries of Cauchy, is really the center of analysis. The analogy between the theory of the functions which Cauchy has called monogenic functions and which are often called analytical functions, and the theory of Laplace's equation which is verified by potentials, is undoubtedly one of the most fruitful analogies in analysis. We know all the advantage Riemann has derived from the theory of potential and from physical intuition in his profound researches upon the functions of a complex variable.

<sup>&</sup>lt;sup>1</sup> Emile Borel, "Comptes Rendus de l'Académie des Sciences de Paris," 29 avril 1912.

If in a decimal number all of whose figures are odd we replace the respective figures 1, 3, 5, 7, 9 by the figures 0, 2, 3, 4, we may consider that number as any number whatever written in the system whose base is 5.

It is therefore natural to ask one's self what new ideas can be brought by molecular theories into this domain of complex variables. Here again we shall be led to replace the very large finite number by the denumerable infinity: it is easy to form series each term of which presents a singular point, the ensemble of the terms of the series thus possessing a denumerable infinitude of singular points. These singular points may, for instance, be so chosen that they coincide with all such points among the points inside of a square whose two coördinates are rational. The most simple series that we can thus form presents itself under the form of the sum of a series of fractions each of which admits of only one pole, which is a simple pole. The physical interpretation, in the domain of reality, of such a series leads us to consider the potential of a system composed of an infinitude of isolated points, the mass concentrated in each of these points being finite (which leads us to admit that the density in each such point is infinite, if the point is considered abstractly as a simple geometrical point without dimensions). We suppose, of course, that the series whose terms denote the values of the masses is convergent, which amounts to saving that the total mass is finite, although concentrated in an infinitude of distinct points—for example, in all the points whose two coördinates are rational numbers.

The potential with which we are now concerned is in the case of a plane what we call a logarithmic potential; we could reason similarly in three-dimensional space: we should then have the Newtonian potential properly so called.

The hypothesis that the attracting masses are simple material points without dimensions is difficult to accept from the physical point of view; one is thus led to perform the analytical operation which consists in dispersing this mass into a small circle (or a small sphere) having this point for

center, without changing the potential outside of this circle (or sphere); we shall call this circle (or sphere) the "sphere of action" of the point which coincides with its center; we shall choose its radius to be proportional to the mass concentrated at its center, so that, if the series formed by the masses converges with sufficient rapidity, we may arrange things in such a manner that the radii of the spheres of action also form a very rapidly converging series, and yet that the maximum density of the attracting mass be finite. It is also easy, if we admit that we can dispose arbitrarily of the distribution of masses and densities, to arrange things in such a way that the distribution in each sphere of action, as well as its derivatives, is reduced to zero over the whole surface of the sphere; the distribution of the density is thus not merely finite, but continuous throughout space.

The hypothesis which we have made concerning the convergence of the series the terms of which are the radii of the spheres of action, implies the convergence of the series the terms of which are the projections of these spheres on any straight line whatever; if, therefore, in this series, we suppress a certain number of the first term, the rest of the series can be made less than any number fixed in advance. From this we conclude that in an interval, however small it may be, taken on the straight line on which we project the spheres. we can find an infinite number of points which belong at the most to a finite number of such projections—namely, those belonging to the spheres S which correspond to the first terms of the series, and which were suppressed in the series in order to make the remainder less than the interval considered. If we consider a plane perpendicular to the straight line and passing through one of these points (this point being chosen, as is possible, distinct from the projections of the

centers of the spheres S, finite in number, concerning which we have just spoken), this plane will at most intersect a finite number of spheres S, without going through their centers, but will be exterior to all the other spheres of action. It is possible to modify the distribution of matter within the spheres S which are finite in number and intersected by the plane in such a manner as to replace these spheres by smaller ones which do not intersect the plane, this operation not modifying the potential outside of the spheres, and the density remaining finite, since the operation relates to only a limited number of spheres. To sum up, it is possible to find a plane perpendicular to any straight line whatever, cutting out of this line any segment whatever given in advance, and such that in all the points of this plane the density shall be zero. Since our potential function is defined by a density everywhere finite and continuous, this potential satisfies the equation of Poisson, which reduces itself to the equation of Laplace wherever the density is zero—that is to say, in all the points of the planes which we have just defined. It was not idle to insist upon this point, for these planes may traverse regions of space in which the given material points are everywhere dense--as are, for example, all the points whose coördinates are rational numbers. We might have feared that there would be no free space between points so closely pressed together, so to speak; we have just seen that this fear was unjustified. The theorem of the theory of ensembles which is necessary and sufficient for demonstrating this result in a rigorous manner is the following: If on a segment of a straight line we have an infinite number of partial segments (in this particular case, the projections of the spheres of action) whose total length is less than the length of the segment, there exist on that segment an infinite number of points which do not pertain to any of the partial seg-

ments. This formulation is almost self-evident, and besides, it would be easy to demonstrate it rigorously.

In the case of the plane we shall replace the spheres by circles and the plane perpendicular at a point of the segment by a perpendicular straight line; we can easily prove that, even in the region where the singular points are everywhere dense, there are points at which an infinite number of such lines intersect, on which the density is zero; at these points the logarithmic potential function satisfies Laplace's equation in two variables. If we study in a similar way the function of a complex variable with poles dense in one region, we define an infinite number of straight lines of continuity, intersecting in all directions, the function admitting of derivatives which are continuous on these lines, and the derivative having the same value in all the directions in each of the points of intersection. To express this fact we shall use the word created by Cauchy for designating functions which admit of a derivative independent of the argument of the increment of the variable; these functions may be called monogenic, but they are not analytical, if we reserve for the word "analytical" the very definite meaning which it has possessed since the labors of Weierstrass.

Without lingering on the physical analogies suggested by the existence of planes which do not intersect the spheres of action of the attracting masses, I should like to insist a little upon the nature of the mathematical problems arising out of the existence of these monogenic but not analytical functions. We know that the essential property of analytical functions is that they are determinate in the whole domain of their existence, when their values are given in one portion, however small it may be, of that domain. Is that property a consequence of analyticity—that is to say, of the existence of the Taylor series with radius of convergence differing from zero

—or of monogeneity—that is to say, of the existence of the unique derivative? This question was meaningless as long as it was possible to confound analyticity with monogeneity; on the other hand, it takes a very clear signification as soon as we have succeeded in constructing non-analytical monogenic functions.

I cannot enter to-day into the detail of the deductions which have led to the solution of this problem; here is the result: it is, indeed, monogeneity which is the essential character to which the fundamental property of analytical functions is due; this fundamental property subsists for the nonanalytical monogenic functions as soon as we specify clearly the nature of the domains in which these functions are considered. I have proposed to call the domains satisfying these distinct conditions "domains of Cauchy." A domain of Cauchy is obtained by cutting off from a continuous domain domains of exclusion analogous to the spheres of action just mentioned, domains which may be infinite in number, but whose sum can be supposed to be less than any given number (just as the spheres or circles of exclusion just considered, whose radii once chosen we can multiply by any number less than unity, while we are free to increase the upper limit of the density at the same time as we decrease the radii of exclusion).

The series formed by these excluded domains should, evidently, be supposed to be convergent; moreover, we ought to suppose that its convergence is more rapid than that of a determinate series which it is not necessary to write here. Under these conditions, which refer only to the domain and not to the function, every function which in Cauchy's domain

<sup>&</sup>lt;sup>1</sup> See Emile Borel, "Définition et domaine d'existence des fonctions monogènes uniformes" (Journal of the International Congress of Mathematicians, Cambridge, England, 1912); "Les fonctions monogènes non-analytiques" (Bulletin de la Société Mathématique de France, 1912).

satisfies the fundamental equation of monogeneity possesses the essential property of the analytical function; we can calculate it throughout its domain of existence by the knowledge of its derivatives at one point (the existence of the first derivative involves the existence of all the derivatives, at least in a certain domain which forms part of the Cauchy domain), and this mode of calculation implies the consequence that, if the monogenic function be zero on an arc however small, it is zero in every point of the domain of Cauchy; two functions, therefore, cannot coincide on an arc without coinciding throughout their domain of existence, in the generalized sense.

I cannot develop the consequences of these results from the point of view of the theory of functions; but I should like, in closing, to submit to you some reflections which they suggest concerning the relations between mathematical and physical continuity.

#### VIII

Most of the equations into which we translate the physical phenomena have certain properties of continuity; the solutions vary in a continuous manner, at least during a certain interval, greater or less in length, when the given quantities vary in a continuous manner. Besides, this property is not absolutely general, and it might happen that the theories of the quanta of emission or absorption may lead us to give more importance than heretofore to exceptional cases; but to-day I do not wish to enter upon this discussion; I limit myself to the general property, verified in a large number of cases.

When we seek to interpret this property in the theory of the potential and of the monogenic functions, we should expect, if for simplification we confine ourselves to the real functions of a single variable, to find a sort of continuous

passage between such of these functions as are analytical in the Weierstrassian sense and those which are entirely discontinuous. Now, this is precisely what does not occur unless we consider non-analytical monogenic functions; as soon as a function ceases to be analytical it no longer possesses any of the essential properties of analytical functions: the discontinuity is sudden. The new monogenic functions permit one to define functions of real variables which might be called quasi-analytical and which constitute in some way a zone of transition between the classical analytical functions and the functions which are not determined by the knowledge of their derivatives in a point. This transitional zone deserves to be studied: it is oftentimes the study of hybrid forms which best teaches us about certain properties of clearly defined species.

We see that the points of contact between molecular physics and mathematics are numerous: I have only been able to point out, in a rapid manner, the most important among them. I am not competent to ask whether the physicists will be able to derive immediate advantage from these analogies: but I am convinced that mathematicians can only gain by investigating them more thoroughly. Mathematical analysis has ever been rejuvenated by contact with nature; it is only because of this permanent contact that it has been able to escape the danger of becoming a pure symbolism, revolving in a circle about itself; thanks to molecular physics, the speculations on discontinuity will assume their full significance, and will develop in a truly fruitful manner. And while it is impossible to foresee the exact applications of these researches, it is not unlikely that the mental habits they foster will not prove useless to those who desire to undertake the task, that cannot long be deferred, of creating an analysis adapted to theoretical researches in the physics of discontinuity.

#### Second Lecture

#### AGGREGATES OF ZERO MEASURE1

I

When we are given a number  $\epsilon$  arbitrarily small, we can inclose all the points of E within intervals whose sum is less than  $\epsilon$ . For an aggregate of two dimensions we have a similar definition, replacing the intervals by the rectangles. Moreover, we see that we may speak of squares instead of rectangles, because if we are given a rectangle we can find a finite number of squares of which the total area differs as little as we please from the area of the rectangle, and such that every point within the rectangle is also within one of these squares. We could also replace squares by circles without altering the generality of the definition.

Aggregates of measure zero play a very important part in the theory of functions of a real and of a complex variable. It is therefore useful to be able to compare the different aggregates of measure zero among themselves. This comparison is aided by the concept of regular aggregates. In the first place, then, we shall define regular aggregates and the fundamental points of these aggregates, and we shall show that every regular aggregate is equivalent to another regular aggregate of which the fundamental points are chosen in a special manner, for example, as the points with rational coördinates. Finally, we shall consider the classification of aggregates of measure zero, with given fundamental points. This classification will be based on the asymptotic decrease of the intervals (or squares) of exclusion.

<sup>&</sup>lt;sup>1</sup> Translated from the French by Professor Griffith Conrad Evans, of the Rice Institute.

An aggregate of measure zero is said to be regular when it can be defined in the following manner:

Let  $A_1$ ,  $A_2$ , ...,  $A_n$ , ... be a denumerable infinity of points, said to be fundamental points. To each integral number h let us make correspond an infinity of squares  $C_1^{(h)}$ ,  $C_2^{(h)}$ , ...,  $C_n^{(h)}$ , ..., of which the areas form a convergent series, such that the square  $C_n^{(h)}$  incloses in its interior  $C_n^{(h+1)}$  and approaches  $A_n$  when h increases indefinitely. Let  $E_h$  be the aggregate of points inside of the squares  $C_n^{(h)}$  (n=1,2,...). The aggregate of points contained in all the  $E_h$  is a regular aggregate (which is evidently of zero measure).

Every aggregate of zero measure can be considered as part of a regular aggregate. In other words, if A is any aggregate of measure zero, we can define a regular aggregate E of zero measure, such that every point of A belongs to E. To prove this proposition let us imagine a sequence of numbers  $\epsilon_1, \epsilon_2, \dots, \epsilon_n$ , decreasing and tending to zero, the series  $\Sigma_{\epsilon_n}$  being supposed convergent. Since the aggregate A is of measure zero, we can define an aggregate  $A^{(h)}$  of squares (with sides parallel to the axes) the sum of whose area is less than  $\epsilon_h$ , and such that every point of A is inside one of these squares  $A^{(h)}$ . We define first the squares  $A^{(1)}$ , then the squares  $A^{(2)}$ ; if there are portions of these squares  $A^{(2)}$  which are outside all the squares  $A^{(1)}$ , we can suppress them as useless. In order to proceed in a perfectly definite manner, we consider the first of the squares  $A^{(1)}$ , say  $A_1^{(1)}$ , and operate successively on the portions of the successive squares  $A^{(2)}$  which are inside  $A_1^{(1)}$ ; we continue in the same way with  $A_2^{(1)}$ , being careful each time to omit the portions already considered, etc. These operations lead us to consider rectangles, each of which may be replaced by an enumerable infinity of squares (in particular cases a finite number). It is sufficient, in order to form the squares according to a definite law, to construct successively the

greatest possible square inside the rectangle, taking as the vertex nearest the origin of coördinates that vertex of the rectangle which is nearest the origin of coördinates. If among the squares so defined there are some which contain no point of the aggregate A we suppress them. We may assume the squares to be arranged in the order of decreasing size (if two of them happen to be equal in size we shall arrange them according to the relative values of the abscissas of their centers; and if these abscissas are equal, according to the value of their ordinates). In the same way we arrange the squares  $A^{(2)}$  (after the required transformations), and so on.

We define an aggregate B of squares which will consist of all the squares  $A^{(1)}$ , and besides a certain number of the squares  $A^{(2)}$ ,  $A^{(3)}$ , .... In the same way  $B^{(2)}$  will include all the squares  $A^{(2)}$  and, besides, a certain number of the squares  $A^{(3)}$ , .... It is clear that the sum of the squares  $B^{(h)}$  is less than  $E_h + E_{h+1} + \cdots$  is finite no matter what h may be and approaches zero when h increases indefinitely. Since all the squares  $A^{(h)}$  will be part of the  $B^{(h)}$ , every point of A is inside of one of the squares  $B^{(h)}$ . In order that the aggregate E defined by the  $B^{(h)}$  may be regular we must be able to number the  $B^{(h)}$ ,  $B_1^{(h)}$ ,  $B_2^{(h)}$ , ...,  $B_n^{(h)}$ , ..., in such a way that  $B_n^{(h+1)}$  shall be less than  $B_n^{(h)}$ .

We achieve this result in the following manner. Consider first the squares  $A^{(1)}$ , if there are any, whose area is greater than  $\epsilon_2$  (we know that there are none whose area is greater than  $\epsilon_1$ , since the sum of all the  $A^{(1)}$  is less than  $\epsilon_1$ ). We designate these squares as  $B_1^{(1)}$ ,  $B_2^{(1)}$ , ...,  $B_{n_1}^{(1)}$ . Let us consider next those remaining squares  $A^{(1)}$  of which the area is greater than  $\epsilon_3$ , and let us denote them by  $B_{n_1+1}$ ,  $B_{n_2+2}$ , ...,  $B_{n_2}^{(1)}$ . Let us take now the squares  $A^{(2)}$  whose area is greater than  $\epsilon_3$ ; they are arranged in a definite order, as we have said. If the first of them is inside one of the  $A^{(1)}$  already numbered,

for example inside  $B_k^{(1)}$ , we shall denote it by  $B_k^{(2)}$ , otherwise we shall denote it at the same time by  $B_{p_1+1}^{(1)}$  and by  $B_{p_1+1}^{(2)}$ . In the same way, if the second of the  $A^{(2)}$  that we take is inside one of the  $A^{(1)}$  already numbered, different from  $B_k^{(1)}$ , say  $B_h^{(1)}$ , we shall denote it by  $B_h^{(2)}$ . If it is not inside any of the  $A^{(1)}$  (it cannot be inside an  $A^{(1)}$  without a number, since its area is greater than  $\epsilon_3$  and the  $A^{(1)}$  without numbers have areas less than  $\epsilon_2$ ), or if it is inside the particular  $B_k^{(1)}$  which has already been utilized, we shall denote it at the same time by  $B_{p_1+2}^{(1)}$  and by  $B_{p_1+2}^{(2)}$ . In this way we manage to define a certain number of new squares  $B^{(1)}$  which we will call  $B_{p_1+1}^{(1)}$ ,  $B_{p_1+2}^{(2)}$ , ...,  $B_{n_1}^{(1)}$ , and a certain number of squares  $B^{(2)}$  which include all the  $A^{(2)}$  of area greater than  $\epsilon_3$ .

Let us consider now the squares  $A^{(1)}$  of area greater than  $\epsilon_4$ , and let us denote them by  $B_{n_1+1}^{(1)}, B_{n_2+2}^{(1)}, \cdots, B_{n_n}^{(1)}$ ; we can proceed in the same way as before for the  $A^{(2)}$  whose areas are greater than  $\epsilon_4$ , and we can then pass on to the  $A^{(3)}$  whose areas are greater than e. Those among them which are inside of the  $B^{(2)}$  already numbered will have the same numbers (each number being given of course but one time). The others will be denoted at the same time by  $B_3^{(1)}$ ,  $B_3^{(2)}$ ,  $B_3^{(3)}$ . We can continue indefinitely in the same way, the  $\epsilon_k$  approaching zero when k increases indefinitely and each operation involving only a finite number of squares. In this way every square belonging to  $A^{(h)}$  will appear in  $B^{(h)}$  in a determinate position. Moreover, it is obvious that  $B_q^{(h)}$  approaches zero no matter what q may be when h increases indefinitely. It is impossible that certain series  $B_{\alpha}^{(1)}$ ,  $B_{\alpha}^{(2)}$ , ...,  $B_q^{(r)}$  should terminate, because that would mean that no one of the squares  $A^{(r+1)}$  is inside  $B_a^{(r)}$ ; that is to say, that  $B_a^{(r)}$ would inclose no point of the aggregate A, which is contrary to our hypothesis. The aggregates of squares  $B^{(h)}$  define, then, a regular aggregate which includes all the points of  $A_1$ , and our theorem is proved.

We notice that in the definition of the regular aggregate E there are certain series  $B_q^{(1)}$ ,  $B_q^{(2)}$ , ..., of which a certain number of the first terms denote squares that coincide among themselves. That, in fact, is no difficulty. We can, however, avoid this circumstance by slightly modifying the definitions of the first  $B_q$  of such a series; if, for instance,  $B_q^{(1)}$ ,  $B_q^{(2)}$ ,  $B_q^{(3)}$  coincide, we can replace  $B_q^{(2)}$  by  $(1 + \epsilon)B_q^{(2)}$ , and  $B_q^{(1)}$  by  $(1 + \epsilon_1)(1 + \epsilon)B_q^{(1)}$  (we designate by  $\alpha C$  a square similarly placed to C, with the ratio  $\alpha$  of similarity). These operations multiply the total extent of the squares  $B_q^{(1)}$  by a factor less than the convergent infinite product  $\Pi(1 + \epsilon_k)$ .

We notice that the regular aggregate E which we have defined is not necessarily the most simple of the regular aggregates of measure zero which include the A, but it is not important that our demonstration should give us the most simple. The essential thing is to show that there exists one; it is then possible to consider without contradiction the collection of all the regular aggregates of measure zero which contain A, and we can choose from this collection if not the simplest (which may not exist, in the same way that the smallest number greater than  $\sqrt{2}$  does not exist), at least an aggregate E whose simplicity is as close as we please to the greatest possible.

From now on we shall consider especially the regular aggregates. Such an aggregate is defined by the fundamental points  $A_n$ , which are limits of the  $B_n^{(h)}$  when h increases indefinitely, and by the magnitudes of the excluding squares  $B_n^{(h)}$  corresponding to  $A_n$ . The derived aggregate of the fundamental points is a closed set A'. In the general case this set is composed of a perfect aggregate and a reducible aggregate. The excluding intervals which correspond

<sup>&</sup>lt;sup>1</sup> It might seem desirable to consider also the relative positions of the  $A_n$  in these squares; but by modifying slightly the definitions we can so arrange that every  $B_n^{(h)}$  has  $A_n$  for its center.

to the points of the reducible aggregate have only in common the points of this reducible aggregate itself. Their study therefore gives us nothing new. The really interesting part of a regular aggregate of zero measure is that which is attached to those points of A' which form a perfect aggregate. We shall have to distinguish cases according to the nature of this perfect aggregate. We shall limit ourselves, however, to the consideration of the case where the aggregate A' contains all the points of a certain area of simple form. The points  $A_n$  will then be dense within this area. All the cases where the area is of a single piece and simply connected may be reduced by conformal representation to the case of the area bounded by a circle. We shall show that if we have two different systems of points  $A_n$  and  $B_n$ , dense within the interior of equal circles and also dense on their circumferences, 2 we can establish between these points a reciprocal continuous one-to-one correspondence in such a way that the ratio between the distance of any two points  $A_p$ ,  $A_q$  and the distance of the corresponding points  $B_p$ ,  $B_q$  will be included between two limits as close to unity as we please. It will follow from this theorem that we shall be able without loss of generality to suppose that the fundamental points of an aggregate of zero measure, when these points are dense within a certain region, coincide with a given dense aggregate in that region - for instance, with the points of rational coordinates.

in the same way.

 $<sup>^1</sup>$  We shall thus leave aside those aggregates of zero measure which we obtain by assuming that A is a perfect linear aggregate which without being linear yet contains no area. For example, we could exclude certain fixed areas around the points with rational coordinates and take for the  $A_n$  the points with algebraic coordinates which did not belong to the excluded areas. We could also build up in some arrangement several similar constructions, or even a denumerable infinity of such constructions superposed, and thus obtain regions which would be quite complicated from The case when neither aggregate has points on the circumference can be treated

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The theorem which we have in view can be expressed as follows: Given two equal circles C and C', and two enumerable aggregates A and B, of which the first is dense in C and on the circumference C, and the second is dense in C' and on the circumference C', and given an arbitrarily small number  $\epsilon$ , then we can number the points of A and B in such a way that to a point on the contour we make correspond a point on the contour, and that we have, whatever p and q may be,

$$1 - \epsilon < \frac{A_p A_q}{B_p B_q} < 1 + \epsilon.$$

We shall say that in this case the two aggregates are similar by  $\epsilon$ .

In order to prove this theorem we shall assume that the points of the two aggregates are arranged provisionally in a determinate order, and we shall consider successively the first point of A, then the first point of B, then the second point of A, then the second point of B, and so on. Thus we shall not miss any point belonging to either of the two aggregates. To each new point that we consider in one aggregate, we shall make correspond a determinate point in the other; and when the turn of this new point comes we shall omit it.

We shall suppose that the centers of the circles C and C' do not belong to the aggregates A and B (nothing would be changed if both of them should belong, for we could make them correspond; and if one of them belonged, but the other not, we could make a conformal transformation, differing little from the identical transformation, which would transform the second circle into an equal circle whose center could then be made to correspond to the center of the first

circle). In this way we can investigate the two circles by considering them superposed and yet distinct. It is possible now to choose two rectangular axes Ox and Oy in such a way that the diameters parallel to the axes contain no points of A or B, and every line parallel to either of the axes contains at most one point of A and one point of B (because the totality of directions of lines which connect the center with points of A or with points of B, or connect the points of A among themselves, or the points of B among themselves, or which are perpendicular to these directions, form an enumerable aggregate). Let us assume an infinite series of positive numbers  $\epsilon_1$ ,  $\epsilon_2$ ,  $\cdots$ ,  $\epsilon_n$ ,  $\cdots$  such that

$$I - \epsilon < \Pi(I - \epsilon_n)$$
,  $\Pi(I + \epsilon_n) < I + \epsilon$ .

The circle C is divided by the diameters parallel to the axes in four equal regions which provisionally we shall call 1, 2, 3, 4; and the circle C' is divided in homologous regions which we shall designate in the same manner.

Consider first  $A_1$ , which may be, for instance, in the region 3: since it cannot be on the diameters, it must be inside this region, unless it be on the circumference (a case which we are for the moment excluding). Let us now designate by  $A'_1$  the point of the region 3 of C' which coincides with  $A_1$ when C' is moved upon C by a translation. If  $A'_1$  happens to belong to B, which is not the general case, we shall call it  $B_1$ . Otherwise we shall define a square with center at  $A_1$ , such that the ratio of the greatest to the least of the shortest possible distances of all the points in the square to points on the boundary of the region 3 shall be less than  $1 + \epsilon_1$ . This shortest distance is parallel to the axes for the rectilinear portions of the boundary and coincides with the radius for the curvilinear portion, and, from our hypothesis in regard to  $A_1$ , is not zero. So the construction of the square is always possible. We now choose  $B_1$  arbitrarily from the points of

B inside this square (if we wish to avoid having to make an arbitrary choice from among a denumerable infinity of points, we can take the point of B whose number is smallest in the provisional classification). Having chosen this point  $B_1$  we construct parallels to the axes passing through  $A_1$  and  $B_1$ , each set of which, with the diameters already drawn, will divide its circle into regions (nine in each) which will correspond two by two. Some of these regions will be rectangles (in this case only one), while the others will be quadrilaterals or triangles of which certain sides are parallels to the axes and one side is an arc of the circle. If we agree to consider as the dimensions of such regions the dimensions of the rectilinear sides, it follows from the construction that the ratio between homologous dimensions of two corresponding regions is included between  $I + \epsilon$  and  $I - \epsilon$ . In the case which we have momentarily excepted, where  $A_1$  is on the circumference, we can take  $B_1$ , also on the circumference, in such a way that the same condition shall be verified with respect to the regions, a construction which is always possible.

Let us turn now to the second point  $B_2$ , taken from the second aggregate. We make correspond to it a point  $A_2$  situated in the homologous region, chosen in such a way that the new regions obtained by drawing parallels to the axes through  $A_2$  and  $B_2$  have homologous sides whose dimensions are included between  $(1 + \epsilon_1)(1 + \epsilon_2)$  and  $(1 - \epsilon_1)(1 - \epsilon_2)$ . This condition necessitates assigning to  $A_2$  a certain area inside this region, and  $A_2$  is chosen inside this region either arbitrarily, or according to some definite law, as has been explained for  $B_1$ , care being taken to have  $A_2$  on the circumference C, if  $B_2$  is on the circumference C.

<sup>&</sup>lt;sup>1</sup> We have, in fact,  $\frac{1}{1+\epsilon_1} > 1-\epsilon_1$ , and, according to our construction, the ratios of homologous sides are included between  $1+\epsilon_1$  and  $1+\epsilon_2$ 

We continue in the same way, taking alternately a point in A and a point in B, making it correspond to some point in the other aggregate. After n operations we shall have at most  $(n + 2)^2$  regions, and the ratio of two homologous dimensions of two regions which correspond will always be included between

and 
$$(1 - \epsilon_1)(1 - \epsilon_2) \cdots (1 - \epsilon_n)$$
 
$$(1 + \epsilon_1)(1 + \epsilon_2) \cdots (1 + \epsilon_n)$$

and therefore between  $I - \epsilon$  and  $I + \epsilon$ . If we continue in this way indefinitely, every point of A and every point of B will have a number, after a finite number of operations, and this number will be at most double the number of the same point in the provisional classification.

This final classification satisfies completely the conditions of our theorem. For, if we consider any two points  $A_p$ ,  $A_q$ , with their corresponding points  $B_p$ ,  $B_q$ , the difference of the abscissa  $x_p$  and  $x_q$  of  $A_p$  and  $A_q$ , when the regional division has progressed far enough (that is, after a number of operations not greater than the larger of the two members p, q), will be equal to the sum of the rectilinear sides of certain regions, and the abscissas  $x_p'$ ,  $x_q'$  of  $B_p$  and  $B_q$  will be equal to the sum of rectilinear sides of the corresponding regions. We shall have then

$$(1) 1 - \epsilon < \frac{x_p' - x_q'}{x_p - x_q} < 1 + \epsilon$$

and similarly

(2) 
$$1 - \epsilon < \frac{y_p' - y_q'}{y_p - y_q} < 1 + \epsilon,$$

from which follows immediately

$$1 - \epsilon < \frac{\sqrt{(x_p' - x_q')^2 + (y_p' - y_q')^2}}{\sqrt{(x_p - x_q)^2 + (y_p - y_q)^2}} < 1 + \epsilon.$$

But this last relation is the statement of our theorem.

We might show in the same way the analogous theorem about the angles  $\alpha$  and  $\beta$  which the lines  $A_pA_q$  and  $B_pB_q$  make with the axis Ox. In fact, we have

$$\tan \alpha = \frac{y_p - y_q}{x_p - x_q} \quad , \quad \tan \beta = \frac{y_p' - y_q'}{x_p' - x_q'},$$

so that from equations (1) and (2) we deduce immediately

$$\frac{1-\epsilon}{1+\epsilon} < \frac{\tan \alpha}{\tan \beta} < \frac{1+\epsilon}{1-\epsilon}$$

If we take the angles  $\alpha$  and  $\beta$  positive, since they are almost of the same value,  $\cot \beta + \tan \alpha$  is greater than or at least equal to 2, and therefore, neglecting  $\epsilon^2$ , we shall have

$$|\alpha-\beta| < |\tan(\alpha-\beta)| = \left|\frac{\frac{\tan \alpha}{\tan \beta} - 1}{\frac{1}{\tan \beta} + \tan \alpha}\right| < \frac{1}{2} \left|\frac{\tan \alpha}{\tan \beta} - 1\right| < \epsilon.$$

The properties of the correspondence which we have shown to exist between two enumerable aggregates A and B, which are dense in equal circles C and C', are worth studying more completely. Here follow some remarks that might be useful in such a study. In the first place we observe that if any partial arrangement of points  $A_{n_1}$ ,  $A_{n_2}$ ,  $\cdots$  approach a limiting point P, there corresponds to it a partial series of points  $B_{n_1}$ ,  $B_{n_2}$ ,  $\cdots$  which approaches a limit P'. The correspondence between P and P' is well defined, — that is, is independent of the partial series that may be chosen. We have in this way a one-to-one correspondence between the points of C and the points of C'.

Let us agree to call the parallels to the axes, drawn through the points of the aggregate, lines of discontinuity. To any point M not on a line of discontinuity corresponds an homologous point M', and the transformation of the region in the

neighborhood of M into the region in the neighborhood of M' may be written in the form

$$x' = (h + \eta)x$$
  
$$y' = (h + \eta')y,$$

where x, y are the coördinates of M, x', y' are the coördinates of M', h, k are constants of value between  $1 - \epsilon$  and  $1 + \epsilon$ , and  $\eta$  and  $\eta'$  are functions of x and y which approach zero when  $x^2 + y^2$  approaches zero. The constants h and k are the two ratios of similitude (parallel to the two axes) of the neighborhoods of M' and M. If the points M' and M lie on a line of discontinuity, the ratio of similitude in the direction perpendicular to this line has not the same value on both sides of the line. At a point M which is the intersection of two lines of discontinuity, there are four values for each ratio of similitude, corresponding respectively to the positive and negative variations of the two coördinates. The ratio of similitude h is thus defined throughout C. It is discontinuous on the lines of discontinuity, but continuous at other points.

If we know nothing about the provisional numbering of the aggregates A and B, we can merely say this about the relation between the provisional numbering and the final numbering: that the final number n is at most twice the provisional number p; for every point numbered provisionally  $A_p$  or  $B_p$  is chosen after at most 2p operations. We cannot, however, give an upper limit to p as a function of n.

It will be possible to determine such a limit, provided that we take care to choose the system of provisional numbering from among those that are sensibly homogeneous. Let us make our meaning clear. By definition, in order to arrange a very large number p of points in a homogeneous manner in a circle C, we shall construct a square grating

such that p of its vertices are inside C; if  $\alpha_n$  is the length of a segment of the grating, we shall put one point in each square of side  $\alpha_p$ , and  $l^2$  in each square of  $l\alpha_{pp}$  exactly if l is an integral number, approximately if l is not. Let us write  $l\alpha_p = \lambda$  and take  $\lambda$  as fixed and p variable. Then for every value of p we can calculate the approximate number of points inside the square of side  $\lambda$ , a number which may be given asymptotically as  $p\lambda^2/\pi r^2$ , r being the radius of the circle C. We shall say that the arrangement of points of the enumerable aggregate  $A_1, A_2, \dots, A_n, \dots$  is asymptotically homogeneous if, for any square of side  $\lambda$ , the number  $\lambda_n$  of points of index less than p inside this square approaches this same symptotic value  $p\lambda^2/\pi r^2$  when p increases indefinitely; i.e., if the ratio  $\pi \lambda_p r^2/p \lambda^2$  between the numbers  $\lambda_p$ and the symptotic value  $p\lambda^2/\pi r^2$  approaches I as p increases indefinitely. We shall say that the arrangement is sensibly homogeneous if this ratio becomes and remains limited by two constants  $\alpha$  and  $\beta(\alpha < 1 < \beta)$  independent of  $\beta$  and of the position of the square of side  $\lambda$ .

In the preceding definition of homogeneous arrangement, nothing was said about the points that happened to be situated on the boundary. If the boundary is a square of side a, the maximum number of points situated on this boundary for a grating of measure a/n is 4n, the total number of points being  $n^2$ . Generally speaking, the number of points on the boundary will be said to be normal if it is of the order of magnitude of the square root of the total number of points. We must observe that this notion of normal depends on the assumption that there are points on the contour. If the points were arranged arbitrarily, in the general case there would be no point on the boundary, and this is indeed the simpler hypothesis. But if there are points on the contour, the case is probably that there is some sort of a relation between the way the contour is chosen and the way the

points are given. Hence it is natural to suppose that the probability that a point falls on an arc of the boundary of unit length is some finite proportion of the probability that a point falls in unit area. This hypothesis is verified, for instance, if the boundary is a circle and if the points of the aggregate are those with rational coördinates. Other such hypotheses might be conceived, related to the theory of numbers.

We must then, in the case where there are points on the boundary, add to the hypothesis that the arrangement is sensibly homogeneous inside, the hypothesis that it is sensibly homogeneous on the boundary.

In many questions, the preceding definition of sensibly homogeneous arrangements is inadequate; it is necessary to add a condition which may be called *intrinsic homogeneity*, because it introduces the relative positions of the points of the aggregate. If we consider the vertices of a grating, which we take as the type of homogeneity (or, say, a net of equilateral triangles), we see that the shortest distance between two vertices is proportional to the inverse square root of the total number of points. We say that a two-dimensional aggregate is intrinsically homogeneous if the shortest distance between any two of its points of number less than p is of the order of magnitude  $1/\sqrt{p}$ . Homogeneity of arrangement and intrinsic homogeneity are thus seen to be independent conceptions, neither being a consequence of the other.

Given a denumerable aggregate, dense within a circle (or square), it is always possible to number its points in such a way as to satisfy the conditions of homogeneity. One of the simplest methods of doing this is as follows. After having numbered some of the points, we trace a grating

An analogous condition should be verified for the shortest distance to the boundary of points very near to this boundary and not lying on it.

fine enough to make a few more squares than points already numbered, and such that one square includes at most one of these points. There will then be some squares that do not contain such points. In each of these we number one point of the aggregate, by choosing it inside a square concentric with the first, and twice smaller, taking the point of smallest subscript in the provisional numbering (thus we are sure of not omitting any point).

Any system of numbering that satisfies both conditions of homogeneity will be spoken of as *normal*. It is easy to verify the fact that the methods of numbering habitually used lead to normal arrangements.

When the two aggregates that are dense in C and C' are numbered normally, it is possible to arrange matters so that the one-to-one correspondence set up between their elements shall be itself normal; *i.e.*, there exist between the provisional numbering, p, and the final numbering, n, inequalities of the form

$$p^{a} < n < p^{\beta},$$

where the exponents  $\alpha$  and  $\beta$  are finite and depend only on the number of dimensions in the aggregate considered, and on the convergent series  $\Sigma \epsilon_n$  which has been used. (In order to be sure that  $\alpha$  and  $\beta$  are finite, there must be a finite quantity h such that  $\lim n^h \epsilon_n = 0$ .)

We divide the aggregate A into two others, A' and A'', still everywhere dense, and the aggregate B, similarly, into B' and B''. It is then easy to show that the correspondence can be set up in such a way that the points of A' correspond to those of B' and the points of A'' to the points of B''. For that, it would not be sufficient of course to apply the general theorem first to A' and B' and then to A and B, because the correspondence thus set up between two points P and P' inside C and C', respectively, would not in

general be the same by means of the two separate correspondences.

This procedure we can extend to the case where A and B each consists of a denumerable infinity of aliquot parts, everywhere dense. We can establish, for instance, a continuous one-to-one correspondence between the rational numbers in a certain interval, and the algebraic numbers in an equal interval, in such a way that to the rational numbers whose denominators consist of h and only h distinct prime factors, correspond the algebraic numbers which are the roots of an irreducible equation of degree h (for h = 1 we get the rational numbers; if we wish to consider only the irrational algebraic numbers we must take irreducible equations of degree h + 1).

III

Let us consider now two regular aggregates of zero measure, of which the fundamental points are precisely the denumerable aggregates A and B inside the circles C and C'. If we suppose that the squares of exclusion belonging to the corresponding fundamental points have as their sides lines which correspond, it is evident that the two aggregates will correspond point by point in the one-to-one correspondence that we have established between the points P inside C and the points P' inside C'. In other words, given a regular aggregate of zero measure of which the fundamental points B are dense in C', we can define a regular aggregate of zero measure of which the fundamental points are the elements of an arbitrary aggregate A, dense in C, in such a way that the two aggregates correspond to each other continuously and in a oneto-one manner (the ratio of similitude being contained between  $1 - \epsilon$  and  $1 + \epsilon$ ).

Hence in order to study regular aggregates of zero measure of which the fundamental points are dense within a certain

region, we can without loss of generality assume that the fundamental points are, for instance, the points with rational coördinates. In particular it is easy to prove this important proposition: Every regular aggregate of zero measure of which the fundamental points are dense within a certain region has the order of the continuum. In other words, if we arrange at pleasure the diminishing of the squares of exclusion in the neighborhood of fundamental points, it is not possible to make this diminution rapid enough so that the fundamental points shall be the only ones of the aggregate.

For simplification let us consider the case of a single dimension; the demonstration is in principle the same for any number of dimensions. Let  $A_n$  be the intervals of exclusion belonging to the points  $A_n$ . For each value of h we can define a positive function  $\phi_h(n)$  increasing with n, such that we shall have

measure 
$$(A_n^h) > \frac{1}{\phi_h(n)}$$
.

On the other hand, if we are given a denumerable succession of increasing functions  $\phi_h(n)$ , it is possible, according to a theorem of Paul du Bois-Reymond, to construct a function  $\phi(n)$  increasing more rapidly than any of the functions  $\phi_h(n)$ . After having found this function  $\phi(n)$ , the theory of continuous functions enables us to define an infinite number of irrational numbers x (an infinity which has the order of the continuum) such that there exists for each of them adenumerable infinity of relations of the form

$$\left|x-\frac{m}{n}\right|<\frac{1}{\phi_h(n)}$$

where m and n are integers. Such a number x, whatever h may be, belongs to at least one of the intervals  $A_n^{(h)}$ ; it is therefore an element of the aggregate defined by the points

 $A_n$  and these intervals of exclusion. In order to define the numbers x and show that their aggregate is of the order of the continuum, it is sufficient to investigate a continuous fraction in which the incomplete quotients increase very rapidly. If we write

$$P_{n+1} = a_n P_n + P_{n-1}$$

$$Q_{n+1} = a_n Q_n + Q_{n-1}$$

$$a_n > \phi(Q_n),$$

and assume that

where  $\phi(n)$  is the function which we have just defined, we shall have, from the nature of the convergents,

$$\left|x - \frac{P_n}{Q_n}\right| < \left|\frac{P_{n+1}}{Q_{n+1}} - \frac{P_n}{Q_n}\right| = \frac{I}{Q_n Q_{n+1}} < \frac{I}{\phi(Q_n)}.$$

But the totality of systems of integers  $a_n$  which verify the relations  $a_n > \phi(Q_n)$  have, themselves, the order of the continuum.<sup>1</sup>

If we wished to have intervals of exclusion which should decrease rapidly enough so that the aggregate of points defined by them would be composed only of the fundamental points, the  $\phi_h(n)$  would have to contain functions increasing more rapidly than any  $\phi(n)$ . According to the theorem of Paul du Bois-Reymond, that is not possible if the indices h are denumerable. It would be necessary then to make belong to any fundamental point a transfinite infinity of intervals of exclusion, the corresponding functions  $\phi_a(n)$  (where  $\alpha$  denotes a transfinite number) being such that every increasing function  $\phi(n)$  is surpassed by one of them. In this way, however, we get outside the domain of definitions expressible in a finite number of words.

In order to classify the regular aggregates of zero measure, it is better to consider rather than the functions  $\phi_h(n)$  which

<sup>&</sup>lt;sup>1</sup> Each  $a_n$  may be odd or even; the aggregate of x then includes an aggregate of the same order as that of the numbers 0.1010110..., written in the binary scale.

we have defined, the functions  $\psi_h(n)$  determined by the relations

$$\sum_{p=n}^{\infty} \text{ measure } A_p^{(h)} = \frac{1}{\psi_h^{(n)}}.$$

The convergence of the series formed by the intervals of exclusion of order h implies that the functions  $\psi_h(n)$  should increase indefinitely with n. After the theorem of Paul du Bois-Reymond, there exists a function  $\psi(n)$  increasing less rapidly than any of these, which nevertheless approaches  $+\infty$  as n approaches  $\infty$ . Hence whatever the value of h, if we take n large enough, we shall have

$$\sum_{p=n}^{\infty} \text{ measure } A_p^h < \frac{1}{\psi(n)};$$

that is to say, that the different series formed by the intervals of exclusion all converge more rapidly than the series

$$\sum \left[\frac{1}{\psi(n)} - \frac{1}{\psi(n+1)}\right].$$

The more rapid the increase of  $\psi(n)$ , the fewer points are included in the aggregate of measure zero, because the intervals of exclusion in that case decrease more rapidly. It is natural, then, to take the function  $\psi(n)$  as defining what we may call the asymptotic order of the regular aggregate of zero measure. These orders can be expressed by means of the notations used for orders of infinity;  $\psi(n) = n^p$  will be said to be of order  $p, \psi(n) = \epsilon^n$  of order  $\omega$ , etc. We meet the aggregates of order  $\omega$  in defining monogenic functions which are not analytic.

IV

Perhaps it is opportune to emphasize a little the general conclusions which follow from this rapid study of aggregates of zero measure.

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Aggregates of zero measure have a fundamental position in the theory of functions. It is, in fact, always possible to inclose the singularities of finite functions in aggregates which are either of zero measure or of measure as small as we like. On the other hand, aggregates which are not of zero measure have a uniform quality, being formed of continuous aggregates either positive or negative. They are heterogeneous with regard to the continuum. Aggregates of zero measure can, however, be sensibly homogeneous with regard to the continuum, that is to say, identical with themselves in intervals as small as we like.

The concept of aggregate of zero measure is so general that we cannot hope to make a profound investigation of the properties of functions without studying minutely this general notion. That is to say, we must not regard all aggregates of zero measure as undifferentiable. The classification based on the asymptotic diminution of intervals of exclusion seems to me to be a first step in this study which faces the students of analysis.

With this question, as with all those where the general notion of increasing functions enters (as, for example, in the theory of the convergence of series with positive terms), difficulties of a transfinite nature are presented which we cannot hope entirely to surmount. But, on the other hand, the problems which are actually met with are generally if not always free of these difficulties (this is the case, for instance, with the usual criteria for the convergence of series of positive terms; for, although theoretically quite special, they are nevertheless practically sufficient for the treatment of the series which are presented in all researches in analysis). We can legitimately hope that it will be the same way with the classification of aggregates of measure zero. Theoretically the complexity of this classification surpasses that of the study of series of positive terms, a study which will never

be finished; but practically, a relatively restricted number of classes will suffice for the needs of analysis.

In closing I should like to direct attention to a notable consequence of the theorem about the correspondence between two denumerable aggregates which are everywhere It might seem natural, passing from the finite to a denumerable infinitive, to suppose that the positions of equilibrium of the centers of gravity of the molecules of a solid body should form a denumerably dense aggregate. But a priori it would seem quite an arbitrary hypothesis to suppose that they should coincide with the points of rational coördinates. This simple arithmetic determination seems to have nothing to do with the physical conception. In fact, it evidently is not necessary. But it is as general as any other. The important point is that the hypothesis verifies the conditions of homogeneity of arrangement and intrinsic homogeneity, as we have stated them. The arithmetic treatment of the approximation of numbers by rational numbers is thus the reflection of the general properties of dense aggregates.

#### Third Lecture

## MONOGENIC UNIFORM NON-ANALYTIC FUNCTIONS 1

# I. THE THEORIES OF CAUCHY, WEIERSTRASS AND RIEMANN

THE integration by d'Alembert of the equation of vibrating strings led to a series of researches out of which the notion of an arbitrary function took shape. Among the geometricians who contributed to clarify the new ideas, there should be mentioned Euler, in the front rank, and besides him Clairaut, Daniel Bernoulli and Lagrange. The question was that of the relation between the analytic and the physical definitions of a function: if a string is displaced arbitrarily from its position of equilibrium, does there exist a formula which represents exactly the initial state of the string? Fourier answered in the affirmative and set out the method of calculation of the coefficients of the trigonometric series which represents an arbitrary function. The views put forward by the genius of Fourier have been confirmed by the vigorous analysis of Lejeune-Dirichlet.

The discovery of Fourier revolutionized the notions prevalent up to that time; it was believed, with Euler, that to every analytic expression there corresponded a curve of which successive parts depended on each other: in order to express this interdependence, Euler created the expression 'continuous function': the sense of this expression has since been modified.

Under the influence of the same ideas Lagrange endeavored to prove that every continuous function can be developed

<sup>&</sup>lt;sup>1</sup> Translated from the French by Professor Percy John Daniell, of the Rice Institute.

in a Taylor series: this series would be the tangible form of the connection, so mysterious till then, between the different arcs of a continuous curve; the knowledge of a small arc would have been sufficient to know the whole curve; but Fourier proved exactly that the problem here was illusory, for the physicist who draws a curve remains at each instant free to modify its aspect; the curve once drawn, it is always possible to represent it in its entirety by a unique analytic expression.

This led to the apparently paradoxical result that there existed no logical reason for regarding two segments of the same straight line, for example, as corresponding to the same function, since it was always permissible likewise to regard as a unique function the ordinate of the continuous curve formed of two different straight lines. At the most it could be said that, in the case of two segments of the same straight line, the formula is simpler than in the case of two segments of different straight lines, but this criterion of simplicity does not seem capable of precise definition, unless one is confined to algebraic functions. The paradox was cleared up by extension of the field of study of functions; Cauchy showed that the properties of real functions could only be well understood if imaginary values of the variable were also studied; the idea of a function of a complex variable became indispensable. Cauchy based this idea on the definition of monogeneity; a function of the complex variable z = x + iyis called monogenic if it has a unique derivative. A function which is monogenic at every point of a region without any exception — that is, not allowing in the region any singular point — can be developed in a Taylor series in the neighborhood of any point in the region; the radius of convergence of the series is equal to the distance from the center to the nearest singular point. From this fundamental theorem

Cauchy deduces the calculation of the integrals of the proper differential equations along any path in the plane.

Cauchy's theory was systematized by Weierstrass and Riemann. Weierstrass defined an analytic function, in an exact manner, by means of elements and thus arrived at the idea of a region of natural existence, an idea which was contained implicitly in Cauchy's work, but which was not mentioned explicitly by him. Riemann conceived a monogenic function a priori independently of any analytic expression and showed the advantages of this geometrical conception.

In reality, the analytic point of view of Weierstrass and the geometric one of Riemann find their most perfect synthesis in Cauchy's fundamental theorem: monogeneity within a circle involves the existence of a Taylor series convergent within the circle. This theorem established a necessary connection between values of the same function as a simple consequence of monogeneity: it is sufficient to know that a function is monogenic within a circle, in order that its value at any interior point should be known by a knowledge of its values in the neighborhood of another point. Since our aim is to define monogenic functions in regions more general than those considered up to the present in the theory of analytic functions, it is necessary to make precise the definition of these new regions.

I shall call a region in which an analytic function can be defined in the sense of Weierstrass a Weierstrassian region or W region. I shall call regions more general than W regions, in which a uniform monogenic function can be defined, Cauchy regions, or C regions, in honor of the creator of the theory of monogenic functions. We shall see that the essential properties of monogenic functions in the C domains which we define are the same as in W regions; this does not

exclude the possibility of defining C' regions more general again than our C regions. In other words, we cannot assert that our generalization covers all uniform monogenic functions: but it brings us to a definition of a more general class than the class W of the analytic functions of Weierstrass.

W regions are characterized by the following properties. Let us call a  $\Gamma$  circle every circle such that all the points within  $\Gamma$  belong to W. Every point P of W is within a  $\Gamma$  circle: the  $\Gamma$  circles corresponding to two points P and Q of W can be reunited by a finite number of  $\Gamma$  circles cutting each other two by two. To every uniform analytic function there corresponds a W region; inversely, M. Runge has shown that to every W region corresponds an infinity of uniform analytic functions having precisely W as the region of existence.

If it is assumed that there is no other process of analytic continuation than the Taylor series, the boundary of the W region is a natural limit of existence of the analytic function, and those portions of the plane, if such exist, which do not belong to W ought to be considered as a lacunar space. On this point Weierstrass has insisted several times, and it has been made conspicuous in the clearest way by M. Henri Poincaré. Let us consider a region D of simple form, such as the interior of a circle, and let us define a function G(z)having D as its lacunar space and another function  $G_1(z)$ defined only within D and having consequently all the rest of the plane as its lacunar space. Let us divide the contour of D into two arcs D' and D''. M. Poincaré shows that it is possible to find two uniform functions F(z) and  $F_1(z)$  existing in the whole plane, except for the singular line D' for Fand D'' for  $F_1$ , and in such a way that

$$F + F_1 = G$$
 outside  $D$ ,  
 $F + F_1 = G_1$  within  $D$ .  
 $\lceil 402 \rceil$ 

If then the functions F and  $F_1$  are regarded as uniform, the function G(z) has the continuation  $G_1(z)$  which has been chosen entirely arbitrarily; it is then proper to discard all ideas of a continuation within the lacunar space. This paradox is apparently cleared up if it is observed that, when a function such as F(z) possesses a singular line D', supposed impassable, this function remains uniform in Weierstrass's sense when there is added a non-uniform function such as  $\log \frac{z-z_0}{z-z_1}$ ,  $z_0$  and  $z_1$  being two points of the line D'. The

remarkable result due to M. Poincaré can then be interpreted by the hypothesis that F(z) and  $F_1(z)$  are not really uniform: but in order that this hypothesis should have a meaning, it is necessary to generalize the definition of continuation, in a way so as to be able to pass in certain cases the impassable cuts of Weierstrass; we shall see very soon how this result can be obtained.

But I wished before now to say some words concerning the ideas of Riemann, although it is specially in the study of non-uniform functions, of which I shall not speak here, that Riemann's theory has shown itself productive.

Cauchy has insisted several times on the importance of monogeneity. If an elementary function obtained by a simple calculation made on z is considered and if, for such a function G(z), the ratio

$$\frac{G(z+\delta z)-G(z)}{\delta z}$$

is calculated, this ratio tends to a determinate limit when  $\delta z$  tends to zero, with any argument. Cauchy expresses this essential fact by calling the function monogenic.

the condition of monogeneity is translated into the two fundamental equations

$$\frac{\partial P}{\partial x} = \frac{\partial Q}{\partial y},$$
$$\frac{\partial P}{\partial y} = -\frac{\partial Q}{\partial x}.$$

Cauchy has shown that these equations, when they are verified in a region of the plane, involve the existence of the Taylor series; that is to say, of that which can be called analyticity in Weierstrass's sense. Cauchy's demonstration assumes the continuity of the derivative; M. Goursat, in a well-known piece of work, has shown that the existence of the first derivative is sufficient, and involves the continuity and existence of all the derivatives; M. Paul Montel has extended this result to cases where the existence of the derivative has not been assumed in a set of points of measure zero. The statement of these researches is outside my scope: I should mention them nevertheless, because they are in a way complementary to the results which I shall state further on. What is sufficient to remember is that, in the W regions, monogenic functions are analytic; for this reason the expression monogenic function is no longer in use by certain geometricians, the expression analytic function being considered equivalent; as our aim is precisely to define monogenic functions which are not analytic, it is important to distinguish clearly between the two expressions.

It is difficult to find out if Cauchy conceived the existence of a monogenic function independently of any analytic expression. In fact, he always reasoned about functions which were defined, implicitly or explicitly through known functions, by means of ordinary or partial differential equations; but his reasoning applies without modification to a function defined in a purely ideal way as a correspondence

between z and G(z). This was the conception of Riemann, and has certainly rendered good service, as much in the field of real variables where it was introduced by Dirichlet, as in the field of complex variables, by accustoming mathematicians to very general methods of reasoning, made once for all and susceptible of application to cases not foreseen at the time when the reasoning was done. In fact, there is no real difference between Cauchy's and Riemann's point of view; to apply considerations like those of Riemann to one determinate function, this function must be defined, that is to say must be distinguishable from other functions; and if this definition is effective, it returns to the category of those which Cauchy admitted. This point belongs to the controversies concerning the axiom of Zermelo: Riemann's point of view is otherwise legitimate, whatever attitude is adopted in this controversy; for those who require a precise definition, it saves one from thinking of all the processes of definition which can be imagined; for those to whom an ideal definition is sufficient, it allows one to treat ideally even those functions which will never be defined practically.

It is by means of Cauchy's fundamental theorem

$$f(\delta) = \frac{1}{2\pi i} \int_{c}^{c} \frac{f(z)dz}{z - \zeta}$$

that it can be shown that monogeneity in a W region involves analyticity in the region. We shall use this theorem also in studying monogenic functions in a region, not W; it will be convenient in order to argue in a general manner about all the possible methods of definition of these functions, to consider them as defined in Riemann's way; that is, to assume that nothing is known about such a function except that it is monogenic. It is necessary to show afterward that a theory thus constructed is not empty, by giving actual examples of functions defined no longer ideally, but explicitly.

I shall restrict myself to stating the definition of C regions in a particular case; if the properties of sets of zero measure studied in the previous lecture are used, it can be seen that this particular statement can be considerably generalized.

Let us consider a W region, and a region within W in which we define a denumerable infinity of fundamental points, everywhere dense; we shall assume that these fundamental points  $A_n$  are the points within the circle |z| = 1 whose coördinates are rational. To each point  $A_n$  is attached a positive number  $r_n$ , and we shall assume that these numbers  $r_n$  tend very rapidly to zero as n increases indefinitely; we shall define later the manner of decrease; it is sufficient here to know that the remainder of the convergent series  $r_1 + r_2 + \cdots + r_n + \cdots$  is less than a quarter of the last term retained; we shall denote by  $C_n$  the region obtained by excluding from the W region the points within circles  $C_n^{(q)}$  defined as follows. Let us consider circles  $S_n^{(q)}$ 

having as their centers the points  $A_n$  and for their radii  $\frac{r_n}{2q}$ ; the circle  $C_1^{(q)}$  has its center at  $A_1$ , and its radius is the smallest of the numbers between  $r_1/2^q$  and  $r_1/2^{q+1}$  and such that it does not cut any of the circles  $S_n^{(q)}(n > 1)$ ; this is possible in virtue of the hypothesis  $r_1 > 4 \sum_{k=1}^{\infty} r_k$ , from which it follows

that  $\frac{r_1}{2^q} - \frac{r_1}{2^{q+1}} > 2 \sum_{k=2}^{\infty} \frac{r_k}{2^k}$ ; the  $S_n^{(q)}$  circles are then either inside

 $C_1^{(q)}$  (including those which touch internally), or outside  $C_1^{(q)}$  (including those which touch externally). We shall take no account of the interior circles, and we shall denote by  $A_{n_1}$  the fundamental point of smallest index corresponding to the exterior circles; the circle  $C_2^{(q)}$  will have its center at  $A_{n_1}$  and its radius the smallest number contained

between  $\frac{r_{n_2}}{2^q}$  and  $\frac{r_{n_2}}{2^{q+1}}$  such that it does not cut any of the

circles  $S_n^{(q)}(n > n_2)$ ; it is exterior to the circle  $C_1^{(q)}$  since it is interior to the circle  $S_2^{(q)}$ , and at the same time exterior to the circles  $S_n^{(q)}$  of index less than  $n_2$ , for these circles are interior to  $C_1^{(q)}$  because of the method by which  $n_2$  was chosen. Similarly the circle  $C_3^{(q)}$  etc. is defined and one sees that if the region obtained by excluding the points inside circles  $C_n^{(q)}$  is denoted by  $C_q$ , and the region obtained by excluding the points of  $C_q$  belong to  $C_{q+1}$ , while all the points of  $C_q$ , all the points of  $C_{q+1}$ ; the consideration of the regions  $C_q$  is then equivalent to that of the regions  $C_{q+1}$  and evades the difficulties which result from intersections of the circles.

The points of the circumference of  $C_n^{(a)}$  are said to constitute the frontier of  $C_q$ ; the points of  $C_q$  which do not belong to this frontier are called *interior* to  $C_q$ ; it is important to observe that we use the word *interior* here in a different sense from the usual one in the theory of W regions. The points of the set  $C_q$ , situated in the interior of the circle of radius I, form a perfect set, which can be considered as the derived set of the set of its frontier points  $C_n^{(q)}$ .

The region C is defined as the set of all points such that each of them is interior to some  $C_q$ : the region C is not perfect, for it does not contain the points  $A_z$ , which are its limiting points. We know that the set (of zero measure) of points which do not belong to C has the power of the continuum. We shall say that a region D is *interior* to C, when all the points of D belong to one and the same  $C_q$ , of fixed index. Among the regions interior to C, we shall consider a little more exclusively the regions  $C_q$ : every point of  $C_q$  is interior to  $C_{q+1}$ .

The region C will be said to belong to the class (C) of

Cauchy regions which we are studying here if the numbers  $r_n$  are such that, for n sufficiently large,

$$\log \log \log \frac{1}{r_n} > n;$$

if this condition is verified for two regions C and C', it is verified for the part common to C and C'.

Together with the regions  $C_p$  and  $C_n$ , we shall consider reduced regions which we shall denote by  $\Gamma_p$  and  $\Gamma$ . To a region C corresponds a determinate system of regions  $C_p$ , and an infinity of systems of reduced regions; the following is the definition of one of these systems. Let us suppose numbers  $\rho_n$  given, tending to zero rapidly as n increases indefinitely, but much less rapidly than  $r_n$ ; more precisely, we shall suppose that

$$\frac{\mathrm{I}}{\rho_n^2} < \log \log \frac{\mathrm{I}}{r_n};$$

and, at the same time, whatever the fixed number  $\alpha$ , that, for n sufficiently large,

$$(3) \qquad \frac{1}{\rho_n} > n^{\alpha};$$

these two conditions (2) and (3) are quite consistent by virtue of (1).1

The regions  $\Gamma_p$  are defined by means of  $\rho_n$  as the  $C_p$ 's by means of  $r_n$ , that is, are limited by circles of radii between  $\frac{\rho_n}{2^p}$  and  $\frac{\rho_n}{2^{p+1}}$  exterior to each other. The region  $\Gamma$  is formed of the set of points *interior* (in the sense indicated above) to each  $\Gamma_p$ . The regions  $\Gamma_p$  are perfect,  $\Gamma$  is not perfect; the set complementary to  $\Gamma$  has zero measure and the power of the continuum.

The set C contains all points of  $\Gamma$  since  $C_p$  contains all

<sup>1 (1) (2)</sup> and (3) could be replaced by wider conditions: my aim here is to simplify the statement.

points of  $\Gamma_p$ : but C contains besides points which do not belong to  $\Gamma$ .

The following theorem is fundamental:

If a function of the coördinates of a point P is defined in C and continuous in every  $C_p$ , the knowledge of its values at all points of  $\Gamma$  involves the knowledge of its values at all points of C.

In other words, two functions continuous in C (that is, defined in all C and continuous in every region interior to C) cannot coincide in all  $\Gamma$  without coinciding in all C; or, finally, a function continuous in C and zero in  $\Gamma$  is zero in C.

In fact, let P be a point of C; this point belonging to a set  $C_p$  interior to C, it is a limiting point of the set formed by the frontier  $^1$  of  $C_p$ ; it is sufficient in order to prove that the function is zero at P, since it is continuous in  $C_p$ , to show that it is zero on each circumference which constitutes this frontier (the remark has already been made that each of these circumferences is *interior* to  $C_{p+1}$ ); then, on one of these circumferences (as on every rectifiable curve traced in the plane), the points which are part of  $\Gamma$  are everywhere dense; the function being continuous on this curve is then zero throughout this curve if it is zero at all points of  $\Gamma$ .

When we speak of a reduced region, we shall assume that we consider a determinate region, the  $\rho_n$ 's being chosen in a precise way, satisfying the inequalities (2) and (3). It might happen that we had to consider at the same time another region  $\Gamma'$  defined by numbers  $\rho'_n$ ; if

$$p_n^{\prime\beta} = \rho_n$$

we say that  $\Gamma'$  is of order  $\beta$  with respect to  $\Gamma$ ; if  $\beta$  is greater than one, the numbers  $\rho'_n$  satisfy the inequalities (2) and (3)

<sup>&</sup>lt;sup>1</sup> We neglect points P which would be interior to C in Weierstrass's sense; for them the proposition is evident, since they are centers of circles inclosing no  $A_n$  in their interior, they are also interior to  $\Gamma$  in Weierstrass's sense.

when the  $\rho_n$ 's satisfy them: in this case the set  $\Gamma'_p$  is interior to  $\Gamma_p$ , for the excluded circles of radii  $\frac{\rho'}{2^p}$  are larger than the circles of radii  $\frac{\rho_n}{2^p}$  (for  $\rho_n$  can always be supposed less than 1).

Let us remark finally that the points of  $C_p$  which lie on any curve whatever, a straight line for instance, form a perfect set, defined by contiguous intervals (in M. Baire's sense), which are the chords intercepted on the straight line by the circles. This set may or may not contain intervals: but in every case it is perfect, and consequently a function continuous in  $C_p$  and zero at all the points which limit the contiguous intervals is zero at all points of the set at the same time with all its derivatives in  $C_p$ .

## II. MONOGENIC FUNCTIONS IN C REGIONS

We shall say that a function F(z) is monogenic in a region such as C if:

- 1°. It is continuous within C (that is, as we have explained, continuous in every  $C_p$ , interior to C; since the set  $C_p$  is perfect, this continuity in  $C_p$  is uniform);
- 2°. At every point P of C, it has a derivative with respect to z, unique and continuous within C. To define the derivative a set  $C_p$  of which P is a part is considered, and denoting by P' any other point of C, the limit of the ratio

(5) 
$$\frac{F(\rho') - F(\rho)}{\overline{\rho \rho'}}$$

is found when the vector  $\overline{\rho\rho}' = z' - z$  tends to zero; if this limit exists for every value of  $\rho$ , it is evidently independent of the value of  $\rho$ , for all points of  $C_p$  belong to  $C_{p+q}$ ; for this reason this limit can be called the derivative of F(z)

within C, that is in every region interior to C. The continuity of the derivative within C is to be understood in the same way as the continuity of the function itself within C: continuity in each  $C_p$  interior to C. This hypothesis of the continuity of the derivative is doubtless superfluous; but it simplifies the argument.

Since the set  $C_p$  is perfect, every function continuous in  $C_p$  is bounded in  $C_p$ .

Let us mention at once an example of the simplest kind of a C region and of a function monogenic in this region.<sup>1</sup>

Let us form the series

$$F(z) = \sum_{n=1}^{\infty} \sum_{p=0}^{n} \sum_{q=0}^{n} \frac{e^{-z}}{z - \frac{p+qi}{n}}.$$

Clearly this series is convergent outside the square T of which the vertices are the points z = 0, i, i, i + i. Inside this square the series has an infinity of poles; in fact, all the points whose coördinates are rational numbers  $x = \frac{p}{n}$ ,  $y = \frac{q}{n}$ .

But if circles having these poles as centers and radii  $\frac{\epsilon}{n^*}$  be considered, the series is absolutely and uniformly convergent at all points outside these circles, whatever the fixed number  $\epsilon$  may be. The same is true if circles  $\Gamma_n^{(h)}$  with centers at the points  $\frac{p}{n}$ ,  $\frac{q}{n}$  and radii  $\frac{1}{h}e^{-\epsilon^{n}}$  are considered, where h is a

fixed integer which we are allowed to increase indefinitely. I shall call  $\Gamma_h$  the set of circles  $\Gamma_n^{(h)}$  and  $C_h$  the set of points which are not inside any of the circles  $\Gamma_n^{(h)}$ . There exists an infinity of curves which cross the circle and of which all the points belong to one same region  $C_h$ .

<sup>&</sup>lt;sup>1</sup> The region C considered here is a little more general than the regions defined above, in the sense that the series  $\sum_{i}^{r}$  converges a little less rapidly.

The function F(z) is evidently monogenic within the region C which is the limit of the  $C_h$ 's; it has in fact at each point of this region a determinate unique derivative, which is obtained by differentiating the series term by term. The value of this derivative is independent of the way in which the increment  $\delta z$  tends to zero, with the reservation, of course, that z and  $z + \delta z$  are inside  $C_h$ .

The study of monogenic functions within a region C requires the extension of Cauchy's fundamental theory to the contour which limits a perfect region  $C_h$ . To this end we shall establish at once the following fundamental property of a function F(z) monogenic in the region C. If we denote by p a fixed number

(6) 
$$\int_{\mathcal{K}} F(z) dz = \sum_{n} \int_{C_{n}^{(p)}} F(z) dz$$

the curve K being any simple curve all of whose points are inside  $C_p$ , the sum  $\Sigma$  referring to all the circles  $C_n^{(p)}$  which are inside  $C_p$ ; the integrals are all taken in the direct sense.

We shall set

(7) 
$$F(z) = P(x, y) + i Q(x, y),$$

so that the equation (6) becomes two equations, of which it is sufficient to demonstrate one; for example,

(8) 
$$\int_{R} P dx - Q dy = \sum_{n} \int_{C_{n}^{(p)}} P dx - Q dy.$$

To prove this relation, we define a function  $P_1(x, y)$ , finite and determinate at all points interior to K, and coinciding with P(x, y) at the points inside K which belong to  $C_p$ ; there remains the definition of  $P_1(x, y)$  inside the circles  $S_n^{(p)}$ ; on the circumference of these circles it coincides with P(x, y). We shall define  $P_1(x, y)$  inside the circle by the condition that on chords of the circle parallel to  $O_p$  it varies

linearly. (Its values at the extremities are known for they coincide with that of P(x, y).) The function  $P_1(x, y)$  thus defined is continuous within K and has at every point a derivative  $\frac{\delta P_1}{\delta y}$ ; this derivative is bounded according to the hypothesis that the derivatives of P are bounded (which is involved by the existence and continuity of the derivatives of F(z)); in fact, at points inside  $C_p$  the derivative of  $P_1$  coincides with the derivative of P; at points inside  $C_p$ , the derivative of  $P_1$  is constant along a chord parallel to Oy and equal to the quotient of the difference of the values of  $P_1$  (that is, of P) at the ends of the chord, divided by the length of this chord. The difference of the values of P is

$$\int_{MN} \frac{\partial P}{\partial x} dx + \frac{\partial P}{\partial y} dy$$

if MN denotes the arc subtended by the chord.

This integral is less than the product of the length of the arc MN and the sum  $\begin{vmatrix} \partial P \\ \partial x \end{vmatrix} + \begin{vmatrix} \partial P \\ \partial y \end{vmatrix}$  and its quotient when it is divided by the chord MN is at most equal to

$$\frac{\pi}{2} \left( \left| \frac{\partial P}{\partial x} \right| + \left| \frac{\partial P}{\partial y} \right| \right)$$

and is consequently bounded at the same time as the derivatives  $\frac{\partial P}{\partial x} \text{ and } \frac{\partial P}{\partial y}.$ 

Similarly the values of  $P_1$  lying between the values of  $P_1$ ,  $P_1$  have the same boundary as  $P_1$ .

<sup>1</sup> The derivative  $\frac{\partial P_1}{\partial y}$  is discontinuous at points on a circumference. This produces no inconvenience; one can modify the definition of  $P_1$  by choosing other curves instead of straight lines. Sufficiently simple results can be obtained by taking the sum of a parabola and a sinusoidal curve.

According to a classical result, denoting by  $^{(R)}$  the area within K

$$\int\!\!\int_{(\kappa)} \frac{\partial P_1}{\partial y} dx dy = -\int_{\kappa} P_1 dx = -\int_{\kappa} P dx$$

since on K,  $P_1$  coincides with P.

Similarly  $Q_1(x, y)$  being defined by means of Q(x, y) in the same way as  $P_1$  by means of P (taking always parallels to Ox in place of parallels to Oy):

$$\int\!\!\int_{(K)} \frac{\partial Q_1}{\partial x} dx dy = \int_K Q_1 dy = \int_K Q dy$$

It follows that

(10) 
$$\int_{\mathbb{R}} P dx - Q dy = - \int \int_{(\mathbb{R})} \left( \frac{\partial P_1}{\partial y} + \frac{\partial Q_1}{\partial x} \right) dx dy.$$

The double integral of the right-hand side reduces to zero for those portions of the area (K) which belong to  $C_p$ , for at a point inside  $C_p$ 

$$\frac{\partial P_1}{\partial y} + \frac{\partial Q_1}{\partial x} = \frac{\partial P}{\partial y} + \frac{\partial Q}{\partial x} = 0.$$

The formula (10) then reduces to

(11) 
$$\int_{\mathbb{R}} P dx - Q dy = -\sum_{n} \int_{C_{n}} \int_{(p)} \left( \frac{\partial P_{1}}{\partial y} + \frac{\partial Q_{1}}{\partial x} \right) dx dy.$$

But the area of  $C_n^{(p)}$  is equal to  $\frac{\pi r_n^2}{4^p}$ ; on the other hand, the moduli of  $\frac{\partial P_1}{\partial y}$  and  $\frac{\partial Q_1}{\partial x}$  are less than a fixed number independent of n (depending on p, but p is fixed); then

$$\left| \int_{\kappa} P dx - Q dy \right| < M \sum_{n} r_{n}^{2}.$$

It is easy to obtain from this the formula (6); since the series  $\Sigma r_n^2$  is, in fact, convergent, we can choose n in such a way that the remainder of this series  $\Sigma_{n+1}^{\infty} r_n^2$  is less than  $\frac{\epsilon}{M}$ .

When the number n has been thus chosen, let us denote by K' the contour formed of the contour K traversed in the direct sense and the circumferences  $C_1^{(p)}$ ,  $C_2^{(p)}$ ,  $\cdots$ ,  $C_n^{(p)}$  traversed in the retrograde sense; we can argue about K' as we have done about K (by completing it if we wish by rectilinear cuts to make it a simple contour); we shall obtain

$$\left| \int_{K'} P dx - Q dy \right| < M \sum_{n=1}^{\infty} r_q^2 < \epsilon$$

that is, the integrals being taken in the direct sense

$$\left| \int_{\mathcal{K}} P dn - Q dy - \sum_{r=1}^{n} \int_{C_r^{(p)}} P dx - Q dy \right| < \epsilon.$$

If  $\epsilon$  is made to tend to zero, n increases indefinitely and from it we obtain the relation (8) from which the relation (6) follows.

We deduce now from (6) Cauchy's fundamental theorem; let x denote a point within a reduced region  $\Gamma_p$  and  $\gamma_q$  a circle with center x within Cp, and with radius between  $\frac{1}{2^{q}}$  and  $\frac{1}{2^{q-1}}$ .

There exists such a circle  $\gamma_q$ , whatever the number q (at least after a certain value of q). In fact x being within  $\Gamma_p$ , whatever n may be,  $a_n$  being the affix of  $A_n$ 

$$|x-a_n|>\frac{1}{2^p}P_n$$

Consequently, the points  $a_n$  for which

$$|x-a_n|<\frac{1}{2^{q-1}}$$

are such that

$$\frac{1}{2^p} P_n < \frac{1}{2^{n-1}}$$

Let us denote by  $n_a$  the smallest value of n after which this inequality (13) is satisfied; all the  $a_n$ 's inside the circle

of center x and radius  $\frac{1}{2^{q-1}}$  have indices greater or equal to  $n_q$ ; the sum  $\frac{1}{2^p}\Sigma r_n$  of the radii of the corresponding circles in  $C_p$  is then extremely small compared to  $\frac{1}{2^p}P_{n_q}$  since the  $r_n$ 's are much smaller than the corresponding  $P_n$ 's; since this sum is extremely small compared to  $\frac{1}{2^{q-1}}$ , there exist circles of center x and radius between  $\frac{1}{2^{q-1}}$  and  $\frac{1}{2^q}$  and which do not cut any of the circles  $C_p^{(p)}$ ; a fortiori they do not cut the circles  $C_n^{(p)}$  whose centers are more distant from x, for the radii  $\frac{r_n}{2^p}$  of these other circles are very small compared with  $\frac{P_n}{2^p}$  and their centers are further from x than  $\frac{P_n}{2^p}$ .

The circle  $\gamma_a$  being thus defined, let us consider the function F(z)

 $f(z) = \frac{F(z)}{z - x}$ 

within the region contained between the contour K and  $\gamma_q$ ; clearly in this region this function is monogenic; we then obtain the relation

$$\int_{R} f(z)dz - \int_{\gamma} f(z) dz = \sum_{C_{\mathbf{n}}^{(p)}} f(z)dz$$

the sum on the right-hand side referring to the  $C_n^{(p)}$ 's which are contained between  $\gamma_n$  and K.

If M denotes the maximum value of |F(z)| within  $C_p$  the maximum value of f(z) on different  $C_n^{(p)}$ 's is evidently  $2^{q+1}M$ ; if q+1 is put in the place of q, an infinity of new terms are introduced on the right-hand side, but it is easily seen that the lengths of the paths of integration (circumferences of the  $C_n^{(p)}$ 's contained between  $\gamma_q$  and  $\gamma_{q+1}$ ) have a

sum of an order much less than  $\frac{1}{2^{q+1}}$ ; the right-hand side is then a convergent series and

$$\lim_{q=\infty} \int_{\gamma_q} f(z) dz = \int_{K} f(z) dz - \sum_{C_n^{(p)}} f(z) dz$$

the sign  $\Sigma$  now referring to all the circumferences  $C_n^p$  which limit  $C_p$ . As for the left-hand side, it follows from the continuity of F(z) at the point x in  $C_p$ , all the  $\gamma_q$ 's being interior to  $C_p$ , that it is equal to  $2 \pi i F(x)$ . The generalized Cauchy formula follows

(14) 
$$2 \pi i F(x) = \int_{K} \frac{F(z) dz}{z - x} - \sum_{n} \int_{C_{n}^{(p)}} \frac{F(z) dz}{z - x}.$$

From this formula the classical consequences can be deduced and in particular the fact that monogeneity (existence of the first derivative) within the region C involves the existence of the derivatives of all orders. This formula (14) shows moreover that non-analytic monogenic functions can be put in the form of series whose terms are analytic functions. It is natural then to look for an associative method of continuation applicable to such sums. The problem is nothing else than the problem of divergent series: to each analytic function corresponds a Taylor development convergent in a circle, but divergent outside this circle; this development is determined by a knowledge of the values of the derivatives. If a series of analytic functions is indefinitely differentiable, its derivatives are expressed linearly by means of the derivations of the terms, and the Taylor series which corresponds to these derivatives is a linear function of the Taylor series corresponding to the different terms of the series. But if the function is not analytic at the point where the series is developed, this Taylor series will be the sum of series whose radii of convergence decrease indefinitely and, in the case we are studying, will have a zero radius of convergence.

The problem of divergent series consists in transforming such a series into a convergent series in such a way that the result coincides with the analytic continuation in the case where this continuation is possible. Thanks to the fine researches of M. Mittag-Leffler, this problem has been resolved for the first time in an entirely satisfactory way; it should be observed that, if it is desired to use these results for the continuation of non-analytic monogenic functions, they must be interpreted either by the language of divergent series, or by an equivalent language if one prefers not to speak of divergent series; but in every case by a new language, specially adapted to the real novelty of the results, and not by the old language of Weierstrassian analytic continuation; that is the only language which may not be used, since it has an absolutely precise meaning, which cannot be modified: Weierstrass's theory is, in some way, so perfect that it can only be departed from by creating a new language: if, as M. Mittag-Leffler proposed, Weierstrass's language were adopted, M. Mittag-Leffler's series would be only a simplified method of calculation containing nothing more from the theoretical point of view than Weierstrass's theory contains.

## III. CONTINUATION BY SERIES (M)

In order to study continuation by M. Mittag-Leffler's series, or series (M), we suppose that the point is interior to a reduced region  $\Gamma'_p$ , of order equal to 2 with respect to  $\Gamma_p$  (the circles of exclusion are defined by numbers  $\rho'_n$  equal to  $\sqrt{\rho_n}$ ); evidently then an infinity of straight lines issuing from the point x can be drawn interior to  $\Gamma_p$ . More precisely, if x belongs to  $\Gamma'$ , within every given angle having its vertex x, a straight line interior to  $\Gamma_{p'}$ , of convenient index, can be found; this index can increase in-

definitely as the angle tends to zero, but is determinate when the angle is given (this follows from the fact that the sum of the angles subtended at x by the circles which limit  $\Gamma_{p'}$  is less than twice the sum of the convergent series  $\sum \left(\frac{\rho_n}{2^{p'}} : \frac{\rho'_n}{2^{p'}}\right)$  and is consequently as small as we please if p' is sufficiently large). We shall suppose, so as not to complicate our notation, that p has been taken equal to p' in the preceding argument (the point x interior to  $\Gamma_p$  is a fortiori interior to  $\Gamma_{p'}$  if p' > p).

We develop F(z) in a series on one of the straight lines which we are about to define, interior to  $\Gamma_p$ . Each of the terms of the right-hand side of (14) is an analytic function on this straight line and can therefore be developed in a series of Mittag-Leffler or (M) polynomials; it is enough to show that the multiple series formed of the set of these series is absolutely convergent, in order to show that it represents  $2 \pi i F(z)$ .

This series is then formed by means of the derivatives of F(z) at the point x (these derivatives exist, as we have remarked, according to (14) for every displacement on the straight line and in  $\Gamma_p$ ), in the same way as the (M) development of an analytic function is formed by means of the derivatives of that function; we assume, to save writing, that x = 0.

I remind the reader of the properties of (M) developments which I have demonstrated in my memoir on series of polynomials and rational fractions ("Acta Matematica," I, xxiv). One finds that

$$\frac{1}{1-z}=\sum G_n(z),$$

 $G_n(z)$ 's being polynomials which it is useless to write again and the series  $\sum |G_n(z)|$  being convergent in the 'star.' A

region  $S(R, \rho)$  is defined as follows: R being > 1 and  $\rho < 1$ , we consider the circle of center o and radius R, the circle of center 1 and radius  $\rho$  and the tangents to this last circle from 0, the points of contact being M and N; the region  $S(R, \rho)$  is bounded by the arc MN less than  $\pi$ , the continuations MM' and NN' of OM and ON as far as the circumference of radius R and the arc M'N' greater than  $\pi$ . In this region, putting  $\frac{3^2 R}{\rho} = \lambda$ ,

(16) 
$$\sum |G_n(z)| < R^{\lambda^{\lambda}}.$$

Consider an integral along one of the circumferences  $C_n^{(p)}$  of radius  $\frac{r_n}{2^p}$ .

$$\int_{C_n^{(p)}} f(z) dz$$

We develop it on a straight line interior to  $\Gamma_p$ , that is outside the circumference having the same center  $a_n$  as  $C_n^{(p)}$  and of radius  $\frac{\rho_n}{2^p}$ . The radius  $\frac{r^n}{2^p}$  being very small compared with  $\rho_n$ , we shall commit no appreciable error by replacing this integral by the majorant function  $\frac{Mr_n}{a_n-x}$ , denoting by M the maximum of |F(z)| in  $C_p$ ,  $2\pi r_n$  being the length of the path of integration (we suppress the factors  $2^p$  which have no influence since p is fixed). If one puts  $x = a_n x'$ ,

$$\frac{Mr_n}{a_n-x}=\frac{Mr_n}{a_n} \quad \frac{1}{1-x^i}.$$

If the point x is inside the region  $S(R, \rho)$  defined by the circle of radius  $\frac{\rho_n}{2^p}$  and center  $A_n$  and by a circle of radius > 1 (2 for example) which contains within it all the regions we are considering, the point  $x' = \frac{x}{a_n}$  will be within the region

 $S\left(\frac{2}{|a_n|}, \frac{\rho_n}{|a_n|}\right)$ , and by developing  $\frac{1}{1-x'}$  we get the inequality

$$\sum |G_n(x')| < \left[\frac{2}{|a_n|}\right]^{\lambda^{\lambda}}$$

putting  $\lambda = \frac{64}{\rho_n}$ ; since  $|a_n|$  is greater than  $\rho_n$  we can write

$$\sum |G_n(x')| < \lambda^{\lambda^{\lambda}}.$$

The development (M) of (17) is, according to (18), when all the terms are replaced by their moduli, less than

But according to (2)
$$\frac{Mr_n}{|a_n|} \lambda^{\lambda^{\lambda}}$$

$$\frac{1}{n} > e^{e^{\frac{1}{\rho_n^2}}}$$

and if *n* is large enough  $\frac{1}{\rho_n^2} > \lambda^{\frac{3}{2}}$  since  $\lambda = \frac{64}{\rho_n}$  and so  $|a_n|$  being  $> \rho_n$ ,

$$M\frac{r_n}{|a_n|}\lambda^{\lambda^{\lambda}} < \lambda M \lambda^{\lambda^{\lambda}} e^{-c^{\lambda^{\frac{3}{2}}}}.$$

This converges very rapidly to zero when n, and consequently  $\lambda$ , increases indefinitely. The absolute convergence of the (M) series is then demonstrated.

Now consider two points  $x_1$  and  $x_2$  belonging to  $\Gamma'$ ; we can construct two angles  $A_1$  and  $A_2$  with vertices at  $x_1$  and  $x_2$ , and such that every half-straight line  $D_1$  within  $A_1$  meets every half-straight line  $D_2$  within  $A_2$  at a point  $x_3$  within the total region considered. We can choose  $D_1$  and  $D_2$  in such a way that these two straight lines belong to the same  $\Gamma_p$  (p being chosen large enough, but afterwards remaining fixed). It will then be possible to calculate the function at  $x_2$  by means of its values and the values of its derivatives at  $x_1$ , by forming only two (M) developments, one with the

origin  $x_1$  and the other with the origin  $x_3$ . If the function is zero at x, as well as all its derivatives, these (M) developments are identically zero and the function is zero at  $x_2$ . From what has been said further back it can be concluded that if a monogenic function is zero at every point of an arc, however small (at all points of this arc interior to C), when there exists on this arc at least one point interior to  $\Gamma_p$  a limit of points interior to  $\Gamma_p$ , the function being zero at all these points is zero, as well as its derivatives, at one point of  $\Gamma_p$  at least, and consequently identically zero in  $\Gamma_p$  (whatever p may be) and identically zero in C. These new monogenic functions possess then the fundamental property of analytic functions.

#### IV. THE LOGARITHMIC POTENTIAL

In the preceding work we have considered singular isolated points, corresponding in the physical point of view to the hypothesis of an infinite density at certain points; a statement can easily be given in which the density is everywhere finite.

Consider a regular uniform analytic function zero at infinity. If  $\Sigma$  is a circle such that all the singular points of the function are inside  $\Sigma$ , if  $\zeta$  is any point outside  $\Sigma$ ,

$$F(\zeta) = \frac{1}{2\pi i} \int_{\Sigma} \frac{F(z)dz}{\zeta - z}$$

the integration being taken in the direct sense.

Let  $\Sigma_1$  and  $\Sigma_2$  be two concentric circles outside  $\Sigma$ , let a be the center of these circles,  $\rho_1$  and  $\rho_2$  their radii.

Evidently, if  $\rho$  is contained between  $\rho_1$  and  $\rho_2$ ,

$$F(\zeta) = \frac{1}{2\pi} \int_0^{2\pi} \frac{F(a + \rho e^{i\alpha}) \rho e^{i\alpha} d\alpha}{\zeta - a - e^{i\alpha}}.$$

$$[422]$$

If we multiply this equality by  $(\rho_2 - \rho)^m (\rho - \rho_1)^n$  and integrate between the limits  $\rho_1$  and  $\rho_2$ , the expression becomes

$$F(\zeta) \int_{\rho_{1}}^{\rho_{2}} (\rho_{2} - \rho)^{m} (\rho - \rho_{1})^{n} d\rho$$

$$= \frac{1}{2 \pi} \int_{a}^{2\pi} \frac{F(a + \rho e^{ia}) (\rho_{2} - \rho)^{m} (\rho - \rho_{1})^{n} \rho e^{ia} d\alpha d\zeta}{\zeta - a - \rho^{ia}}$$
Put
$$\int_{r_{1}}^{r_{2}} (\rho_{2} - \rho)^{m} (\rho - \rho_{1})^{n} dr = \frac{1}{2 \pi} H$$

$$a + \rho e^{ia} = x + iy.$$
Then
$$F(\zeta) = \frac{1}{H} \int \int_{c_{1}c_{2}} \frac{F(x + iy) (\rho_{2} - \rho)^{m} (\rho - \rho_{1})^{n}}{\zeta - x - iy} e^{ia} dx dy$$
or putting
$$\zeta = \xi + i\eta$$
(19)
$$F(\zeta) = \theta(\xi, \eta) = \int \int_{c_{1}c_{2}} \frac{\phi(x, y) dx dy}{\xi + in - x - iy}$$

the region of integration being the ring contained between the circles  $C_1$  and  $C_2$ .

We shall define the function  $\phi(x, y)$  outside this ring by giving it the value zero; the whole plane can then be taken as the region of integration. The function  $\phi(x, y)$  is bounded and continuous in the whole plane; its derivatives are also bounded, at least as far as order m on  $C_1$  and as far as order n on  $C_2$ ; by an artifice analogous to that which we are about to employ, it would be easy to arrange matters so that all the derivatives would be continuous; in general it is enough to know that the derivatives are continuous as far as some order, fixed beforehand.

If the function F(z) has a singular point a,  $\rho_1$  can be made to tend to zero and if, further, the product  $\rho^m F(z)$  remains finite for z = a, the formula holds for  $\rho_1 = 0$ ; if this product does not remain finite, in the formula we replace  $(\rho - \rho_1)^m$ 

by  $e^{-\frac{1}{\rho}}$  or  $e^{-e^{\rho}}$  etc. Further, in the case of a unique singular point, the circle  $C_2$  can be drawn with a radius as small as we please, after the circle  $C_1$  has been reduced to zero.

It is easy to deduce from this that every regular analytic uniform function, zero at infinity, can be represented in every region D interior to its region of existence W, and approaching W as nearly as we wish, by an expression of the form

(20) 
$$F(\zeta) = \theta(\xi, \eta) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{\phi(x, y) dx dy}{\xi + i\eta - x - iy}$$

the function  $\phi(x, y)$  being bounded and, further, zero at all points of D (this hypothesis involves the fact that  $\phi(x, y)$  is zero at infinity, since the point at infinity belongs to D).

Inversely every expression of the form (20) in which  $\phi(x, y)$  is a bounded function, zero at infinity, and continuous in the whole plane, as well as its derivatives (at least up to order m), represents a function which is monogenic at every point where  $\phi(x, y)$  is zero; for by a simple calculation,

 $\frac{\partial \theta}{\partial \xi} + i \frac{\partial \theta}{\partial \eta} = 2 \pi \phi(\xi, \eta).$ 

If the points where  $\phi(x,y)$  is zero form a W region, the theory of analytic functions shows us that the function  $\theta(\xi,\eta)$  is determined at every point of W by the knowledge of its values in the neighborhood of any particular point of W. The problem of the general determination of the region of existence of monogenic functions can then be set as follows: to find the conditions which  $\phi(x,y)$  should satisfy in order that this fundamental property of  $\theta(\xi,\eta)$  should hold; that is, that the knowledge of this function on an arc of a curve where it is monogenic allows the calculation of its value in the whole region of monogeneity.

Consider, for example, the series of rational fractions with simple poles. Denoting by  $C_a$  a circle with center a and radius  $\rho$ , and taking a point  $\zeta$  outside this circle for which

$$|z-a|=r,$$
 
$$\frac{1}{\xi-a}=\int\int_{C_a}\frac{3(\rho-r)}{\pi\rho^3}\frac{dxdy}{\zeta-z}.$$

When the point  $\zeta$  is inside the circle  $C_a$ , the integral is easily calculated; putting  $\left|\frac{\zeta-a}{\varrho}\right| = \lambda$  its value is

$$\frac{3\lambda^2-2\lambda^3}{\zeta-a}.$$

The function

$$\theta(\xi, \eta; a) = \int \int_{C_a} \frac{3(\rho - r)}{\pi \rho^3} \frac{dxdy}{\xi + i\eta - x - iy}$$

is then bounded in the whole plane; outside  $C_a$  it is monogenic and coincides with the analytic function  $\frac{1}{r-a}$ .

Evidently an infinity of functions  $\theta_n(\xi, \eta)$  can be defined in a similar way, such that the equation

$$\theta_n(\xi, \eta) = \frac{1}{\zeta - a_n}$$

holds for every point  $\zeta = \xi + i\eta$  outside the circle  $C_n$  with center  $a_n$  and radius  $\rho_n$ , these functions being moreover bounded and continuous in the whole plane; if the  $|a_n|$ 's are bounded and if the coefficients  $A_n$  are such that the series

$$\sum \frac{|A_n|}{\rho_n^3}$$

is convergent, the series

$$\theta(\xi, \eta) = \sum \theta_n(\xi, \eta)$$

will be absolutely and uniformly convergent in the whole plane, and will be represented by an integral of the form

(21) 
$$\theta(\xi, \eta) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{\phi(x, y) dx dy}{\xi + i\eta - x - iy},$$

the function  $\phi(x, y)$  being the sum of a series everywhere convergent whose respective terms are zero outside various circles  $C_n$ ; this function  $\phi(x, y)$  is then zero at all points exterior to all these circles and the function  $\theta(\xi, \eta)$  is monogenic at these points. If the radii  $\rho_n$  are replaced by  $\epsilon \rho_n$ ,  $\epsilon$  being as small as we please, the function  $\phi(x, y)$  is zero in a more and more extended region; it remains bounded, but its bound increases indefinitely as  $\epsilon$  tends to zero. We are thus led to consider a priori a function such as (21) and to study it in the region C where  $\phi(x, y)$  is zero. It is natural to suppose the region C to be simply connected; we limit ourselves to the case where this region C consists of W regions (these regions may reduce to a zero as a limiting case) and of a finite or infinite number of straight lines  $\Delta$ , in such a way that any two points can be reunited by a polygonal line with a finite number of sides.

An important idea is then that of the order of infinity of the function  $\phi(x, y)$  in the neighborhood of the straight lines. By a calculation analogous to that which has just been developed, the convergence of the (M) developments can be shown by making the hypothesis that  $\phi(x, y)$  is not only zero upon the straight lines (which is the necessary condition of monogeneity) but tends very rapidly to zero in the neighborhood of each straight line. More precisely, if  $\sigma$  denotes the distance of the point (x, y) from the straight line  $\Delta$  considered, it is assumed that the product

$$e^{\frac{1}{\sigma^2}}\phi(x, y)$$

tends uniformly to zero as  $\sigma$  tends to zero. By means of this hypothesis, it can be affirmed that the function  $\theta(\xi, \eta)$  is determined in the whole region of its existence by the knowledge of its values at any point of this region. This hypothesis contains as a special case the condition satisfied

by analytic functions in W regions, for if a straight line is within W, the function  $\phi(x, y)$  is identically zero at all points whose distances from the straight line are less than a number  $\sigma$ , chosen conveniently.

The region C can be reduced to the real axis; that is the case of the function

$$\phi(\xi) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{e^{-\frac{1}{cx^2}} dx \, dy}{(x^2 + y^2)(\xi - x - iy)}.$$

The Taylor development

$$\theta(\xi) = \theta(\xi_0) + (\xi - \xi_0)\theta'(\xi_0) + \cdots$$

diverges for any value of  $\xi_0$  but is summable (M), whatever  $\xi_0$  may be, for every value of  $\xi$ , its sum being equal to the function  $\theta(\xi)$ . The function  $\theta(\xi)$  will be called quasi-analytic.

Calculations of double integrals of form (21) lead easily to expressions of the same form; similarly in differentiating, transforming the double integral by integration by parts, it is only necessary to assume the existence of the derivatives of  $\phi(x, y)$  exactly to the order of the derivatives of  $\theta(\xi, \eta)$  which it is desired to calculate. To calculate the product, if we put

(22) 
$$\theta(\xi, \eta) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{\phi(x, y) dx dy}{\zeta - z}.$$

(23) 
$$\theta_1(\xi, \eta) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{\phi_1(x, y) dx dy}{\zeta - z},$$

the product becomes

$$\theta(\xi, \eta)\theta_1(\xi, \eta) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{\phi(x, y)\phi_1(x_1, y_1)dxdy}{(\zeta - z)(\zeta - z_1)} \frac{dx}{\zeta} \frac{dx_1dy_1}{(\zeta - z)(\zeta - z_1)},$$
or since
$$\frac{1}{(\zeta - z)(\zeta - z_1)} = \frac{1}{z - z_1} \left(\frac{1}{\zeta - z} - \frac{1}{\zeta - z_1}\right),$$

if we put

$$\psi(x, y) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{\phi(x_1, y_1) dx_1 dy_1}{z - z_1},$$

$$\psi_1(x, y) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{\phi_1(x_1, y_1) dx_1 dy_1}{z - z_1},$$

(24) 
$$\theta(\xi, \eta)\theta_1(\xi, \eta) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \left[ \frac{\phi(x, y)\psi_1(x, y) + \psi(x, y)\phi_1(x, y)}{\zeta - z} \right] \frac{dxdy}{\zeta - z}.$$

We can then put in the form of a double integral (24) every polynomial P in terms of one or more functions of  $\theta(\xi, \eta)$  and their derivatives; if the regions of existence have a simply connected common region the differential equation obtained by equating P to zero cannot be satisfied in any portion of this region without being satisfied in the whole region C.

#### V. CONCLUSION

The results we are establishing suppress the absolutely sharp demarcation established by Weierstrass's theory between real analytic functions and real non-analytic functions. I do not wish to develop the consequences of this fact from the point of view of the theory of functions; I prefer to insist a little on its importance from the point of view of the relations between mathematics and physics. It is a necessary postulate in the application of mathematics to experimental sciences, that sufficiently slight variations in the data ought not to influence the results appreciably; for, if it were not so, since the experimental data are never known with vigorous precision, one could not foresee any phenomenon. But certain mathematical properties are at least apparently discontinuous, depending for example on

the fact that some number is rational or irrational. Thus the solutions of the equation

$$\frac{d^2y}{dx^2} + m^2y = \cos nx$$

are of a different nature according as the ratio  $\frac{m}{n}$  is commensurable or incommensurable. Nevertheless, in this case, if  $\frac{m}{n}$  varies continuously, the solution y varies very little in an interval of variation of x large compared with the length of the periods. It is not always thus, certainly, but the cases in which there is no continuity have been little studied; the equation

$$\left(\frac{dx}{dt}\right)^2 + x^2 = a^2 + 2 \lambda t$$

can be given as an example in which the solutions vary discontinuously as  $\lambda$  becomes equal to zero; but this equation does not come under the Hamiltonian type.

It is important to know whether the properties of harmonic functions (that is, of potentials) vary continuously when the definition of the functions itself varies continuously. This has no place in Weierstrass's theory; the introduction of quasi-analytic functions restores continuity; a distribution of attracting masses infinitely near to Ox leads, if the density is sufficiently slight in the neighborhood of Ox, to properties of the potential on Ox which are not dissimilar from the case where the density is zero in the neighborhood of Ox.

EMILE BOREL.

## THE BREVIARY OF ÆSTHETIC'

I

#### "WHAT IS ART?"

In reply to the question, "What is art?", it might be said jocosely (but this would not be a bad joke) that art is what everybody knows it to be. And indeed, if it were not to some extent known what it is, it would be impossible even to ask that question, for every question implies a certain knowledge of what is asked about, designated in the question and therefore known and qualified. A proof of this is to be found in the fact that we often hear expressed just and profound ideas in relation to art by those who make no profession of philosophy or of theory, by laymen, by artists who do not like to reason, by the ingenuous, and even by the common people: these ideas are sometimes implicit in judgments concerning particular works of art, but at others assume altogether the form of aphorisms and of definitions. Thus it happens that there arises the belief in the possibility of making blush, at will, any proud philosopher who should believe himself to have "discovered" the nature of art. by placing before his eyes or making ring in his ears propositions taken from the most superficial books or phrases of the most ordinary conversation, and shewing that they already most clearly contained his vaunted discovery.

And in this case the philosopher would have good reason to blush—that is, had he ever nourished the illusion of introducing into universal human consciousness, by means of his

<sup>&</sup>lt;sup>1</sup> A monograph prepared for the inauguration of the Rice Institute, by Benedetto Croce, Senator of the Kingdom of Italy, Member of several Royal Commissions, Editor of "La Critica." Translated from the Italian by Douglas Ainslie, B.A. Oxon., of The Athenæum, London, England.





doctrines, something altogether original, something extraneous to this consciousness, the revelation of an altogether new world. But he does not blush, and continues upon his way, for he is not ignorant that the question as to what is art (as indeed every philosophical question as to the nature of the real, or in general every question of knowledge), even if by its use of language it seem to assume the aspect of a general and total problem, which it is claimed to solve for the first and last time, has always, as a matter of fact, a circumscribed meaning, referable to the particular difficulties that assume vitality at a determined moment in the history of thought. Certainly, truth does walk the streets, like the esprit of the well-known French proverb, or like metaphor, "queen of tropes" according to rhetoricians, which Montaigne discovered in the babil of his chambrière. But the metaphor used by the maid is the solution of a problem of expression proper to the feelings that affect the maid at that moment; and the obvious affirmations that by accident or intent one hears every day as to the nature of art, are solutions of logical problems, as they present themselves to this or that individual, who is not a philosopher by profession, and yet as man is also to some extent a philosopher. And as the maid's metaphor usually expresses but a small and vulgar world of feeling compared with that of the poet, so the obvious affirmation of one who is not a philosopher solves a problem small by comparison with that which occupies the philosopher. The answer as to what is art may appear similar in both cases, but is different in both cases owing to the different degree of richness of its intimate content; because the answer of the philosopher worthy of the name has neither more nor less than the task of solving in an adequate manner all the problems as to the nature of art that have arisen down to that moment in the course of history;

whereas that of the layman, since it revolves in a far narrower space, shews itself to be impotent outside those limits. Actual proof of this is also to be found in the force of the eternal Socratic method, in the facility with which the learned, by pressing home their questions, leave those without learning in open-mouthed confusion, though these had nevertheless begun by speaking well; but now finding themselves, in the course of the inquiry, in danger of losing what small knowledge they possessed, they have no resource but to retire into their shell, declaring that they do not like "subtleties."

The philosopher's pride is solely based therefore upon the greater intensity of his questions and answers; a pride not unaccompanied with modesty—that is, with the consciousness that if his sphere be wider, or the largest possible, at a determined moment, yet it is limited by the history of that moment, and cannot pretend to a value of totality. or what is called a definite solution. The ulterior life of the spirit, renewing and multiplying problems, does not so much falsify, as render inadequate preceding solutions, part of them falling among the number of those truths that are understood, and part needing to be again taken up and integrated. A system is a house, which, as soon as it has been built and decorated, has need of continuous labour, more or less energetic, in order to keep it in repair (subject as it is to the corrosive action of the elements); and at a certain moment there is no longer any use in restoring and propping up the system, we must demolish and reconstruct it from top to bottom. But with this capital difference: that in the work of thought, the perpetually new house is perpetually maintained by the old one, which persists in it, almost by an act of magic. As we know, those superficial or ingenuous souls that are ignorant of this magic are terrified at it; so much so,

that one of their tiresome refrains against philosophy is that it continually undoes its work, and that one philosopher contradicts another: as though man did not always make and unmake his houses, and as though the architect that follows did not always contradict the architect that precedes; and as though it were possible to draw the conclusion from this making and unmaking of houses and from this contradiction among architects, that it is useless to make houses!

The answers of the philosopher, though they have the advantage of greater intensity, also carry with them the dangers of greater error, and are often vitiated by a sort of lack of good sense, which has an aristocratic character, in so far as it belongs to a superior sphere of culture, and even when meriting reproof, is the object not only of disdain and derision, but also of secret envy and admiration. This is the foundation of the contrast, that many delight to illustrate, between the mental equilibrium of ordinary people and the extravagances of philosophers; since, for example, it is clear that no man of good sense would have said that art is a reflexion of the sexual instinct, or that it is something maleficent and deserves to be banned from well-ordered republics. These absurdities have, however, been uttered by philosophers and even by great philosophers. the innocence of the man of common sense is poverty, the innocence of the savage; and though there have often been sighs for the life of the savage, and a remedy has been called for to rescue good sense from philosophies, it remains a fact that the spirit, in its development, courageously affronts the dangers of civilisation and the momentary loss of good sense. The researches of the philosopher in relation to art must tread the paths of error in order to find the path of truth, which does not differ from, but is, those very paths of error which contain a clue to the labyrinth.

The close connection of error and truth arises from the fact that a complete and total error is inconceivable, and, since it is inconceivable, does not exist. Error speaks with two voices, one of which affirms the false, but the other denies it; it is a colliding of yes and no, which is called contradiction. Therefore, when we descend from general considerations to the examination of a theory that has been condemned as erroneous in its definite particulars, we find the cure in the theory itself—that is, the true theory, which grows out of the soil of error. Thus it happens that those very people who claim to reduce art to the sexual instinct, in order to demonstrate their thesis have recourse to arguments and meditations which, instead of uniting, separate art from that instinct; or that he who would expel poetry from the well-constituted republic, shudders in so doing, and himself creates a new and sublime poetry. There have been historical periods in which the most crude and perverted doctrines of art have dominated; yet this did not prevent the habitual and secure separation of the beautiful from the ugly at those periods, nor the very subtle discussion of the theme when the abstract theory was forgotten and particular cases were studied. Error is always condemned, not by the mouth of the judge, but ex ore suo.

Owing to this close connection with error, the affirmation of the truth is always a process of strife, by means of which it keeps freeing itself in error from error; whence arises another pious but impossible desire, namely, that which demands that truth should be directly exposed, without discussion or polemic; that it should be permitted to proceed majestically alone upon its way: as if this stage parade were the symbol suited to truth, which is thought itself, and, as thought, ever active and in labour. Indeed, nobody succeeds in exposing a truth, save by criticising the different solutions

of the problem with which it is connected; and there is no philosophical treatise, however weak, no little scholastic manual or academic dissertation, which does not collect at its beginning or contain in its body a review of opinions, historically given or ideally possible, which it wishes to oppose or to correct. This fact, though frequently realised in a capricious and disorderly manner, just expresses the legitimate desire to pass in review all the solutions that have been attempted in history or are possible of achievement in idea (that is, at the present moment, though always in history), in such a way that the new solution shall include in itself all the preceding labour of the human spirit.

But this demand is a logical demand, and as such intrinsic to every true thought and inseparable from it; and we must not confound it with a definite literary form of exposition, in order that we may not fall into the pedantry for which the scholastics of the Middle Ages and the dialecticians of the school of Hegel in the nineteenth century became celebrated, which is very closely connected with the formalistic superstition, and represents a belief in the marvellous virtue of a certain sort of extrinsic and mechanical philosophical exposition. We must, in short, understand it in a substantial, not in an accidental sense, respecting the spirit, not the letter, and proceed with freedom in the exposition of our own thought, according to time, place, and person. Thus, in these rapid lectures intended to provide as it were a guide to the right way of thinking out problems of art, I shall carefully refrain from narrating (as I have done elsewhere) the whole process of liberation from erroneous conceptions of art, mounting upwards from the poorest to the richest: and I shall cast far away, not from myself, but from my readers, a part of the baggage with which they will charge themselves when, prompted thereto by the sight of

the country passed over in our bird's flight, they shall set themselves to accomplish more particular voyages in this or that part of it, or to cross it again from end to end.

However, connecting the question which has given occasion to this indispensable prologue (indispensable for the purpose of removing from my discourse every appearance of pretentiousness, and also all blemish of inutility),—the question as to what is art,—I will say at once, in the simplest manner, that art is vision or intuition. The artist produces an image or a phantasm; and he who enjoys art turns his gaze upon the point to which the artist has pointed, looks through the chink which he has opened, and reproduces that image in himself. "Intuition," "vision," "contemplation," "imagination," "fancy," "figurations," "representations," and so on, are words continually recurring, like synonyms, when discoursing upon art, and they all lead the mind to the same conceptual sphere which indicates general agreement.

But this reply, that art is intuition, obtains its force and meaning from all that it implicitly denies and from which it distinguishes art. What negations are implicit in it? I shall indicate the principal, or at least those that are the most important for us at this present moment of our culture.

It denies, above all, that art is a physical fact: for example, certain determined colours, or relations of colours; certain definite forms of bodies; certain definite sounds, or relations of sounds; certain phenomena of heat or of electricity—in short, whatsoever be designated as "physical." The inclination toward this error of physicising art is already present in ordinary thought, and as children who touch the soap-bubble and would wish to touch the rainbow, so the human spirit, admiring beautiful things, hastens spontaneously to trace out the reasons for them in external nature, and proves that it must think, or believes that it

should think, certain colours beautiful and certain other colours ugly, certain forms beautiful and certain other forms ugly. But this attempt has been carried out intentionally and with method on several occasions in the history of thought: from the "canons" which the Greek theoreticians and artists fixed for the beauty of bodies, through the speculations as to the geometrical and numerical relations of figures and sounds, down to the researches of the æstheticians of the nineteenth century (Fechner, for example), and to the "communications" presented in our day by the inexpert, at philosophical, psychological, and natural science congresses, concerning the relations of physical phenomena with art. And if it be asked why art cannot be a physical fact, we must reply, in the first place, that physical facts do not possess reality, and that art, to which so many devote their whole lives and which fills all with a divine joy, is supremely real; thus it cannot be a physical fact, which is something unreal. This sounds at first paradoxical, for nothing seems more solid and secure to the ordinary man than the physical world; but we, in the seat of truth, must not abstain from the good reason and substitute for it one less good, solely because the first should have the appearance of a lie; and besides, in order to surpass what of strange and difficult may be contained in that truth, to become at home with it, we may take into consideration the fact that the demonstration of the unreality of the physical world has not only been proved in an indisputable manner and is admitted by all philosophers (who are not crass materialists and are not involved in the strident contradictions of materialism), but is professed by these same physicists in the spontaneous philosophy which they mingle with their physics, when they conceive physical phenomena as products of principles that are beyond experience, of atoms or of ether,

or as the manifestation of an Unknowable: besides, the matter itself of the materialists is a supermaterial principle. Thus physical facts reveal themselves, by their internal logic and by common consent, not as reality, but as a construction of our intellect for the purposes of science. Consequently, the question whether art be a physical fact must rationally assume this different signification: that is to say, whether it be possible to construct art physically. And this is certainly possible, for we indeed carry it out always, when, turning from the sense of a poem and ceasing to enjoy it, we set ourselves, for example, to count the words of which the poem is composed and to divide them into syllables and letters; or, disregarding the æsthetic effect of a statue, we weigh and measure it: a most useful performance for the packers of statues, as is the other for the typographers who have to "compose" pages of poetry; but most useless for the contemplator and student of art, to whom it is neither useful nor licit to allow himself to be "distracted" from his proper object. Thus art is not a physical fact in this second sense, either; which amounts to saying that when we propose to ourselves to penetrate its nature and mode of action, to construct it physically is of no avail.

Another negation is implied in the definition of art as intuition: if it be intuition, and intuition is equivalent to theory in the original sense of contemplation, art cannot be a utilitarian act; and since a utilitarian act aims always at obtaining a pleasure and therefore at keeping off a pain, art, considered in its own nature, has nothing to do with the useful and with pleasure and pain, as such. It will be admitted, indeed, without much difficulty, that a pleasure as a pleasure, any sort of pleasure, is not of itself artistic; the pleasure of a drink of water that slakes thirst, or a walk in the open air that stretches our limbs and makes our blood

circulate more lightly, or the obtaining of a longed-for post that settles us in practical life, and so on, is not artistic. Finally, the difference between pleasure and art leaps to the eyes in the relations that are developed between ourselves and works of art, because the figure represented may be dear to us and represent the most delightful memories, and at the same time the picture may be ugly; or, on the other hand, the picture may be beautiful and the figure represented hateful to our hearts, or the picture itself, which we approve as beautiful, may also cause us rage and envy, because it is the work of our enemy or rival, for whom it will procure advantage and on whom it will confer new strength: our practical interests, with their relative pleasures and pains, mingle and sometimes become confused with art and disturb, but are never identified with, our æsthetic interest. At the most it will be affirmed, with a view to maintaining more effectively the definition of art as the pleasurable, that it is not the pleasurable in general, but a particular form of the pleasurable. But such a restriction is no longer a defence, it is indeed an abandonment of that thesis; for given that art is a particular form of pleasure, its distinctive character would be supplied, not by the pleasurable, but by what distinguishes that pleasurable from other pleasurables, and it would be desirable to turn the attention to that distinctive elementmore than pleasurable or different from pleasurable. Nevertheless, the doctrine that defines art as the pleasurable has a special denomination (hedonistic æsthetic), and a long and complicated development in the history of æsthetic doctrines: it shewed itself in the Græco-Roman world, prevailed in the eighteenth century, reflowered in the second half of the nineteenth, and still enjoys much favour, being especially well received by beginners in æsthetic, who are above all struck by the fact that art causes pleasure. The life of this

doctrine has consisted of proposing in turn one or another class of pleasures, or several classes together (the pleasure of the superior senses, the pleasure of play, of consciousness of our own strength, of criticism, etc., etc.), or of adding to it elements differing from the pleasurable, the useful for example (when understood as distinct from the pleasurable), the satisfaction of cognoscitive and moral wants, and the like. And its progress has been caused just by this restlessness, and by its allowing foreign elements to ferment in its bosom, which it introduces through the necessity of somehow bringing itself into agreement with the reality of art, thus attaining to its dissolution as hedonistic doctrine and to the promotion of a new doctrine, or at least to drawing attention to its necessity. And since every error has its element of truth (and that of the physical doctrine has been seen to be the possibility of the physical "construction" of art as of any other fact), the hedonistic doctrine has its eternal element of truth in the placing in relief the hedonistic accompaniment, or pleasure, common to the æsthetic activity as to every form of spiritual activity, which it has not at all been intended to deny in absolutely denying the identification of art with the pleasurable, and in distinguishing it from the pleasurable by defining it as intuition.

A third negation, effected by means of the theory of art as intuition, is that art is a moral act; that is to say, that form of practical act which, although necessarily uniting with the useful and with pleasure and pain, is not immediately utilitarian and hedonistic, and moves in a superior spiritual sphere. But the intuition, in so far as it is a theoretic act, is opposed to the practical of any sort. And in truth, art, as has been remarked from the earliest times, does not arise as an act of the will; good will, which constitutes the honest man, does not constitute the artist. And since it is not the result of an

act of will, so it escapes all moral discrimination, not because a privilege of exemption is accorded to it, but simply because moral discrimination cannot be applied to art. An artistic image portrays an act morally praiseworthy or blameworthy; but this image, as image, is neither morally praiseworthy nor blameworthy. Not only is there no penal code that can condemn an image to prison or to death, but no moral judgment, uttered by a rational person, can make of it its object: we might just as well judge the square moral or the triangle immoral as the Francesca of Dante immoral or the Cordelia of Shakespeare moral, for these have a purely artistic function, they are like musical notes in the souls of Dante and of Shakespeare. Further, the moralistic theory of art is also represented in the history of æsthetic doctrines, though much discredited in the common opinion of our times. not only on account of its intrinsic demerit, but also, in some measure, owing to the moral demerit of certain tendencies of our times, which render possible, owing to psychological dislike, that refutation of it which should be made—and which we here make—solely for logical reasons. The end attributed to art, of directing the good and inspiring horror of evil, of correcting and ameliorating customs, is a derivation of the moralistic doctrine; and so is the demand addressed to artists to collaborate in the education of the lower classes, in the strengthening of the national or bellicose spirit of a people, in the diffusion of the ideals of a modest and laborious life; and so on. These are all things that art cannot do, any more than geometry, which, however, does not lose anything of its importance on account of its inability to do this; and one does not see why art should do so, either. That it cannot do these things was partially perceived by the moralistic æstheticians also; who very readily effected a transaction with it, permitting it to provide pleasures that

were not moral, provided they were not openly dishonest, or recommending it to employ to a good end the dominion that, owing to its hedonistic power, it possessed over souls, to gild the pill, to sprinkle sweetness upon the rim of the glass containing the bitter draught-in short, to play the courtezan (since it could not get rid of its old and inborn habits), in the service of holy church or of morality: meretrix ecclesiæ. On other occasions they have sought to avail themselves of it for purposes of instruction, since not only virtue but also science is a difficult thing, and art could remove this difficulty and render pleasant and attractive the entrance into the ocean of science-indeed, lead them through it as through a garden of Armida, gaily and voluptuously, without their being conscious of the lofty protection they had obtained, or of the crisis of renovation which they were preparing for themselves. We cannot now refrain from a smile when we talk of these theories, but should not forget that they were once a serious matter corresponding to a serious effort to understand the nature of art and to elevate the conception of it; and that among those who believed in it (to limit ourselves to Italian literature) were Dante and Tasso, Parini and Alfieri, Manzoni and Mazzini. And the moralistic doctrine of art was and is and will be perpetually beneficial by its very contradictions; it was and will be an effort, however unhappy, to separate art from the merely pleasing, with which it is sometimes confused, and to assign to it a more worthy post: and it, too, has its trueside, because, if art be beyond morality, the artist is neither this side of it nor that, but under its empire, in so far as he is a man who cannot withdraw himself from the duties of man, and must look upon art itself—art, which is not and never will be moral—as a mission to be exercised as a priestly office.

Again (and this is the last and perhaps the most important of all the general negations that it suits me to recall in relation to this matter), with the definition of art as intuition, we deny that it has the character of conceptual knowledge. Conceptual knowledge, in its true form, which is the philosophical, is always realistic, aiming at establishing reality against unreality, or at lowering unreality by including it in reality as a subordinate moment of reality itself. tuition means, precisely, indistinction of reality and unreality, the image with its value as mere image, the pure ideality of the image; and opposing the intuitive or sensible knowledge to the conceptual or intelligible, the æsthetic to the noetic, it aims at claiming the autonomy of this more simple and elementary form of knowledge, which has been compared to the dream (the dream, and not the sleep) of the theoretic life, in respect to which philosophy would be the waking. And indeed, whoever should ask, when examining a work of art, whether what the artist has expressed be metaphysically and historically true or false, asks a question that is without meaning, and commits an error analogous to his who should bring the airy images of the fancy before the tribunal of morality: without meaning, because the discrimination of true and false always concerns an affirmation of reality, or a judgment, but it cannot fall under the head of an image or of a pure subject, which is not the subject of a judgment, since it is without qualification or prediscate. It is useless to object that the individuality of the image cannot subsist without reference to the universal, of which that image is the individuation, because we do not here deny that the universal, as the spirit of God, is everywhere and animates all things with itself, but we deny that the universal is rendered logically explicit and is thought , in the intuition. Useless also is the appeal to the principle

of the unity of the spirit, which is not broken, but, on the contrary, strengthened by the clear distinction of fancy from thought, because from the distinction comes opposition, and from opposition concrete unity.

Ideality (as has also been called this character that distinguishes the intuition from the concept, art from philosophy and from history, from the affirmation of the universal and from the perception or narration of what has happened) is the intimate virtue of art: no sooner are reflection and judgment developed from that ideality, than art is dissipated and dies: it dies in the artist, who becomes a critic; it dies in the contemplator, who changes from an entranced enjoyer of art to a meditative observer of life.

But the distinction of art from philosophy (taken widely as including all thinking of the real) brings with it other distinctions, among which that of art from myth occupies the foremost place. For myth, to him who believes in it, presents itself as the revelation and knowledge of reality as opposed to unreality,—a reality that drives away other beliefs as illusory or false. It can become art only for him who no longer believes in it and avails himself of mythology as a metaphor, of the austere world of the gods as of a beautiful world, of God as of an image of sublimity. Considered, then, in its genuine reality, in the soul of the believer and not of the unbeliever, it is religion and not simple fancy: and religion is philosophy, philosophy in process of becoming, philosophy more or less imperfect, but philosophy, as philosophy is religion, more or less purified and elaborated, in continuous process of elaboration and purification, but religion or thought of the Absolute or Eternal. Art lacks the thought that is necessary ere it can become myth and religion, and the faith that is born of thought; the artist neither believes nor disbelieves in his image: he produces

it. And, for a different reason, the concept of art as intuition excludes, on the other hand, the conception of art as the production of classes and types, species and genera, or again (as a great mathematician and philosopher had occasion to say of music), as an exercise of unconscious arithmetic; that is, it distinguishes art from the positive sciences and from mathematics, in both of which appears the conceptual form, though without realistic character, as mere general representation or mere abstraction. But that ideality which natural and mathematical science would seem to assume, as opposed to the world of philosophy, of religion and of history, and which would seem to approximate it to art (and owing to which scientists and mathematicians of our day are so ready to boast of creating worlds, of fictiones, resembling the fictions and figurations of the poets, even in their vocabulary), is gained with the renunciation of concrete thought, by means of generalisation and abstraction, which are capricious, volitional decisions, practical acts, and, as practical acts, extraneous and inimical to the world of art. Thus it happens that art manifests much more repugnance toward the positive and mathematical sciences than toward philosophy, religion and history, because these seem to it to be fellow-citizens of the same world of theory or of knowledge, whereas those others shock it with the brutality toward contemplation of the practical world. Poetry and classification, and, worse still, poetry and mathematics, appear to be as little in agreement as fire and water: the esprit mathématique and the esprit scientifique, the most declared enemies of the esprit poétique; those periods in which the natural sciences and mathematics prevail (for example, the intellectualism of the eighteenth century) seem to be the least fruitful in poetry.

And since this vindication of the alogical character of art

is, as I have said, the most difficult and important of the negations included in the formula of art-intuition, the theories that attempt to explain art as philosophy, as religion, as history, or as science, and in a lesser degree as mathematics, occupy the greater part of the history of æsthetic science and are adorned with the names of the greatest philosophers. Schelling and Hegel afford examples of the identification or confusion of art with religion and philosophy in the eighteenth century; Taine, of its confusion with the natural sciences: the theories of the French verists. of its confusion with historical and documentary observation; the formalism of the Herbartians, of its confusion with mathematics. But it would be vain to seek pure examples of these errors in any of these authors and in the others that might be mentioned, because error is never pure, for if it were so, it would be truth. Thus the doctrines of art that for the sake of brevity I shall term "conceptualistic" contain elements of dissolution, the more copious and efficacious by as much as the spirit of the philosopher who professed them was energetic, and therefore nowhere are they so copious and efficacious as in Schelling and Hegel, who thus had so lively a consciousness of artistic production as to suggest by their observations and their particular developments a theory opposed to that maintained in their systems. Furthermore, the very conceptualistic theories are superior to the others previously examined, not only in so far as they recognise the theoretic character of art, but also carry with them their contribution to the true doctrine, owing to the claim that they make for a determination of the relations (which, if they be of distinction, are also of unity) between fancy and logic, between art and thought.

And here we can already see how the simplest formula, that "art is intuition,"—which, translated into other sym-

bolical terms (for example, that "art is the work of fancy"), is to be found in the mouths of all those who daily discuss art, and is to be found in older terms ("imitation," "fiction," "fable," etc.) in so many old books,—pronounced now in the text of a philosophical discourse, becomes filled with a historical, critical, and polemical content, of which I can hardly here give any example. And it will no longer cause astonishment that its philosophical conquest should have cost an especially great amount of toil, because that conquest is like setting foot upon a little hill long fought for in battle. easy ascent by the thoughtless pedestrian in time of peace is a very different matter; it is not a simple resting-place on a walk, but the symbol and result of the victory of an army. The historian of æsthetic follows the steps of its difficult progress, in which (and this is another magical act of thought) the conqueror, instead of losing strength through the blows that his adversary inflicts upon him, acquires new strength through these very blows, and reaches the sighed-for eminence, repulsing his adversary, and yet in his company. Here I cannot do more than record in passing the importance of the Aristotelian concept of mimesis (arising in opposition to the Platonic condemnation of poetry), and the attempt made by the same philosopher to distinguish poetry and history: a concept that was not sufficiently developed, and perhaps not altogether mature in his mind, and therefore long misunderstood, but which was yet to serve, after many centuries, as the point of departure for modern æsthetic thought. And I will mention in passing the ever-increasing consciousness of the difference between logic and fancy, between judgment and taste, between intellect and genius, which became ever more lively during the course of the seventeenth century, and the solemn form which the contest between Poetry and Metaphysic assumed in the "Scienza

Nuova" of Vico; and also the scholastic construction of an Æsthetica, distinct from a Logica, as Gnoseologia inferior and Scientia cognitionis sensitivæ, in Baumgarten, who, however, remained involved in the conceptualistic conception of art, and did not carry out his project; and the Critique of Kant directed against Baumgarten and all the Leibnitzians and Wolffians, which made it clear that intuition is intuition and not a "confused concept"; and romanticism, which perhaps better developed the new idea of art, announced by Vico, in its artistic criticism and in its histories than in its systems; and, finally, the criticism inaugurated in Italy by Francesco de Sanctis, who caused art as pure form, or pure intuition, to prevail over all utilitarianism, moralism, and conceptualism (to adopt his vocabulary).

But doubt springs up at the feet of truth, "like a young shoot,"—as the terzina of father Dante has it,—doubt, which is what drives the intellect of man "from mount to mount." The doctrine of art as intuition, as fancy, as form, now gives rise to an ulterior (I have not said an "ultimate") problem, which is no longer one of opposition and distinction toward physics, hedonistic, ethic and logic, but the field of images itself, which sets in doubt the capacity of the image to define the character of art and is in reality occupied with the mode of separating the genuine from the spurious image, and of enriching in this way the concept of the image and of art. What function (it is asked) can a world of pure images possess in the spirit of man, without philosophical, historical, religious or scientific value, and without even moral or hedonistic value? What is more vain than to dream with open eyes in life, which demands, not only open eyes, but an open mind and a nimble spirit? Pure images! But to nourish oneself upon pure images is called by a name of little honour, "to dream," and there is usually added to

this the epithet of "idle." It is a very insipid and inconclusive thing; can it ever be art? Certainly, we sometimes amuse ourselves with the reading of some sensational romance of adventure, where images follow images in the most various and unexpected way; but we thus enjoy ourselves in moment's of fatigue, when we are obliged to kill time, and with a full consciousness that such stuff is not art. Such instances are of the nature of a pastime, a game; but were art a game or a pastime, it would fall into the wide arms of hedonistic doctrine, ever open to receive it. And it is a utilitarian and hedonistic need that impels us sometimes to relax the bow of the mind and the bow of the will, and to stretch ourselves, allowing images to follow one another in our memory, or combining them in quaint forms with the aid of the imagination, in a sort of waking sleep, from which we rouse ourselves as soon as we are rested; and we sometimes rouse ourselves just to devote ourselves to the work of art. which cannot be produced by a mind relaxed. Thus either art is not pure intuition, and the claims put forward in the doctrines which we believed we had above confuted, are not satisfied, and so the confutation itself of these doctrines is troubled with doubts; or intuition cannot consist in a simple act of imagination.

In order to render the problem more exact and more difficult, it will be well to eliminate from it at once that part to which the answer is easy, and which I have not wished to neglect, precisely because it is usually united and confused with it. The intuition is the product of an image, and not of an incoherent mass of images obtained by recalling former images and allowing them to succeed one another capriciously, by combining one image with another in a like capricious manner, joining a horse's neck to a human head, and thus playing a childish game. Old Poetic availed itself

above all of the concept of unity, in order to express this distinction between the intuition and imagining, insisting that whatever the artistic work, it should be simplex et unum; or of the allied concept of unity in variety—that is to say, the multiple images were to find their common centre unit of union in a comprehensive image: and the æsthetic of the nineteenth century created with the same object the distinction, which appears in not a few of its philosophers. between fancy (the peculiar artistic faculty) and imagination (the extra-artistic faculty). To amass, select, cut up, combine images, presupposes the possession of particular images in the spirit; and fancy produces, whereas imagination is sterile, adapted to extrinsic combinations and not to the generation of organism and life. The most profound problem, contained beneath the rather superficial formula with which I first presented it, is, then: What is the office of the pure image in the life of the spirit? or (which at bottom amounts to the same thing), How does the pure image come into existence? Every inspired work of art gives rise to a long series of imitators, who just repeat, cut up in pieces, combine, and mechanically exaggerate that work, and by so doing play the part of imagination toward or against the fancy. But what is the justification, or what the genesis, of the work of genius, which is afterward submitted (a sign of glory!) to such torments? In order to make this point clear, we must go deeply into the character of fancy or pure intuition.

And the best way to prepare this deeper study is to recall to mind and to criticise the theories with which it has been sought to differentiate artistic intuition from merely incoherent imagination (while taking care not to fall into realism or conceptualism), to establish in what the principle of unity consists, and to justify the productive character of the

fancy. The artistic image (it has been said) is such, when it unites the intelligible with the sensible, and represents an idea. Now "intelligible" and "idea" cannot mean anything but concept (nor has it a different meaning with those who maintain this doctrine); even though it be the concrete concept or idea, proper to lofty philosophical speculation, which differs from the abstract concept or from the representative concept of the sciences. But in any case, the concept or idea always unites the intelligible to the sensible, and not only in art, for the new concept of the concept, first stated by Kant and (so to speak) immanent in all modern thought, heals the breach between the sensible and the intelligible worlds, conceives the concept as judgment, and the judgment as synthesis a priori, and the synthesis a priori as the word become flesh, as history. Thus that definition of art leads the fancy back to logic and art to philosophy, contrary to intention; and is at most valid for the abstract conception of science, not for the problem of art (the æsthetic and teleological "Critique of Judgment" of Kant had precisely this historical function of correcting what of abstract there yet remained in the "Critique of Pure Reason"). To seek a sensible element for the concept, beyond that which it has already absorbed in itself as concrete concept, and beyond the words in which it expresses itself, would be superfluous. If we persist in this search, it is true that we abandon the conception of art as philosophy or history, but only to pass to the conception of art as allegory. And the unsurmountable difficulties of the allegory are well known, as its frigid and anti-historical character is known and universally felt. Allegory is the extrinsic union, the conventional and arbitrary juxtaposition of two spiritual acts, a concept or thought and an image, where it is assumed that this image must represent that concept. And not only is the individual char-

acter of the artistic image not explained by this, but, in addition, a duality is purposely created, because thought remains thought and image image in this juxtaposition, without relation between themselves; so much so, that in contemplating the image, we forget the concept without any disadvantage, --indeed, with advantage,—and in thinking the concept, we dissipate, also with advantage, the superfluous and tiresome image. Allegory enjoyed much favour in the Middle Ages, that mixture of Germanism and Romanism, of barbarism and culture, of bold fancy and of acute reflection; but it was the theoretic element in, and not the effective reality of, the same mediæval art which, where it is art, drives allegory away from or resolves it in itself. This need for the solution of allegorical dualism leads to the refining of the theory of intuition, in so far as it is allegory of the idea. into the other theory, of the intuition as—symbol; for the idea does not stand by itself in the symbol, thinkable separately from the symbolising representation, nor does the symbol stand by itself, representable in a lively manner without the idea symbolised. The idea is all reduced to representation (as said the æsthetician Vischer, if to anyone belongs the blame of the very prosaic comparison for so poetic and metaphysical a theme), like a lump of sugar melted in a glass of water, which exists and acts in every molecule of water, but is no longer to be found as a lump of sugar. But the idea that has disappeared, the idea that has become entirely representative, the idea that we can no longer succeed in seizing as idea (save by extracting it, like sugar from sugared water), is no longer idea, and is only the sign that the unity of the artistic image has not yet been achieved. Certainly art is symbol, all symbol—that is, all significant: but symbol of what? What does it mean? The intuition is truly artistic, it is truly intuition, and not a chaotic mass of

images, only when it has a vital principle that animates it, making it all one with itself; but what is this principle?

The answer to such a question may be said to result from the examination of the greatest ideal strife that has ever taken place in the field of art (and is not confined to the epoch that took its name from it and in which it was predominant): the strife between romanticism and classicism. Giving the general definition, here convenient, and setting aside minor and accidental determinations, romanticism asks of art, above all, the spontaneous and violent effusion of the affections, of love and hate, of anguish and jubilation, of desperation and elevation; and is willingly satisfied and pleased with vaporous and indeterminate images, broken and allusive in style, with vague suggestions, with approximate phrases, with powerful and troubled sketches: while classicism loves the peaceful soul, the wise design, figures studied in their characteristics and precise in outline, ponderation, equilibrium, clarity; and resolutely tends toward representation, as the other tends toward feeling. And whoever puts himself at one or the other point of view finds crowds of reasons for maintaining it and for confuting the opposite point of view; because (say the romantics), What is the use of an art, rich in beautiful images, which, nevertheless, does not speak to the heart? And if it do speak to the heart, what is the use if the images be not beautiful? And the others will say, What is the use of the shock of the passions, if the spirit do not rest upon a beautiful image? And if the image be beautiful, if our taste be satisfied, what matters the absence of those emotions which can all of them be obtained outside art, and which life does not fail to provide, sometimes in greater quantity than we desire?—But when we begin to feel weary of the fruitless defence of both partial views; above all, when we turn

away from the ordinary works of art produced by the romantic and classical schools, from works convulsed with passion or coldly decorous, and fix them on the works, not of the disciples, but of the masters, not of the mediocre, but of the supreme, we see the contest disappear in the distance and find ourselves unable to call the great portions of these works, romantic or classic or representative, because they are both classic and romantic, feelings and representations, a vigorous feeling which has become all most brilliant representation. Such, for example, are the works of Hellenic art, and such those of Italian poetry and art: the transcendentalism of the Middle Ages became fixed in the bronze of the Dantesque terzina; melancholy and suave fancy, in the transparency of the songs and sonnets of Petrarch; sage experience of life and badinage with the fables of the past, in the limpid ottava rima of Ariosto; heroism and the thought of death, in the perfect blank-verse hendecasyllabics of Foscolo; the infinite variety of everything, in the sober and austere songs of Giacomo Leopardi. Finally (be it said in parenthesis and without intending comparison with the other examples adduced), the voluptuous refinements and animal sensuality of international decadentism have received their most perfect expression in the prose and verse of an Italian, D'Annunzio. All these souls were profoundly passionate (all, even the serene Lodovico Ariosto, who was so amorous, so tender, and so often represses his emotion with a smile); their works of art are the eternal flower that springs from their passions.

These expressions and these critical judgments can be theoretically resumed in the formula, that what gives coherence and unity to the intuition is feeling: the intuition is really such because it represents a feeling, and can only appear from and upon that. Not the idea, but the feeling, is

what confers upon art the airy lightness of the symbol: an aspiration enclosed in the circle of a representation—that is art; and in it the aspiration alone stands for the representation, and the representation alone for the aspiration. Epic and lyric, or drama and lyric, are scholastic divisions of the indivisible: art is always lyrical—that is, epic and dramatic in feeling. What we admire in genuine works of art is the perfect fanciful form which a state of the soul assumes; and we call this life, unity, solidity of the work of art. What displeases us in the false and imperfect forms is the struggle of several different states of the soul not vet unified, their stratification, or mixture, their vacillating method, which obtains apparent unity from the will of the author, who for this purpose avails himself of an abstract plan or idea, or of extra-æsthetic, passionate emotion. A series of images which seem to be, each in turn, rich in power of conviction, leaves us nevertheless deluded and diffident, because we do not see them generated from a state of the soul, from a "sketch" (as the painters call it), from a motive; and they follow one another and crowd together without that precise intonation, without that accent, which comes from the heart. And what is the figure cut out from the background of the picture or transported and placed against another background, what is the personage of drama or of romance outside his relation with all the other personages and with the general action? And what is the value of this general action if it be not an action of the spirit of the author? The secular disputes concerning dramatic unity are interesting in this connection; they are first applied to the unity of "action" when they have been obtained from an extrinsic determination of time and place, and this finally applied to the unity of "interest," and the interest would have to be in its turn dissolved in the interest of the spirit of

the poet—that is, in his intimate aspiration, in his feeling. The negative issue of the great dispute between classicists and romanticists is interesting, for it resulted in the negation both of the art which strives to distract and illude the soul as to the deficiency of the image with mere feeling, with the practical violence of feeling, with feeling that has not become contemplation, and of the art which, by means of the superficial clearness of the image, of drawing correctly false, of the word falsely correct, seeks to deceive as to its lack of inspiration and its lack of an æsthetic reason to justify what it has produced. A celebrated sentence uttered by an English critic, and become one of the commonplaces of journalism, states that "all the arts tend to the condition of music": but it would have been more accurate to say that all the arts are music, if it be thus intended to emphasise the genesis of æsthetic images in feeling, excluding from their number those mechanically constructed or realistically ponderous. another not less celebrated utterance of a Swiss semi-philosopher, which has had the like good or bad fortune of becoming trivial, discovers that "every landscape is a state of the soul": which is indisputable, not because the landscape is landscape, but because the landscape is art.

Artistic intuition, then, is always lyrical intuition: this latter being a word that is not present as an adjective or definition of the first, but as a synonym, another of the synonyms that can be united to the several that I have mentioned already, and which, all of them, designate the intuition. And if it be sometimes convenient that instead of appearing as a synonym, it should assume the grammatical form of the adjective, that is only to make clear the difference between the intuition-image, or nexus of images (for what is called image is always a nexus of images, since imageatoms do not exist any more than thought-atoms), which

constitutes the organism, and, as organism, has its vital principle, which is the organism itself,—between this, which is true and proper intuition, and that false intuition which is a heap of images put together in play or intentionally or for some other practical purpose, the connection of which, being practical, shows itself to be not organic, but mechanic, when considered from the æsthetic point of view. But the word *lyric* would be redundant save in this explicative or polemical sense; and art is perfectly defined when it is simply defined as *intuition*.

## II PREJUDICES RELATING TO ART

THERE can be no doubt that the process of distinction of art from the facts and the acts with which it has been and is confused, which I have summarily traced, necessitates no small mental effort; but this effort is rewarded with the freedom which it affords of handling the many fallacious distinctions which disfigure the field of æsthetic. These, although they do not present any difficulty in thinking out (indeed, at first they seduce by their very facility and deceitful self-evidence), yet imply the other and greater annoyance of preventing all profound understanding, and indeed of making it impossible to understand anything as to what art truly is. It is true that many people, in order to retain the power of repeating vulgar and traditional distinctions, voluntarily resign themselves to this ignorance. We, on the contrary, now prefer to throw them all away, as a useless hindrance in the new task to which the new theoretic position that we have attained invites and leads us, and to enjoy the greater facility which comes from feeling rich. Wealth is not only to be obtained by acquiring many objects, but, on the contrary, by getting rid of all those that represent economic debt.

Let us begin with the most famous of these economic debts in the circle of æsthetic: the distinction between content and form, which has caused a division of schools even in the nineteenth century: the schools of the æsthetic of the content (Gehaltsæsthetik) and that of the æsthetic of form (Formæsthetik). The problems from which these

opposed schools arose were, in general, the following: Does art consist solely of the content, or solely of the form, or of content and form together? What is the character of the content, what that of the æsthetic form?-It was answered, on the one hand, that art, the essence of art, is all contained in the content, defined as that which pleases, or as what is moral, or as what raises man to the heaven of religion or of metaphysic, or as what is historically correct, or. finally, as what is naturally and physically beautiful. And, on the other hand, that the content is indifferent, that it is simply a pcg or hook from which beautiful forms are suspended, which alone beatify the asthetic spirit: unity, harmony, symmetry, and so on. And on both sides it was attempted to attract the element that had previously been excluded from the essence of art as subordinate and secondary: those for the content admitted that it was an advantage to the content (which, according to them, was really the constitutive element of the beautiful) to adorn itself with beautiful forms also, and to present itself as unity, symmetry, harmony, etc.: and the formalists, in their turn, admitted that if art did not gain by the value of its content, its effect did, not a single value, but the sum of two values being in this case offered. These doctrines, which attained their greatest scholastic bulk in Germany with the Hegelians and the Herbartians, is also to be found more or less everywhere in the history of æsthetic, ancient, mediæval, modern, and most modern; and is what amounts to most in common opinion, for nothing is more common than to hear that a drama is beautiful in "form," but a failure in "content"; that a poem is "most nobly" conceived, but "executed in ugly verse"; that a painter would have been greater did he not waste his power as a designer and as a colourist, upon "small and unworthy themes," instead of selecting, on the contrary, those

of a historical, patriotic, or sociological character. It may be said that fine taste and true critical sense of art have to defend themselves at every step against the perversions of judgment arising from these doctrines, in which philosophers become the crowd, and the crowd feels itself philosophical, because in agreement with those crowd-philosophers. The origin of these theories is no secret for us, because, even in the brief sketch that we have given, it is quite clear that they have sprung from the trunk of hedonistic, moralistic, conceptualistic, or physical conceptions of art: they are all doctrines which, failing to perceive what makes art art, were obliged somehow to regain art, which they had allowed to escape them, and to reintroduce it in the form of an accessory or accidental element; the upholders of the theory of the content conceived it as an abstract formal element, the formalists as the abstract element of the content. What interests us in those æsthetics is just this dialectic, in which the theorists of the content become formalists against their will, and the formalists upholders of the theory of the content; thus each passes over to occupy the other's place, but to be restless there and to return to their own, which gives rise to the same restlessness. The "beautiful forms" of Herbart do not differ in any way from the "beautiful contents" of the Hegelians, because both are nothing. And we become yet more interested to observe their efforts to get out of prison, and the blows with which they weaken its doors or its walls, and the air-holes which some of those thinkers succeed in opening.—Their efforts are clumsy and sterile, like those of the theorists of the content (they are to be seen in a repulsive form in the Philosophie des Schönen of Hartmann), who, by adding stitch to stitch, composed a net of "beautiful contents" (beautiful, sublime, comic, tragic, humouristic, pathetic, idyllic, sentimental, etc., etc.),

in which very coarse net they tried to enclose every form of reality, even that which they had called "ugly." failed to perceive that their æsthetic content, thus made to enclose little by little the whole of reality, has no longer any character that distinguishes it from other contents, since there is no content beyond reality; and that therefore their fundamental theory was thus fundamentally negated. These contradictory and ingenuous explosions resemble those of other formalistic theorists of the content who maintained the concept of an æsthetic content, but defined it as that "which interests man," and made the interest relating to man to lie in his different historical situations—that is, relative to the individual. This was another way of denying the initial assumption, for it is very clear that the artist would not produce art, did he not interest himself in something which is the datum or the problem of his production, but that this something becomes art only because the artist, by becoming interested in it, makes it so.—These are evasions of formalists, who after having limited art to abstract beautiful forms, void of all content and only to be summed up with contents, timidly introduced among beautiful forms that of the harmony of content with form; or more resolutely declared themselves partisans of a sort of eclecticism, which makes art to consist of a sort of "relation" of the beautiful content with the beautiful form, and, with an incorrectness worthy of eclectics, attributed to terms outside the relation qualities which they assume only within the relation.

For the truth is really this: content and form must be clearly distinguished in art, but must not be separately qualified as artistic, precisely because their relation only is artistic—that is, their unity, understood not as an abstract, dead unity, but as concrete and living, which is that of the synthesis a priori; and art is a true asthetic synthesis a priori of feeling

and image in the intuition, as to which it may be repeated that feeling without image is blind, and image without feeling is void. Feeling and image do not exist for the artistic spirit outside the synthesis; they will have existence from another point of view in another plane of knowledge, and feeling will be the practical aspect of the spirit that loves and hates, desires and dislikes, and the image will be the inanimate residue of art, the withered leaf, prev of the wind of imagination and of amusement's caprice. this has no concern with the artist or the asthetician: just as art is no vain fancying, so is it not tumultuous passionality, but the uplifting of that act by means of another act, or, if it be preferred, the substitution of that tumult for another tumult, that of the longing to create and to contemplate for the joys and the sorrows of artistic creation. It is therefore indifferent, or a question of terminological opportunity, whether we should present art as content or as form, provided it be always recognised that the content is formed and the form filled, that feeling is figurative feeling and the figure a figure that is felt. And it is only owing to historical deference toward him who better than others caused the concept of the autonomy of art to be appreciated, and wished to affirm this autonomy with the word "form," thus opposing alike the abstract theory of the content of the philosophisers and moralists and the abstract formalism of the academicians, -in deference, I say, to De Sanctis, and also because of the ever active polemic against the attempts to absorb art in other modes of spiritual activity,—that the æsthetic of the intuition can be called "Æsthetic of form." It is useless to refute an objection that certainly might be made (but rather with the sophistry of the advocate than with the acuteness of the scientist), namely, that the æsthetic of the intuition also, since it describes the content of art as feeling

or state of the soul, qualifies it outside the intuition, and seems to admit that a content, which is not feeling or a state of the soul, does not lend itself to artistic elaboration, and is not an æsthetic content. Feeling, or the state of the soul, is not a particular content, but the whole universe seen sub specie intuitionis; and outside it there is no other content conceivable that is not also a different form of the intuitive form; not thoughts, which are the whole universe sub specie cogitationis; not physical things and mathematical beings, which are the whole universe sub specie schematismic et abstractionis; not wills, which are the whole universe sub specie volitionis.

Another not less fallacious distinction (to which the words "content" and "form" are also applied) separates intuition from expression, the image from the physical translation of the image. It places on one side phantasms of feeling, images of men, of animals, of landscapes, of actions, of adventures, and so on; and on the other, sounds, tones, lines, colours, and so on; calling the first the external, the second the internal element of art: the art properly so-called, the other technique. It is easy to distinguish internal and external, at least in words, especially when no minute enquiry is made as to the reasons and motives for the distinction, and when the distinction is just thrown down there without any service being demanded of it; so easy that by never thinking about it the distinction may eventually come to seem to thought indubitable. But it becomes a different question when, as must be done with every distinction, we pass from the act of distinguishing to that of establishing relation and unifying, because this time we run against desperate obstacles. What has here been distinguished cannot be unified, because it has been badly distinguished; how can something external and extraneous to the internal become united to the internal

and express it? How can a sound or a colour express an image without sound and without colour? How can the bodiless express a body? How can the spontaneity of fancy and of reflection and even technical action coincide in the same When the intuition has been distinguished from the expression, and the one has been made different from the other, no ingenuity of terms can reunite them; all the processes of association, of habit, of mechanicising, of forgetting, of instinctification, proposed by the psychologists and laboriously developed by them, allow the scissure to reappear at the end: on one side the expression, on the other the image. And there does not seem to be any way of escape, save that of taking refuge in the hypothesis of a mystery which, according to poetical or mathematical tastes, will assume the appearance of a mysterious marriage or of a mysterious psychophysical parallelism. The first is a parallelism incorrectly overcome; the second, a marriage deferred to distant ages or to the obscurity of the unknowable.

But before having recourse to mystery (a refuge to which there is always time to fly), we must enquire whether the two elements have been correctly distinguished, and if an intuition without expression be conceivable. It may happen that the thing is as little existing and as inconceivable as a soul without a body, which has truly been as much talked of in philosophies as in religions, but to have talked about it is not the same thing as to have experienced and conceived it. In reality, we know nothing but expressed intuitions: a thought is not thought for us, unless it be possible to formulate it in words; a musical fancy, only when it becomes concrete in sounds; a pictorial image, only when it is coloured. We do not say that the words must necessarily be declaimed in a loud voice, the music performed, or the picture painted upon wood or canvas; but it is certain that

when a thought is really thought, when it has attained to the maturity of thought, the words run through our whole organism, soliciting the muscles of our mouth and ringing internally in our ears; when music is truly music, it trills in the throat and shivers in the fingers that touch ideal notes: when a pictorial image is pictorially real, we are impregnated with lymphs that are colours, and maybe, where the colouring matters were not at our disposition, we might spontaneously colour surrounding objects by a sort of irradiation, as is said of certain hysterics and of certain saints, who caused the stigmata upon their hands and feet by means of an act of imagination! Thought, musical fancy, pictorial image, did not indeed exist without expression, they did not exist at all previous to the formation of this expressive state of the spirit. To believe in their pre-existence is ingenuousness. if it be ingenuous to have faith in those impotent poets, painters, or musicians who always have their heads full of poetic, pictorial, and musical creations, and only fail to translate them into external form, either because, as they say, they are impatient of expression, or because technique is not sufficiently advanced to afford sufficient means for their expression: many centuries ago it offered sufficient means to Homer, Pheidias, and Apelles, but it does not suffice for them, who, if we are to believe them, carry in their mighty heads an art greater than those others! Sometimes, too, ingenuousness arises from the illusion due to keeping a bad account with ourselves that, having imagined, and consequently expressed, some few images, we already possess in ourselves all the other images that must form part of a work, which we do not yet possess, as well as the vital nexus that should connect them, which is not yet formed and therefore is not expressed.

Art, understood as intuition, according to the concept that

I have exposed, having denied the existence of a physical world outside of it, which it looks upon as simply a construction of our intellect, does not know what to do with the parallelism of the thinking substance and of substance extended in space, and has no need to promote impossible marriages, because its thinking substance—or, better, its intuitive act—is perfect in itself, and is that same fact which the intellect afterwards constructs as extended. And inasmuch as an image without expression is inconceivable, by just so much is an image which shall be also expression conceivable, and indeed logically necessary; that is, which shall be really an image. If we take from a poem its metre, its rhythm, and its words, poetical thought does not, as some opine, remain behind: there remains nothing. The poetry is born, like those words, that rhythm, and that metre. Nor could expression be compared with the epidermis of organisms, unless it be said (and perhaps this may not be false even in physiology) that all the organism in every cell's cell is also epidermis.

I should, however, be wanting to my methodological convictions and to my intention of doing justice to errors (and I have already done justice to the distinction of form and content by demonstrating the truth at which they aimed and failed to grasp), were I not to indicate what truth may also be active at the base of the false distinction of the indistinguishable, intuition and expression. Fancy and technique are rationally distinguished, though not as elements of art; and they are related and united between themselves, though not in the field of art, but in the wider field of the spirit in its totality. Technical or practical problems to be solved, difficulties to be vanquished, are truly present to the artist, and there is truly something which, without being really physical, and being, like everything real, a spiritual act, can be meta-

phoricised as physical in respect to the intuition. What is this something? The artist, whom we have left vibrating with expressed images which break forth by infinite channels from his whole being, is a whole man, and therefore also a practical man, and as such takes measures against losing the result of his spiritual labour, and in favour of rendering possible or easy, for himself and for others, the reproduction of his images; hence he engages in practical acts which assist that work of reproduction. These practical acts are guided, as are all practical acts, by knowledge, and for this reason are called technical; and, since they are practical, they are distinguished from contemplation, which is theoretical, and seem to be external to it, and are therefore called physical: and they assume this name the more easily in so far as they are fixed and made abstract by the intellect. Thus writing and phonography are united with words and music, canvas and wood and walls covered with colours, stone cut and incised, iron and bronze and other metals melted and moulded to certain shapes by sculpture and architecture. So distinct among themselves are the two forms of activity that it is possible to be a great artist with a bad technique, a poet who corrects the proofs of his verses badly, an architect who makes use of unsuitable material or does not attend to statics, a painter who uses colours that deteriorate rapidly: examples of these weaknesses are so frequent that it is not worth while to cite any of them. But what is impossible is to be a great poet who writes verses badly, a great painter who does not give tone to his colours, a great architect who does not harmonise his lines, a great composer who does not harmonise his notes; and, in short, a great artist who cannot express himself. It has been said of Raphael that he would have been a great painter even if he had not possessed hands; but certainly not that he would have been

a great painter if the sense of design and colour had been wanting to him.

And (be it noted in passing, for I must condense as I proceed) this apparent transformation of the intuitions into physical things-altogether analogous with the apparent transformation of wants and economic labour into things and into merchandise—also explains how people have come to talk not only of "artistic things" and of "beautiful things," but also of "a beautiful of nature." It is evident that, besides the instruments that are made for the reproduction of images, objects already existing can be met with, whether produced by man or not, which perform such a service that is to say, are more or less adapted to fixing the memory of our intuitions; and these things take the name of "natural beautics," and exercise their fascination only when we know how to understand them with the same soul with which the artist or artists have taken and appropriated them, giving value to them and indicating the "point of view" from which we must look at them, thus connecting them with their own intuitions. But the always imperfect adaptability, the fugitive nature, the mutability of "natural beauties" also justify the inferior place accorded to them, compared with beauties produced by art. Let us leave it to rhetoricians or madmen to affirm that a beautiful tree, a beautiful river, a sublime mountain, or even a beautiful horse or a beautiful human figure, are superior to the chisel-stroke of Michelangelo or the verse of Dante; but let us say, with greater propriety, that "Nature" is stupid compared with Art, and that she is "mute," if man does not make her speak.

A third distinction, which also labours to distinguish the indistinguishable, is attached to the concept of the æsthetic expression, and divides it into two moments of expression abstractly considered, *propriety* and beauty of expression

sion, or adorned expression, founding upon these the classification of two orders of expression, naked and ornate. This is a doctrine of which traces may be found in all the various domains of art, but which has not been developed in any one, of them to the same extent as in that of words, where it bears a celebrated name and is called "Rhetoric," and has had a very long history, from the Greek rhetoricians to our own day. It persists in the schools, in treatises, and even in æsthetics of scientific pretensions, not to mention in common belief (as is natural), though in our day it has lost much of its primitive vigour. Men of lofty intellect have accepted it, or let it live, for centuries, owing to the force of inertia or of tradition; the few rebels have hardly ever attempted to reduce their rebellion to a system and to cut out the error at its roots. The injury done by Rhetoric, with its idea of "ornate" as differing from, and of greater value than, "naked" speech, has not been limited solely to the circle of æsthetic, but has appeared also in criticism, and even in literary education, because, just as it was incapable of explaining perfect beauty, so it was adapted to provide an apparent justification for vitiated beauty, and to encourage writing in an inflated, affected, and improper form. However, the division which it introduces and on which it relies is a logical contradiction, because, as is easy to prove, it destroys the concept itself, which it undertakes to divide into moments, and the objects, which it undertakes to divide into classes. An appropriate expression, if appropriate, is also beautiful, beauty being nothing but the determination of the image, and therefore of the expression; and if it be wished to indicate by calling it naked that there is something wanting which should be present, then the expression is inappropriate and deficient, either it is not or is not yet expression. On the other hand, an ornate expression, if it

be expressive in every part, cannot be called ornate, but as naked as the other, and as appropriate as the other; if it contain inexpressive, additional, extrinsic elements, it is not beautiful, but ugly, it is not or is not yet expression; to be so, it must purify itself of external elements (as the other must be enriched with the elements that are wanting).

Expression and beauty are not two concepts, but a single concept, which it is permissible to designate with either synonymous vocable: artistic fancy is always corporeal, but it is not obese, being always clad with itself and never charged with anything else, or "ornate." Certainly a problem was lurking beneath this falsest of distinctions, the necessity of making a distinction; and the problem (as can be deduced from certain passages in Aristotle, and from the psychology and gnoscology of the Stoics, and as we see it, intensified in the discussions of the Italian rhetoricians of the seventeenth century) was concerned with the relations between thought and fancy, philosophy and poetry, logic and æsthetic (dialectic and rhetoric, or, as was still said at the time, the "open" and the closed "fist"). "Naked" expression referred to thought and to philosophy, "ornate" expression to fancy and to poetry. But it is not less true that this problem as to the distinction between the two forms of the theoretical spirit could not be solved in the field of one of them, intuition or expression, where nothing will ever be found but fancy, poetry, æsthetic; and the undue introduction of logic will only project there a deceitful shadow, which will darken and hamper intelligence, depriving it of the view of art in its fulness and purity, without giving it that of logicity and of thought.

But the greatest injury caused by the rhetorical doctrine of "ornate" expression to the theoretical systematisation of the forms of the human spirit, concerns the treatment of lan-

guage, because, granted that we admit naked and simply grammatical expressions, and expressions that are ornate or rhetorical, language becomes an aggregate of naked expressions and is handed over to grammar, and, as an ulterior consequence (since grammar finds no place in rhetoric and æsthetic), to logic, where the subordinate office of a semeiotic or ars significandi is assigned to it. Indeed, the logistic conception of language is closely united and proceeds pari passu with the rhetorical doctrine of expression; they appeared together in Hellenic antiquity, and they still exist, though disputed, in our time. Rebellions against the logicism of the doctrine of language have rarely appeared, and have had as little efficacy as those against rhetoric; and only in the romantic period (traversed by Vico a century before) has a lively consciousness been formed by certain thinkers as to the fantastic or metaphoric nature of language, and its closer connection with poetry than with logic. Yet since a more or less inartistic idea of art persisted even among the best (conceptualism, moralism, hedonism, etc.), there remained a very powerful impediment to the identification of language and art. This identification appears to be as unavoidable as it is easy, having established the concept of art as intuition and of intuition as expression, and therefore implicitly its identity with language: always assuming that language be conceived in its full extension, without arbitrary restrictions to so-called articulate language and without arbitrary exclusion of tonic, mimetic, and graphic; and in all its intension—that is, taken in its reality, which is the act of speaking itself, without falsifying it with the abstractions of grammars and vocabularies, and with the foolish belief that man speaks with the vocabulary and with grammar. Man speaks at every instant like the poet, because, like the poet, he expresses his impressions and his

feelings in the form called conversational or familiar, which is not separated by any abyss from the other forms called prosaic, poetic-prosaic, narrative, epic, dialogue, dramatic, lyric, melic, song, and so on. And if it do not displease man in general to be considered poet and always poet (as he is by force of his humanity), it should not displease the poet to be united with common humanity, because this union alone explains the power which poetry, understood in the loftiest and in the narrowest sense, wields over all human souls. Were poetry a language apart, a "language of the gods," men would not understand it: and if it clevate them, it elevates them not above, but within themselves: true democracy and true aristocracy coincide in this field also. Coincidence of art and language, which implies, as is natural, coincidence of æsthetic and of philosophy of language, definable the one by the other and therefore identical,—this I ventured to place twelve years ago in the title of a treatise of mine on Æsthetic, which has truly not failed of its effect upon many linguists and philosophers of Æsthetic in Italy and outside Italy, as is shewn by the copious "literature" which it has produced. This identification will benefit studies on art and poetry by purifying them of hedonistic, moralistic, and conceptualistic residues, still to be found in such quantity in literary and artistic criticism. But the benefit which it will confer upon linguistic studies will be far more inestimable, for it is urgent that they should be disencumbered of physiological, psychological, and psychophysiological methods, now the fashion, and be freed from the ever returning theory of the conventional origin of language, which has the inevitable correlative of the mystical theory as its inevitable reaction. It will no longer be necessary to construct absurd parallelisms even for language, or to promote mysterious nuptials between sign and image: when language is no longer con-

ceived as a sign, but as an image which is significant—that is, a sign in itself, and therefore coloured, sounding, singing, articulate. The significant image is the spontaneous work of the human spirit, whereas the sign, wherewith man agrees with man, presupposes language; or if it be wished, nevertheless, to explain language by signs, it recommends us to call upon God, as upon the giver of the first signs—that is, to presuppose language in another way, by consigning it to the Unknowable.

I shall conclude my account of the prejudices relating to art with that one of them which is most usual, because it is mingled with the daily life of criticism, namely, history of art: prejudice of the possibility of distinguishing several or many particular forms of art, each one determinable in its own particular concept and within its limits, and furnished with its proper laws. This erroneous doctrine is embodied in two systematic series, one of which is known as the theory of literary and artistic kinds (lyric, drama, romance, epic and romantic poem, idyll, comedy, tragedy; sacred, civil-life, familiar, from life, still-life, landscape, flower and fruit painting; heroic, funereal, costume, sculpture: church, operatic, chamber music; civil, military, ecclesiastic architecture, etc., etc.), and the other as theory of the arts (poetry, painting, sculpture, architecture, music, art of the actor, gardening, etc., etc.). One of these sometimes figures as a subdivision of another. This prejudice. of which it is easy to trace the origin, has its first notable monuments in Hellenic culture, and persists in our days. Many æstheticians still write treatises on the æsthetic of the tragic, the comic, the lyric, the humorous, and æsthetics of painting, of music, or of poetry (these last are still called by the old name of "poetics"); and, what is worse (though but little attention is paid to these æstheticians who are im-

pelled to write through solitary dilettantism or academic profession), critics, in judging works of art, have not altogether abandoned the habit of judging them according to the genus or particular form of art to which, according to the above æstheticians, they should belong; and, instead of clearly stating whether a work be beautiful or ugly, they proceed to reason their impressions, saying that it well observes, or wrongly violates, the laws of the drama, or of romance, or of painting, or of bas-relief. It is also very common in all countries to treat artistic and literary history as history of kinds, and to present the artists as cultivating this or that kind; and to divide the work of an artist, which always has unity of development, whatever form it take, whether lyric, romance or drama, into as many compartments as there are kinds; so that Lodovico Ariosto, for example, appears now among the cultivators of the Latin poetry of the Renaissance, now among the authors of the first Latin satires, now among those of the first comedies, now among those who brought the poem of chivalry to perfection: as though Latin poetry, satire, comedy, and poem were not always the same poet, Ariosto, in his experiments, in his logic, and in the manifestations of his spiritual development.

It is not to be denied that the theory of kinds and of the arts has not had, and does not now possess, its own internal dialectic and its autocriticism, or irony, according as we may please to call it; and no one is ignorant that literary history is full of these cases of an established style, against which an artist of genius offends in his work and calls forth the reprobation of the critics: a reprobation which does not, however, succeed in suffocating the admiration for, and the popularity of, his work, so that finally, when it is not possible to blame the artist and it is not wished to blame the critic of kinds, the

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or accepts beside it a new kind, like a legitimated bastard, and the compromise lasts, by force of inertia, until a new work of genius comes to upset again the fixed rule. An ironylof the doctrine is also the impossibility, in which the theoreticians find themselves, of logically fixing the boundaries between the kinds and the arts; all the definitions that they have produced, when examined rather more closely, either evaporate in the general definition of art, or shew themselves to be an arbitrary raising to the rank of kinds and rules particular works of art irreducible to rigorous logical terms. Absurdities resulting from the effort to determine rigorously what is indeterminable, owing to the contradictory nature of the attempt, are to be found even among the great ones, even in Lessing, who arrives at this extravagant conclusion, that painting represents "bodies": bodies, not actions and souls, not the action and the soul of the painter! They are also to be found among the questions that logically arise from that illogic: thus, a definite field having been assigned to every kind and to every art, what kind and what art is superior? Is painting superior to sculpture, drama to lyric? And again, the forces of art having been thus divided, would it not be advisable to reunite them in a type of work of art which shall drive away other forces, as a coalition of armies drives away a single army: will not the work, for instance, in which poetry, music, scenic art, decoration, are united, develop a greater æsthetic force than a Lied of Goethe or a drawing of Leonardo? These are questions, distinctions, judgments, and definitions which arouse the revolt of the poetic and artistic sense, which loves each work for itself, for what it is, as a living creature, individual and incomparable, and knows that each work has its individual law. Hence has arisen the disagreement between the

affirmative judgment of artistic souls and the negative one of professional critics, between the negation of the former and the affirmation of the latter; and the professional critics pass for pedants, not without good reason, although artistic souls are in their turn "disarmed prophets"—that is, incapable of reasoning and of deducing the correct theory immanent in their judgments, and of opposing it to the pedantic theory of their adversaries.

That correct theory is precisely an aspect of the conception of art as intuition, or lyrical intuition; and, since every work of art expresses a state of the soul, and the state of the soul is individual and always new, the intuition implies infinite intuitions, which it is impossible to place in pigeonholes as kinds, unless these be infinite pigeonholes, and therefore not pigeonholes of kinds, but of intuitions. And since, on the other hand, individuality of intuition implies individuality of expression, and a picture is distinct from another picture, not less than from a poem, and picture and poem are not of value because of the sounds that beat the air and the colours refracted by the light, but because of what they can tell to the spirit, in so far as they enter into it, it is useless to have recourse to abstract means of expression, to construct the other series of kinds and classes: which amounts to saying that any theory of the division of the arts is without foundation. The kind or class is in this case one only, art itself or the intuition, whereas single works of art are infinite: all are original, each one incapable of being translated into the other (since to translate, to translate with artistic skill, is to create a new work of art), each one uncontrolled by the intellect. No intermediate element interposes itself philosophically between the universal and the particular, no series of kinds or species, of generalia. Neither the artist who produces art, nor the spectator who contemplates

it, has need of anything but the universal and the individual, or, better, the universal individuated: the universal artistic activity, which is all contracted or concentrated in the representation of a single state of the soul.

Nevertheless, if the pure artist and the pure critic, and also the pure philosopher, are not occupied with *generalia*. with classes or kinds, these retain their utility on other grounds; and this utility is the true side of those erroneous theories, which I will not leave without mention. It is certainly useful to construct a net of generalia, not for the production of art, which is spontaneous, nor for the judgment of it, which is philosophical, but to collect and to some extent circumscribe the infinite single intuitions, for the use of the attention and of memory, in order to group together to some extent the innumerable particular works of art. These classes will always be formed, as is natural, either by means of the abstract imagination or the abstract expression, and therefore as classes of states of the soul (literary and artistic kinds) and classes of means of expression (art). Nor does it avail to object here that the various kinds and arts are arbitrarily distinguished, and that the general dichotomy is itself arbitrary; since it is admitted without difficulty that the procedure is certainly arbitrary, but the arbitrariness becomes innocuous and useful from the very fact that every pretension of being a philosophical principle and criterion for the judgment of art is removed from it. Those kinds and classes render easy the knowledge of art and education in art, offering to the first, as it were, an index of the most important works of art, to the second a collection of most important information suggested by the practice of art. thing depends upon not confounding hints with reality, and hypothetic warnings or imperatives with categoric imperatives: a confusion which multiple and continuous

temptations are certainly apt to induce, whence it is easy to be dominated by them, but not at all inevitable. Books of literary origin, rhetoric, grammar (with their divisions into parts of speech and their grammatical and syntactical laws), of the art of musical composition, of metre, of painting, and so on, contain the principal hints and collections of precepts. Tendencies toward a definite expression of art are manifested in them either only in a secondary manner,—and in this case it is art that is still abstract, art in elaboration (the poetic arts of classicism or romanticism, purist or popular grammars, etc.),—or as tendencies toward the philosophical comprehension of their argument, and then they give rise to the divisions into kinds and into arts, an error which I have criticised: an error which, by its contradictions, opens the way to the true doctrine of the individuality of art.

Certainly this doctrine produces at first sight a sort of bewilderment: individual, original, untranslatable, unclassifiable intuitions seem to escape the rule of thought, which would seem unable to dominate them without placing them in relation with one another; and this appears to be precisely forbidden by the doctrine that has been developed, which has rather the air of being anarchic or anarchoid than liberal and liberistic.

A little piece of poetry is æsthetically equal to a poem; a tiny little picture or a sketch, to an altar picture or an affresco; a letter is a work of art, no less than a romance; even a fine translation is as original as an original work! These propositions will be indubitable, because logically deduced from verified premises; they will be true, although (and this is without doubt a merit) paradoxical, or at variance with vulgar opinions: but will they not be in want of some complement? There must be some mode of arranging, subordinating, connecting, understanding, and domi-

nating the dance of the intuitions, if we do not wish to bewilder our wits with them.

And there is indeed such a mode, for when we denied theoretic value to abstract classifications we did not intend to deny it to that genetic and concrete classification which is not, indeed, a "classification" and is called History. In history each work of art takes the place that belongs to it—that and no other: the ballade of Guido Cavalcanti and the sonnet of Cecco Angioleri, which seem to be the sigh or the laughter of an instant; the "Commedia" of Dante, which seems to resume in itself a millennium of the human spirit; the "Maccheronee" of Merlin Cocaio at the close of the Middle Ages, with their noisy laughter; the elegant Cinquecento translation of the *Eneid* by Annibal Caro; the dry prose of Sarpi; and the Jesuitic-polemical prose of Danielo Bartoli: without the necessity of judging that to be not original which is original, because it lives; that to be small which is neither great nor small, because it escapes measure: or we can say great and small, if we will, but metaphorically, with the intention of manifesting certain admirations and of noting certain relations of importance (quite other than arithmetic or geometrical). And in history, which is ever becoming richer and more definite, not in pyramids of empirical concepts, which become more and more empty the higher they rise and the more subtle they become, is to be found the link of all works of art and of all intuitions, because in history they appear organically connected among themselves, as successive and necessary stages of the development of the spirit, each one a note of the eternal poem which harmonises all single poems in itself.

#### III

### THE PLACE OF ART IN THE SPIRIT AND IN HUMAN SOCIETY

THE dispute as to the dependence or independence of art was at its hottest in the romantic period, when the motto of "art for art's sake" was coined, and as its apparent antithesis that other of "art for life"; and from that time it was discussed, to tell the truth, rather among men of letters or artists than philosophers. It has lost interest in our day, fallen to the rank of a theme with which beginners amuse or exercise themselves, or of an argument for academic orations. However, even previous to the romantic period, and indeed in the most ancient documents containing reflections upon art, are to be found traces of it; and philosophers of Æsthetic themselves, even when they appear to neglect it (and they do indeed neglect it in its vulgar form), really do consider it, and indeed may be said to think of nothing else. Because, to dispute as to the dependence or the independence, the autonomy or the heteronomy of art does not mean anything but to enquire whether art is or is not, and, if it is, what it is. An activity whose principle depends upon that of another activity is, effectively, that other activity, and retains for itself an existence that is only putative or conventional: art which depends upon morality, upon pleasure, or upon philosophy is morality, pleasure, or philosophy; it is not art. If it be held not to be dependent, it will be advisable to investigate the foundation of its independence—that is to say, how art is distinguished from morality, from pleasure, from philosophy, and from all

other things; what it is—and to posit whatever it may be as truly autonomous and independent. It may chance to be asserted, on the other hand, by those very people who affirm the concept of the original nature of art, that although it preserve its peculiar nature, yet its place is below another activity of superior dignity, and (as used at one time to be said) that it is a handmaid to ethic, a minister to politics, and a dragoman to science; but this would only prove that there are people who have the habit of contradicting themselves or of allowing discord among their thoughts: dazed folk whose existence truly does not call for any sort of proof. For our part, we shall take care not to fall into so dazed a condition; and having already made clear that art is distinguished from the physical world and from the practical, moral, and conceptual activity as intuition, we shall give ourselves no further anxiety, and shall assume that with that first demonstration we have also demonstrated the independence of art.

But another problem is implicit in the dispute as to dependence or independence; of this I have hitherto purposely not spoken, and I shall now proceed to examine it. Independence is a concept of relation, and in this aspect the only absolute independence is the Absolute, or absolute relation; every particular form and concept is independent on one side and dependent on another, or both independent and dependent. Were this not so, the spirit, and reality in general, would be either a series of juxtaposed absolutes, or (which amounts to the same thing) a series of juxtaposed nullities. The independence of a form implies the matter to which it is applied, as we have already seen in the development of the genesis of art as an intuitive formation of a sentimental or passionate material; and in the case of absolute independence, since all material and aliment would be want-

ing to it, form itself, being void, would become nullified. But since the recognised independence prevents our thinking one activity as submitted to the principle of another, the dependence must be such as to guarantee the independence. But this would not be guaranteed in the hypothesis that one activity should be made to depend upon another, in the same way as that other upon it, like two forces which counterbalance each other, and of which the one does not conquer the other; because, if it do not conquer it, we have reciprocal arrest and static; if it conquer the other, pure and simple dependence, which has already been excluded. Hence, considering the matter in general, it appears that there is no other way of thinking the simultaneous independence and dependence of the various spiritual activities than that of conceiving them in the relation of condition and conditioned, in which the conditioned surpasses the condition and presupposes it, and, becoming again in its turn condition, gives rise to a new conditioned, thus constituting a series of developments. No other defect could be attributed to this series than that the first of the series would be a condition without a previous conditioned, and the last conditioned which would not become in its turn condition, thus causing a double rupture of the law of development itself. Even this defect is healed if the last be made the condition of the first and the first the condition of the last; that is to say, if the series be conceived as reciprocal action, or, better (and abandoning all naturalistic phraseology), as a circle. This conception seems to be the only way out of the difficulties with which the other conceptions of the spiritual life are striving, both that which makes it consist of an assemblage of independent and unrelated faculties of the soul, or of independent and unrelated ideas of value, and that which subordinates all these in one and resolves them in that one,

which remains immobile and impotent; or, more subtly, conceives them as necessary grades of a linear development which leads from an irrational first to a last that would wish to be most rational, but is, however, superrational, and as such also irrational.

But it will be opportune not to insist upon this somewhat abstract scheme, and rather consider the manner in which it becomes actual in the life of the spirit, beginning with the æsthetic spirit. For this purpose we shall again return to the artist, or man-artist, who has achieved the process of liberation from the sentimental tumult and has objectified it in a lyrical image—that is, has attained to art. He finds his satisfaction in this image, because he has worked and moved in this direction: all know more or less the joy of the complete expression which we succeed in giving to our own psychical impulses, and the joy in those of others, which are also ours, when we contemplate the works of others, which are to some extent ours, and which we make ours. But is the satisfaction definite? Was only the man-artist impelled toward the image? Toward the image and toward another at the same time; toward the image in so far as he is manartist, toward another in so far as he is artist-man; toward the image on the first plane, but, since the first plane is connected with the second and third planes, also toward the second and third, although immediately toward the first and mediately toward the second and third? And now that he has reached the first plane, the second appears immediately behind it, and becomes a direct aim from indirect that it was before; and a new demand declares itself, a new process begins. Not, be it well observed, that the intuitive power gives place to another power, as though taking its turn of pleasure or of service; but the intuitive power itself-or, better, the spirit itself, which at first seconed to be, and in a

certain sense was, all intuition—develops in itself the new process, which comes forth from the vitals of the first. "One soul is not kindled at another" in us (I shall avail myself again on this occasion of Dante's words), but the one soul, which first is all collected in one single "virtue," and which "seems to obey no longer any power," satisfied in that virtue alone (in the artistic image), finds in that virtue, together with its satisfaction, its dissatisfaction: its satisfaction, because it gives to the soul all that it can give and is expected from it; its dissatisfaction, because, having obtained all that, and having satiated the soul with its ultimate sweetness,— "what is asked and thanked for,"—satisfaction is sought for the new need caused by the first satisfaction, which was not able to arise without that first satisfaction. And we all know also, from continual experience, the new want which lurks behind the formation of images. Ugo Foscolo has a love-affair with the Countess Arese; he knows with what sort of love and with what sort of woman he has to do, as can be proved from the letters he wrote, which are to be read in print. Nevertheless, during the moments that he loves her, that woman is his universe, and he aspires to possess her as the highest beatitude, and in the enthusiasm of his admiration would render the mortal woman immortal, would transfigure this earthly creature into one divine for the time to come, achieving for her a new miracle of love. And indeed he already finds her rapt to the empyrean, an object of worship and of prayers:

And thou, divine one, living in my hymns, Shalt receive the vows of my Insubrian descendants.

The ode All' amica risanata would not have taken shape in the spirit of Foscolo unless this metamorphosis of love had been desired and longed for with the greatest seriousness

(lovers and even philosophers, if they have been in love, can witness that these absurdities are seriously desired); and the images with which Foscolo represents the fascination of his goddess-friend, so rich in perils, would not have presented themselves so vividly and so spontaneously as they did. But what was that impetus of the soul which has now become a magnificent lyrical representation? Was all of Foscolo, the soldier, the patriot, the man of learning, moved with so many spiritual needs, expressed in that aspiration? Did it act so energetically within him as to be turned into action, and to some extent to give direction to his practical life? Foscolo, who had not been wanting of insight in the course of his love, as regards his poetry also from time to time became himself again when the creative tumult was appeased, and again acquired full clearness of vision. He asks himself what he really did will, and what the woman deserved. It may be that a slight suspicion of scepticism had insinuated itself during the formation of the image, if our ears be not deceived in seeming to detect here and there in the ode some trace of elegant irony toward the woman, and of the poet toward himself. This would not have happened in the case of a more ingenuous spirit, and the poetry would have flowed forth quite ingenuously. Foscolo the poet, having achieved his task and therefore being no longer poet, now wishes to know his real condition. He no longer forms the image, because he has formed it; he no longer fancies, but perceives and narrates ("that woman," he will say later of the "divine one," "had a piece of brain instead of a heart"); and the lyrical image changes, for him and for us, into an autobiographical extract, or perception.

With perception we have entered a new and very wide spiritual field; and, truly, words are not strong enough to satirise those thinkers who, now as in the past, confound

image and perception, making of the image a perception (a portrait or copy or imitation of nature, or history of the individual and of the times, etc.), and, worse still, of the perception a kind of image apprehensible by the "senses." But perception is neither more nor less than a complete judgment, and as judgment implies an image and a category or system of mental categories which must dominate the image (reality, quality, etc.); and in respect of the image, or a priori asthetic synthesis of feeling and fancy (intuition), it is a new synthesis, of representation and category, of subject and predicate, the a priori logical synthesis, of which it would be fitting to repeat all that has been said of the other, and, above all, that in it content and form, representation and category, subject and predicate, do not appear as two elements united by a third, but the representation appears as category, the category as representation, in indivisible unity: the subject is subject only in the predicate, and the predicate is predicate only in the subject. Nor is perception a logical act among other logical acts, or the most rudimentary and imperfect of them; for he who is able to extract from it all the treasures it contains would have no need to seek beyond it for other determinations of logicity, because consciousness of what has really happened, which in its eminently literary forms takes the name of history, and consciousness of the universal, which in its eminent forms takes the name of system or philosophy, spring from perception, which is itself this synthetic gemination: and philosophy and history constitute the superior unity, which philosophers have discovered, for no other reason than the synthetic connection of the perceptive judgment, whence they are born and in which they live, identifying philosophy and history, and which men of good sense discover in their own way, though they always observe that ideas suspended

in air are phantoms, are facts which occur—real facts—what alone is true, and alone worthy of being known. Finally, perception (the variety of perceptions) explains why the human intellect strives to emerge from them and to impose upon them a world of types and of laws, governed by mathematical measures and relations; which is the reason of the formation of the natural sciences and mathematics, in addition to philosophy and history.

It is not here my task to give a sketch of Logic, as I have been or am giving a sketch of Æsthetic; and therefore, refraining from determining and developing the theory of Logic, and intellectual, perceptive, and historical knowledge, I shall resume the thread of the argument, not proceeding on this occasion from the artistic and intuitive spirit, but from the logical and historical, which has surpassed the intuitive and has elaborated the image in perception. Does the spirit find satisfaction in this form? Certainly: all know the very lively satisfactions of knowledge and science; all know, from experience, the desire which takes possession of one to discover the countenance of reality, concealed by our illusions; and even though that countenance be terrible, the discovery is never unaccompanied with profound pleasure, due to the satisfaction of possessing the truth. But does such satisfaction differ in being complete and final from that afforded by art? Does not dissatisfaction perhaps appear side by side with the satisfaction of knowing reality? This, too, is most certain; and the dissatisfaction of having known manifests itself (as indeed all know by experience) in the desire for action: it is well to know the real state of affairs, but we must know it in order to act; by all means let us know the world, but in order that we may change it: tempus cognoscendi, tempus destruendi, tempus renovandi. No man remains stationary in knowledge, not

even sceptics or pessimists who, in consequence of that knowledge, assume this or that attitude, adopt this or that form of life. And that very fixing of acquired knowledge, that "retaining" after "understanding," without which (still quoting Dante) "there can be no science," the formation of types and laws and criteria of measurement, the natural sciences and mathematics, to which I have just referred, were a surpassing of the act of theory by proceeding to the act of action. And not only does everyone know from experience, and can always verify by comparison with facts, that this is indeed so; but on consideration, it is evident that things could not proceed otherwise. There was a time (which still exists for not a few unconscious Platonicians, mystics, and ascetics) when it was believed that to know was to elevate the soul to a god, to an Idea, to a world of ideas, to an Absolute placed above the phenomenal human world; and it was natural that when the soul, becoming estranged from itself by an effort against nature, had attained to that superior sphere, it returned confounded to earth, where it could remain perpetually happy and inactive. thought, which was no longer thought, had for counterpoise a reality that was not reality. But since (with Vico, Kant, Hegel, and other heresiarchs) knowledge has descended to earth, and is no longer conceived as a more or less pallid copy of an immobile reality, but remains always human, and produces, not abstract ideas, but concrete concepts which are syllogisms and historical judgments, perceptions of the real, the practical is no longer something that represents a degeneration of knowledge, a second fall from heaven to earth, or from paradise to hell, nor something that can be resolved upon or abstained from, but is implied in theory itself, as a demand of theory; and as the theory, so the practice. Our thought is historical thought of a his-

torical world, a process of development of a development; and hardly has a qualification of reality been pronounced, when the qualification is already of no value, because it has itself produced a new reality, which awaits a new qualification. A new reality, which is economic and moral life, turns the intellectual into the practical man, the politician, the saint, the man of business, the hero, and elaborates the a priori logical synthesis into the practical a priori synthesis but this is nevertheless always a new feeling, a new desiring, a new willing, a new passionality, in which the spirit can never rest, and solicits above all as new material a new intuition, a new lyricism, a new art.

And thus the last term of the series reunites itself (as I stated at the beginning) with the first term, the circle is closed, and the passage begins again: a passage which is a return of that already made, whence the Vichian concept expressed in the word "return," now become classic. the development which I have described explains the independence of art, and also the reasons for its apparent dependence, in the eyes of those who have conceived errongous doctrines (hedonistic, moralistic, conceptualistic, etc.), which I have criticised above, though noting, in the course of criticism, that in each one of them could be found some reference to truth. If it be asked, which of the various activities of the spirit is real, or if they be all real, we must reply that none of them is real; because the only reality is the activity of all these activities, which does not reside in any one of them in particular: of the various syntheses that we have one after the other distinguished, - asthetic synthesis, logical synthesis, practical synthesis, ... the only real one is the synthesis of syntheses, the Spirit, which is the true Absolute, the actus But from another point of view, and for the same reason, all are real, in the unity of the spirit, in the eternal

going and coming, which is their eternal constancy and reality. Those who see in art the concept, history, mathematics, the type, morality, pleasure, and everything else, are right, because these and all other things are contained within it, owing to the unity of the spirit; indeed, the presence in it of them all, and the energetic unilaterality alike of art as of any other particular form, tending to reduce all activities to one, explains the passage from one form to another, the completing of one form in the other, and it explains development. But those same people are wrong (owing to the distinction, which is the inseparable moment of unity) in the way that they find them all equally abstract or equally confused. Because concept, type, number, measure, morality, utility, pleasure and pain are in art as art, either antecedent or consequent; and therefore are there presupposed (sunk and forgotten there, to adopt a favourite expression of De Sanctis) or as presentiments. Without that presumption, without that presentiment, art would not be art; but it would not be art either (and all the other forms of the spirit would be disturbed by it), if it were desired to impose those values upon art as art, which is and never can be other than pure intuition. The artist will always be morally blameless and philosophically uncensurable, even though his art should indicate a low morality and philosophy: in so far as he is an artist, he does not act and does not reason, but poetises, paints, sings and, in short, expresses himself: were we to adopt a different criterion, we should return to the condemnation of Homeric poetry, in the manner of the Italian critics of the Seicento and the French critics of the time of the fourteenth Louis, who turned up their noses at what they termed "the manners" of those inebriated, vociferating, violent, cruel and ill-educated heroes. The criticism of the philosophy underlying Dante's poem

is certainly possible, but that criticism will enter the subterranean parts of the art of Dante as though by undermining, and will leave intact the soil on the surface, which is the art; Nicholas Macchiavelli will be able to destroy the Dantesque political ideal, recommending neither an emperor nor an international pope as greyhound of liberation, but a tyrant or a national prince; but he will not have eradicated that aspiration from Dante's poem. In like manner, it may be advisable not to show and not to permit to boys and young men the reading of certain pictures, romances, and plays; but this recommendation and act of forbidding will be limited to the practical sphere and will affect, not the works of art, but the books and canvases which serve as instruments for the reproduction of the art, which, as practical works, paid for in the market at a price equivalent to so much corn or gold, can also themselves be shut up in a cabinet or cupboard, and even be burnt in a "pyre of vanities," à la Savonarola. To confound the various phases of development in an ill-understood impulse for unity, to make morality dominate art, when and so far as art surpasses morality, or art dominate science, when and so far as science dominates or surpasses art, or has already been itself dominated and surpassed by life: this is what unity well understood, which is also rigorous distinction, should prevent and reject.

And it should prevent and reject it also, because the established order of the various stages of the circle makes it possible to understand not only the independence and the dependence of the various forms of the spirit, but also the preservation of this order of the one in the other. It is well to mention one of the problems which present themselves in this place, or rather to return to it, for I have already referred to it fugitively: the relation between fancy

and logic, art and science. This problem is substantially the same as that which reappears as the search for the distinction between poetry and prose; at any rate, since (and the discovery was soon made, for it is already found in the "Poetic" of Aristotle) it was recognised that the distinction cannot be drawn as between the metrical and the unmetrical, since there can be poetry in prose (for example, romances and plays) and prose in metre (for example, didascalic and philosophic poems). We shall therefore conduct it with the more profound criterion, which is that of image and perception, of intuition and judgment, which has already been explained; poetry will be the expression of the image, prose that of the judgment or concept. But the two expressions, in so far as expressions, are of the same nature, and both possess the same æsthetic value; therefore, if the poet be the lyrist of his feelings, the prosaist is also the lyrist of his feelings,—that is, poet,—though it be of the feelings which arise in him from or in his search for the concept. And there is no reason whatever for recognising the quality of poet to the composer of a sonnet and of refusing it to him who has composed the "Metaphysic," the "Somma Teologia," the "Scienza Nuova," the "Phenomenology of the Spirit," or told the story of the Peloponnesian wars, of the politics of Augustus and Tiberius, or the "universal history": in all of those works there is as much passion and as much lyrical and representative force as in any sonnet or poem. For all the distinctions with which it has been attempted to reserve the poetic quality for the poet and to deny it to the prosaist, are like those stones, carried with great effort to the top of a steep mountain, which fall back again into the valley with ruinous results. Yet there is a just apparent difference, but in order to determine it, poetry and prose must not be separated in the manner of

naturalistic logic, like two co-ordinated concepts simply opposed the one to the other; we must conceive them in development as a passage from poetry to prose. And since the poet, in this passage, not only presupposes a passionate material, owing to the unity of the spirit, but preserves the passionality and elevates it to the passionality of a poet (passion for art), so the thinker or prosaist not only preserves that passionality and elevates it to a passionality for science, but also preserves the intuitive force, owing to which his judgments come forth expressed together with the passionality that surrounds them, and therefore they retain their artistic as well as their scientific character. We can always contemplate this artistic character, assuming its scientific character, or separating it therefrom and from the criticism of science, in order to enjoy the æsthetic form which it has assumed; and this is also the reason why science belongs, though in different aspects, to the history of science and to the history of literature, and why, among the many different kinds of poetry enumerated by the rhetoricians, it would at the least be capricious to refuse to number the "poetry of prose," which is sometimes far purer poetry than much pretentious poetry of poetry. And it will be well that I should mention again a new problem of the same sort, to which I have already alluded in passing: namely, the connection between art and morality, which has been denied to be immediate identification of the one with the other, but which must now be reasserted, and to note that, since the poet preserves the passion for his art when free from every other passionality, so he preserves in his art the consciousness of duty (duty toward art), and every poet, in the act of creation, is moral, because he accomplishes a sacred function.

And finally, the order and logic of the various forms of the spirit, making the one necessary for the other and

therefore all necessary, reveal the folly of negating the one in the name of the other: the error of the philosopher (Plato), or of the moralist (Savonarola or Proudhon), or of the naturalist and practical man (there are so many of these that I do not quote names!), who refute art and poetry; and, on the other hand, the error of the artist who rebels against thought, science, practice, and morality, as did so many "romantics" in tragedy, and as do so many "decadents" in comedy in our day. These are errors and follies to which also we can afford a caress in passing (always keeping in view our plan of not leaving anyone quite disconsolate), for it is evident that they have a positive content of their own in their very negativity, as rebellion against certain false concepts or certain false manifestations of art and of science, of practice and of morality (Plato, for example, combating the idea of poetry as "wisdom"; Savonarola, the not austere and therefore corrupt civilisation of the Italian Renaissance so soon to be dissolved), etc. But it is madness to attempt to prove that were philosophy without art, it would exist for itself, because it would be without what conditions its problems, and air to breathe would be taken from it, in order to make it prevail alone against art; and that practice is not practice, when it is not set in motion and revived by aspirations, and, as they say, by "ideals," by "dear imagining," which is art; and, on the other hand, that art without morality, art that usurps with the decadents the title of "pure beauty," and before which is burnt incense, as though it were a diabolic idol worshipped by a company of devils, owing to the lack of morality in the life from which it springs and which surrounds it, is decomposed as art, and become caprice, luxury, and charlatanry; the artist no longer serves it, but it serves the private and futile interests of the artist as the vilest of slaves.

Nevertheless, objection has been taken to the idea of the circle in general, which affords so much aid in making clear the connection of dependence and independence of art and of the other spiritual forms, on the ground that it thinks the work of the spirit as a tiresome and melancholy doing and undoing, a monotonous turning upon itself, not worth the trouble of effecting. Certainly there is no metaphor but leaves some side open to parody and caricature; but these, when they have gladdened us for the moment, oblige us to return seriously to the thought expressed in the metaphor. And the thought is not that of a sterile repetition of going and coming, but a continuous enrichment in the going of the going and the coming of the coming. The last term, which again becomes the first, is not the old first, but presents itself with a multiplicity and precision of concepts, with an experience of life lived, and even of works contemplated, which was wanting to the old first term; and it affords material for a more lofty, more refined, more complex and more mature art. Thus, instead of being a perpetually even revolution, the idea of the circle is nothing but the true philosophical idea of progress, of the perpetual growth of the spirit and of reality in itself, where nothing is repeated, save the form of the growth; unless it should be objected to a man walking, that his walking is a standing still, because he always moves his legs in the same time!

Another objection, or rather another movement of rebellion against the same idea, is frequently to be observed, though not clearly self-conscious: the restlessness, existing in some or several, the endcavour to break and to surpass the circularity that is a law of life, and to attain to a region of repose from movement, so full of anxiety; withdrawn henceforward from the ocean and standing upon the shore, to turn back and contemplate the tossing billows. But I have

already had occasion to state of what this repose consists: an effectual negation of reality, beneath the appearance of elevation and sublimation; and it is certainly attained, but is called death; the death of the individual, not of reality, which does not die, and is not afflicted by its own motion, but enjoys it. Others dream of a spiritual form, in which the circle is dissolved, a form which should be Thought of thought, unity of the Theoretical and of the Practical, Love, God, or whatever other name it may bear; they fail to perceive that this thought, this unity, this Love, this God, already exists in and for the circle, and that they are uselessly repeating a search already completed, or are repeating metaphorically what has already been discovered, in the myth of another world, where the very drama of the only world should be repeated.

I have hitherto outlined this drama, as it truly is, ideal and extratemporal, employing such terms as first and second, solely with a view to verbal convenience and in order to indicate logical order:-ideal and extratemporal, because there is not a moment and there is not an individual in whom it is not all performed, as there is no particle of the universe unbreathed upon by the Spirit of God. But the ideal, indivisible moments of the ideal drama can be seen as if divided in empirical reality, like an impure and embodied symbol of the ideal distinction. Not that they are really divided (ideality is the true reality), but they appear to be so empirically to him who looks upon them with a view to classification, for he possesses no other way of determining in the types the individuality of the facts that have attracted his attention, save that of enlarging and of exaggerating ideal distinctions. Thus the artist, the philosopher, the historian, the naturalist, the mathematician, the man of business, the good man, seem to live separated from one

another; and the spheres of artistic, philosophical, historical, naturalistic, mathematical culture, and those of economic and ethic and of the many institutions connected with them, to be distinct from one another; and finally, the life of humanity is divided into epochs in the ages, in which one or the other or only some of the ideal forms are represented: epochs of fancy, of religion, of speculation, of natural sciences, of industrialism, of political passions, of moral enthusiasms, of pleasure sceking, and so on; and these epochs have their more or less perfect goings and comings. But the eye of the historian discovers the perpetual difference in the uniformity of individuals, of classes, and of epochs; and the philosophical consciousness, unity in difference; and the philosopher-historian secs ideal progress and unity, as also historical progress, in that difference.

. But let us, too, speak as empiricists for a moment (so that since empiricism exists it may be of some use), and let us ask ourselves to which of the specimens belongs our epoch, or that from which we have just emerged: what is its prevailing characteristic? To this there will be an immediate and universal reply that it is and has been naturalistic in culture, industrial in practice; and philosophical greatness and artistic greatness will at the same time both be denied to it. But since (and here empiricism is already in danger) no epoch can live without philosophy and without art, our epoch, too, has possessed both, so far as it was capable of possessing them. And its philosophy and its art—the former mediately, the latter immediately—find their places in thought, as documents of what our epoch has truly been in its complexity and interests; by interpreting these, we shall be able to clear the ground upon which must arise our duty.

Contemporary art, sensual, insatiable in its desire for enjoyments, furrowed with turbid attempts at an ill-un-

derstood aristocracy, which reveals itself as a voluptuous ideal or an ideal of arrogance and of cruelty, sometimes sighing for a mysticism which is also egoistic and voluptuous, without faith in God and without faith in thought, incredulous and pessimistic, - and often very powerful in its rendering of such states of the soul: this art,-vainly condemned by moralists,—when understood in its profound motives and in its genesis, asks for action, which will certainly not be directed toward condemning, repressing, or rearranging art, but toward directing life more energetically toward a more healthy and more profound morality, which will be mother of a nobler art, and, I would also say, of a nobler philosophy. A more noble philosophy than that of our epoch, incapable of accounting not only for religion, for science, and for itself, but for art itself, which has again become a profound mystery, or rather a theme for horrible blunders by positivists, neocriticists, psychologists, and pragmatists, who have hitherto represented contemporary philosophy, and have relapsed (perhaps in order to acquire new strength and to mature new problems!) into the most childish and most crude conceptions of art.

#### IV

#### **QRITICISM AND THE HISTORY OF ART**

ARTISTIC and literary criticism is often looked upon by A artists as a morose and tyrannical pedagogue who gives capricious orders, imposes prohibitions, and grants permissions, thus aiding or injuring their works by wilfully deciding upon their fate. And so the artists either shew themselves submissive, humble, flattering, adulatory, toward it, while hating it in their hearts; or, when they do not obtain what they want, or their loftiness of soul forbids that they should descend to those arts of the courtier, they revolt against it, proclaiming its uselessness, with imprecations and mockery, comparing (the remembrance is personal) the critic to an ass that enters the potter's shop and breaks in pieces with quadrupedante ungulæ sonitu the delicate products of his art set out to dry in the sun. This time, to tell the truth, it is the artists' fault, for they do not know what criticism is, expecting from it favours which it is not in a position to grant, and injuries which it is not in a position to inflict: since it is clear that since no critic can make an artist of one who is not an artist, so no critic can ever undo, overthrow, or even slightly injure an artist who is really an artist, owing to the metaphysical impossibility of such an act: these things have never happened in the course of history, they do not happen in our day, and we can be sure that they will never happen in the future. But sometimes it is the critics themselves, or the self-styled critics, who do actually present themselves as pedagogues, as oracles, as guides of art, as legislators, seers, and prophets; they command artists to

do this or that, they assign themes to them and declare that certain subjects are poetical, and certain others not; they are discontented with the art at present produced, and would prefer one similar to that prevailing at this or that epoch of the past, or at another of which they declare they catch a glimpse in the near or remote future; they will reprove Tasso for not being Ariosto, Leopardi for not being Metastasio, Manzoni for not being Alfieri, D'Annunzio because he is not Berchet or Fra Jacopone; and they describe the great artist of the future, supplying him with ethic, philosophy, history, language, metric, with architectonic and colouristic processes, and with whatever it may seem to them that he stands in need. And this time it is clear that the blame lies with the critic; and the artists are right in behaving toward such brutality in the way that we behave toward beasts, which we try to tame, to illude and to delude, in order that they may serve us; or we drive them away and send them to the slaughter-house when they are no longer good for any service. But for the honour of criticism we must add that those capricious critics are not so much critics as artists: artists who have failed and who aspire to a certain form of art, which they are unable to attain, either because their aspiration was contradictory, or because their power was not sufficient and failed them; and thus, preserving in their soul the bitterness of the unrealised ideal, they can speak of nothing else, lamenting everywhere its absence, and everywhere invoking its presence. And sometimes, too, they are artists who are anything but failures, -indeed, most felicitous artists,—but, owing to the very energy of their artistic individuality, incapable of emerging from themselves in order to understand forms of art different from their own, and disposed to reject them with violence; they are aided in this negation by the odium figulinum, the jealousy

of the artist for the artist, which is without doubt a defect, but one with which too many excellent artists appear to be stained for us to refuse to it some indulgence similar to that accorded to the defects of women, so difficult, as we know, to separate from their good qualities. Other artists should calmly reply to these artist-critics: "Continue doing in your art what you do so well, and let us do what we can do": and to the artists who have failed and improvised themselves critics: "Do not claim that we should do what you have failed in doing, or what is work of the future, of which neither you nor we know anything." As a fact, this is not the usual reply, because passion forms half of it; but this is indeed the logical reply, which logically terminates the question, though we must foresee that the altercation will not terminate, but will indeed last as long as there are intolerant artists and failures—that is to say, for ever.

And there is another conception of criticism, which is expressed in the magistrate and in the judge, as the foregoing is expressed in the pedagogue or in the tyrant; it attributes to criticism the duty, not of promoting and guiding the life of art,-which is promoted and guided, if you like to call it so, only by history; that is, by the complex movement of the spirit in its historical course,—but simply to separate, in the art which has already been produced, the beautiful from the ugly, and to approve the beautiful and reprove the ugly with the solemnity of a properly austere and conscientious sentence. But I fear that the blame of uselessness will not be removed from criticism, even with this other definition, although perhaps the motive of this blame may to some extent be changed. Is there really need of criticism in order to distinguish the beautiful from the ugly? The production itself of art is never anything but this distinguishing, because the artist arrives at purity of expression precisely by elimi-

nating the ugly which menaces to invade it; and this ugliness is his tumultuous human passions striving against the pure passion of art: his weaknesses, his prejudices, his convenience, his laissez faire, his haste, his having one eye on art and another on the spectator, on the editor, on the impresario—all of them things that impede the artist in the physiological bearing and normal birth of his image-expression, the poet of the verse that rings and creates, the painter of sure drawing and harmonious colour, the composer of melody, and introduces into their work, if care be not taken to defend themselves against it, sonorous and empty verses, incorrections, lack of harmony, discordances. And since the artist, at the moment of producing, is a very severe judge of himself from whom nothing escapes,—not even that which escapes others,—others also discern, immediately and very clearly, in the spontaneity of contemplation, where the artist has been an artist and where he has been a man, a poor man; in what works, or in what parts of works, lyrical enthusiasm and creative fancy reign supreme, and in what they have become chilled and have yielded their place to other things, which pretend to be art, and therefore (considered from the aspect of this pretence) are called "ugly." What is the use of the sentence of criticism, when the sentence has already been given by genius and by taste? Genius and taste are legion, they are people, they are general and secular consensus of opinion. So true is this, that the sentences of criticism are always given too late; they consecrate forms that have already been solemnly consecrated with universal applause (pure applause must not, however, be confounded with the clapping of hands and with social notoriety, the constancy of glory with the caducity of fortune), they condemn ugliness already condemned, grown wearisome and forgotten, or still praised in words, but with a bad conscience,

through prejudice and obstinate pride. Criticism, conceived as a magistrate, kills the dead or blows air upon the face of the living, who is quite lively, in the belief that its breath is that of the God who brings life; that is, it performs a useless task, because this has previously been performed. I ask myself what critics have established the greatness of Dante, of Shakespeare, or of Michelangelo: if, among the legions who have acclaimed and do acclaim these great men, there are or have been men of letters and professional critics, their acclamation does not differ in this case from that of youth and of the people, who are all equally ready to open their hearts to the beautiful, which speaks to all, save sometimes, when it is silent, on discovering the surly countenance of a critic-judge.

And so there arises a third conception of criticism; the criticism of interpretation or comment, which makes itself small before works of art and limits itself to the duty of dusting, placing in a good light, furnishing information as to the period at which a picture was painted and what it represents, explaining linguistic forms, historical allusions, the presumptions of fact and of idea in a poem; and in both cases, its duty performed, permits the art to act spontaneously within the soul of the onlooker and of the reader, who will then judge of it according as his intimate taste tells him to judge. In this case the critic appears as a cultivated cicerone or as a patient and discreet schoolmaster: "Criticism is the art of teaching to read," is the definition of a famous critic: and the definition has not been without its echo. Now no one contests the utility of guides to museums or exhibitions, or of teachers of reading, still less of erudite guides and masters who know so many things hidden from the majority and are able to throw so much light on subjects. Not only has the art that is most remote from us need of

this assistance, but also that of the nearest past, called contemporary, which, although it treats of subjects and presents forms that seem to be obvious, is yet not always sufficiently obvious; and sometimes a great effort is requisite in order to prepare people to feel the beauty of a little poem or of some work of art, though born but yesterday. Prejudices, habits and forgetfulness form hedges barring the approach to that work: the expert hand of the interpreter and of the commentator is required to remove them. Criticism in this sense is certainly most useful, but we do not see why it should be called criticism when that sort of work already possesses its own name of interpretation, comment, or exegesis. To call this criticism is at best useless, for it is equivocal.

It is equivocal because criticism demands to be, wishes to be and is something different: it does not wish to invade art, nor to rediscover the beauty of the beautiful, or the ugliness of the ugly, nor to make itself small before art, but rather to make itself great before art which is great and, in a certain sense, above it. What, then, is legitimate and true criticism?

• First of all, it is at once all three of the things that I have hitherto explained; that is to say, all these three things are its necessary conditions, without which it would not arise. Without the moment of art (and, as we have seen, that criticism which affirms itself to be productive or an aid to production, or as repressing certain forms of production to the advantage of certain other forms, is, in a certain sense, art against art), the experience of art would be wanting to the critic, art created within his spirit, severed from nonart, and enjoyed in preference to that. And finally, this experience would be wanting without exegesis, without the removal of the obstacles to reproductive fancy, which supply the spirit with those presumptions of historical knowledge

of which it has need, and which are the wood to burn in the fire of fancy.

But here, before going further, it will be well to resolve a grave doubt which has been agitated and is still agitated, both in philosophical literature and in ordinary thought, and which certainly, where justified, would not only compromise the possibility of criticism, of which I am discoursing, but also of reproductive fancy itself, or taste. Is it truly possible to collect, as does exegesis, the materials required for reproducing the work of art of others (or our own past work of art, when we search our memory and consult our papers in order to remember what we were when we produced it), and to reproduce that work of art in our fancy in its genuine features? Can the collection of the material required be ever complete? And however complete it be. will the fancy ever permit itself to be chained by it in its labour of reproduction? Will it not act as a new fancy, introducing new material? Will it not be obliged to do so. owing to its impotence truly to reproduce the other and the past? Is the reproduction of the individual, of the individuum ineffabile, conceivable, when every sane philosophy teaches that the universal alone is eternally reproducible? Will not the reproduction of the works of art of others or of the past be consequently a simple impossibility; and will not what is usually alleged as an undisputed fact in ordinary conversation, and is the expressed or implied presupposition in every dispute upon art, be perhaps (as was said of history in general) une fable convenue?

Truly, when we consider the problem rather from without, it will seem most improbable that the firm belief which all possess in the comprehension and intelligence of art is without foundation,—all the more, if we observe that these very people who deny the possibility of reproductions in

abstract theory—or, as they call it, the absoluteness of taste -are yet most tenacious in maintaining their own judgments of taste, and very clearly realise the difference there is between the affirmation that wine pleases or displeases me because it agrees or disagrees with my physiological organism, and the affirmation that a poem is beautiful, and another a pastiche: the second order of judgments (as Kant shows in a classical analysis) carries with it the uncoercible pretension to universal validity; souls become passionate about it; and in days of chivalry there were even those who maintained the beauty of the "Gerusalemme," sword in hand, whereas no one that we know has ever been killed maintaining, sword in hand, that wine was pleasant or unpleasant. To object that works artistically base have yet pleased many or someone, and if not others, their author, is not valid, because their having pleased is not set in doubt (since nothing can be born in the soul without the consent of the soul, and consequently without a correlative pleasure); but it is doubted whether that pleasure were æsthetic, and were founded upon a judgment of taste and beauty. And passing from extrinsic scepticism to intrinsic consideration, it should be said that the objection to the conceivability of the asthetic reproduction is founded upon a reality conceived in its turn as a shock of atoms, or as abstractly monadistic, composed of monads without communication among themselves and harmonised only from without. But that is not reality: reality is spiritual unity, and in spiritual unity nothing is lost, everything is an eternal possession. Not only the reproduction of art, but, in general, the memory of any fact (which is indeed always reproduction of intuitions), would be inconceivable without the unity of the real; and if we had not been ourselves Cæsar and Pompey,-that is, that universal which was once determined as Cæsar and Pompey and is

now determined as ourselves, they living in us,—we should be unable to form any idea of Cæsar and Pompey. And further, the doctrine that individuality is irreproducible and the universal only reproducible is certainly a doctrine of "sound" philosophy, but of sound scholastic philosophy, which separated universal and individual, making the latter an accident of the former (dust carried along by time), and did not know that the true universal is the universal individuated, and that the only true effable is the so-called ineffable, the concrete and individual. And finally, what does it matter if we have not always ready the material for reproducing with full exactitude all works of art or any work of art of the past? Fully exact reproduction is, like every human work, an ideal which is realised in infinity, and therefore is always realised in such a manner that it is admitted at every instant of time by the conformation of reality. Is there a suggestion in a poem of which the full signification escapes us? No one will wish to affirm that that suggestion, of which we now have a crepuscular vision that fails to satisfy, will not be better determined in the future by means of research and meditation and by the formation of favourable conditions and sympathetic currents.

Therefore, inasmuch as taste is most sure of the legitimacy of its discussions, by just so much is historical research and interpretation indefatigable in restoring and preserving and widening the knowledge of the past; not mentioning that relativists and sceptics, both in taste and in history, utter their desperate cries from time to time, which do not reduce anyone, not even themselves, as we have seen, to the effectual desperation of not judging.

Closing here this long but indispensable parenthesis and taking up the thread of the discourse, art, historical exegesis, and taste, if they be conditions of criticism, are not yet criti-

cism. Indeed, nothing is obtained by means of that triple presupposition, save the reproduction and enjoyment of the image-expression; that is to say, we return and place ourselves neither more nor less than in the place of the artist-producer in the act of producing his image. Nor can we escape from those conditions, as some boast of doing, by proposing to ourselves to reproduce in a new form the work of the poet and the artist by providing its equivalent; hence they define the critic: artifex additus artifici. Because that reproduction in a new garment would be a translation, or a variation, another work of art, to some extent inspired by the first; and if it were the same, it would be a reproduction pure and simple, a material reproduction, with the same words, the same colours, and the same tones—that is, useless. The critic is not artifex additus artifici, but philosophus additus artifici: his work is not achieved, save when the image received is both preserved and surpassed; it belongs to thought, which we have seen surpass and illumine fancy with new light, make the intuition perception, qualify reality, and therefore distinguish reality from unreality. In this perception, this distinction, which is always and altogether criticism or judgment, the criticism of art, of which we are now especially treating, originates with the question: whether and in what measure the fact, which we have before us as a problem, is intuition—that is to say, is real as such; and whether and in what measure, it is not such—that is to say, is unreal: reality and unreality, which in art are called beauty and ugliness, as in logic they are called truth and error, in economy gain and loss, in ethic good and evil. Thus the whole criticism of art can be reduced to this briefest proposition, which further serves to differentiate its work from that of art and taste (which, considered in themselves, are logically mute), and from exegetical erudition (which lacks logi-

cal synthesis, and is therefore also logically mute): "There is a work of art a," with the corresponding negative: "There is not a work of art a."

It seems to be a trifle, for the definition of art as intuition seemed to be neither more nor less than a trifle, but it has on the contrary been since seen how many things it included in itself, how many affirmations and how many negations: so many that, although I have proceeded and proceed in a condensed manner. I have not been able and will not be able to afford more than brief mention of them. proposition or judgment of the criticism of art, "The work of art a is," implies, above all, like every judgment, a subject (the intuition of the work of art a) to conquer which is needed the labour of exegesis and of fantastic reproduction, together with the discernment of taste: we have already seen how difficult and complicated this is, and how many go astray in it, through lack of fancy, or owing to slightness and superficiality of culture. And it further implies, like every judgment, a predicate, a category, and in this case the category of art, which must be conceived in the judgment, and which therefore becomes the concept of art. And we have also seen, as regards the concept of art, to what difficulties and complications it gives rise, and how it is a possession always unstable, continually attacked and ambushed, and continually to be defended against assaults and ambushes. Criticism of art, therefore, develops and grows, declines and reappears, with the development, the decadence, and the reappearance of the philosophy of art; and each can compare what it was in the Middle Ages (when it may almost be said that it was not) with what it became in the first half of the nineteenth century with Herder, with Hegel, and with the Romantics, in Italy with De Sanctis; and in a narrower field, what it was with De Sanctis, and what it became in the

following period of naturalism, in which the concept of art became clouded and finally confused with physic and with physiology, and even with pathology. And if disagreements as to judgments depend for one half, or less than half, upon lack of clearness as to what the artist has done, lack of sympathy and taste for another half, or more than half, this arises from the small clearness of ideas upon art; whence it often happens that two individuals are substantially at one as to the value of a work of art, save that the one approves what the other blames, because each refers to a different definition of art.

And owing to this dependence of criticism upon the concept of art, as many forms of false criticism are to be distinguished as there are false philosophies of art; and, limiting ourselves to the principal forms of which we have already discoursed, there is a kind of criticism which, instead of reproducing and characterising art, breaks in pieces and classifies it; there is another, moralistic, which treats works of art like actions in respect of ends which the artist proposes or should have proposed to himself; there is hedonistic criticism, which presents art as having attained or failed to attain to pleasure and amusement; there is also the intellectualistic form, which measures progress according to the progress of philosophy, knows the philosophy but not the passion of Dante, judges Ariosto feeble because he has a feeble philosophy, Tasso more serious because his philosophy is more serious, Leopardi contradictory in his pessimism. There is that criticism usually called psychological, which separates content from form, and instead of attending to works of art, attends to the psychology of the artists as men; and there is the other form, which separates form from content and is pleased with abstract forms because, according to cases and to individual sympathies, they recall

antiquity or the Middle Ages; and there is yet another, which finds beauty where it finds rhetorical ornaments; and finally there is that which, having fixed the laws of the kinds and of the arts, receives or rejects works of art according as they approach or retreat from the models which they have formed. I have not enumerated them all, nor had I the intention of so doing, nor do I wish to expound the criticism of criticism, which could be nothing but a repetition of the already traced criticism and dialectic of Æsthetic; and already here and there will have been observed the beginnings of inevitable repetition. It would be more profitable to summarise (if even a rapid summary did not demand too much space) the history of criticism, to place the historicalnames in the ideal positions that I have indicated, and to shew how criticism of models raged above all during the Italian and French classical periods, conceptualistic criticism in German philosophy of the nineteenth century, that of moralistic description at the period of religious reform or of the Italian national revival, psychology in France with Sainte-Beuve and many others; how the hedonistic form had its widest diffusion among people in society, among boudoir and journalistic critics; that of classifications, in schools, where the duty of criticism is believed to have been successfully fulfilled when the so-called origin of metres and literary and artistic kinds and their representatives has been investigated.

But the forms which I have briefly described are forms of criticism, however erroneous; though this cannot, in truth, be said of other forms which raise their banners and combat among themselves, under the names of "æsthetic criticism" and "historical criticism." These I beg leave to baptise, on the contrary, as they deserve, pseudo-æsthetic criticism (or æsthetistic), and pseudo-historical criticism (or historisti-

cal). These two forms, though very much opposed, have a common hatred of philosophy in general, and of the concept of art in particular: against any intervention of thought in the criticism of art, which in the opinion of the former is the affair of artistic souls; in the opinion of the latter, of the erudite. In other words, they debase criticism below criticism, the former limiting it to pure taste and enjoyment of art, the latter to pure exceptical research or preparation of materials for reproduction by the fancy. What Æsthetic, which implies thought and concept of art, can have to do with pure taste without concept is difficult to say; and what history can have to do with disconnected erudition relative to art, which is not organisable as history because without a concept of art and ignorant of what art is (whereas history demands always that we should know that of which we narrate the history), is yet more difficult to establish; at the most we could note the reasons for the strange "fortune" which those two words have experienced. But there would be no harm in those names or in the refusal to exercise criticism, provided that the upholders of both should remain within the boundaries assigned by themselves, these enjoying works of art, those collecting material for exegesis; and they might leave criticism to him who should wish to criticise, or satisfy themselves with speaking ill of it without touching problems which properly belong to criticism. In order to attain to such an attitude of reserve it would be necessary neither more nor less than that the æsthetes should never open their mouths in ecstasy about art, that they should silently degustate their joys, and, at the most, that when they met their like they should understand one another, as animals are said to do (who knows, though, if it be true!) without speaking: their countenance unconsciously bearing an expression of ravishment, their arms outstretched in an attitude of

wonder, or their hands joined in a prayer of thanksgiving for the joy experienced, should suffice for everything. Historians, for their part, might certainly speak: speak of codices, of corrections, of chronical and of topical dates, of political facts, of biographical occurrences, of sources of works, of language, of syntaxes, of metres, but never of art, which they serve, but to whose countenance, as simple crudites, they cannot raise their eyes, as the maid-servant does not raise them to look upon her mistress, whose clothes she nevertheless brushes and whose food she prepares: sic vos. non vobis. But go and ask of men such abstentions, sacrifices, and heroisms, however extravagant in their ideas and fanatic in their extravagances! In particular, go and ask those who, for one or another reason, are occupied with art all their lives, not to talk of or to judge art! But the mute æsthetisticians talk of, judge, and argue about art, and the inconclusive historicians do the same; and since in thus talking they are without the guide of philosophy and of the concept of art, which they despise and abhor, and yet have nced of a concept,—when good sense does not fortunately happen to suggest the right one to them, without their being aware of it, - they wander among all the various preconceptions, moralistic and hedonistic, intellectualistic and contentistic, formalistic and rhetorical, physiological and academical, which I have recorded, now relying upon this one, now upon that, now confounding them all and contaminating one with the other. And the most curious spectacle (though to be foreseen by the philosopher) is that the æsthetisticians and historicians, those irreconcilable adversaries, although they start from opposite points, yet agree so well that they end by uttering the same fatuities; and nothing is more amusing than to meet again the most musty intellectualistic and moralistic ideas in the pages of deeply moved lovers of

art (so deeply moved as to hate thought), and in the most positive historians (so positive as to fear compromising their positivity by attempting to understand the object of their researches, which chances this time to be called art).

True criticism of art is certainly asthetic criticism, but not because it disdains philosophy, like pseudo-æsthetic, but because it acts as philosophy and as conception of art; it is historical criticism, not because, like pseudo-history, it deals with the extrinsic of art, but because, after having availed itself of historical data for fantastic reproduction (and till then it is not yet history), when fantastic reproduction has been obtained, it becomes history, by determining what is that fact which has been reproduced in the fancy, and so characterising the fact by means of the concept, and establishing what exactly is the fact that has occurred. Thus, the two things at variance in spheres inferior to criticism coincide in criticism; and "historical criticism of art" and "asthetic criticism" are the same: it is indifferent which word we use, for each may have its special use solely for reasons of convenience, as when, for instance, it is desired to call special attention, with the first, to the necessity of the understanding of art; with the second, to the historical objectivity of its consideration. Thus the problem discussed by certain methodologists is solved, namely, whether history enter into the criticism of art as means or as end: since it is henceforth clear that history adopted as a means is not history, precisely because it is a means, but is exegetic material; and that which enters it as end is certainly history, though it does not enter it as a particular element, but as its constituent whole: which precisely describes the word "end."

But if criticism of art be historical criticism, it follows that it will not be possible to limit the duty of discerning the beautiful and the ugly to simple approval and refusal in

the immediate consciousness of the artist when he produces, or of the man of taste when he contemplates; it must widen and clevate itself to what is called explanation. And since in the world of history (which is, indeed, the only world) negative or privative facts do not exist, what seems to taste to be ugly and repugnant, because not artistic, will be neither ugly nor repugnant to historical consideration, because it knows that what is not artistic yet is something else, and has its right to existence as truly as it has existed. The virtuous Catholic allegory composed by Tasso for his "Gerusalemme" is not artistic, nor the patriotic declamation of Niccolini and Guerrazzi, nor the subtleties and conceits which Petrarch introduced into his poems; but Tasso's allegory is one of the manifestations of the work of the Catholic counter-reform in the Latin countries: the declamations of Niccolini and of Guerrazzi were violent attempts to rouse the souls of Italians against the priest and the stranger, representing adhesion to the manner of that arousing: the subtleties and conceits of Petrarch, the cult of traditional troubadour elegance, revived and enriched in the new Italian civilisation; that is to say, they are all practical facts, very significant historically and worthy of respect. We can well continue to talk of the beautiful and of the ugly, in the field of historical criticism, through vivacity of language, or in order to chime with current parlance; provided that we shew at the same time, or hint, or let be understood, or at least do not exclude, the positive content, both of that beautiful and of that ugly, which will never be so radically condemned in its ugliness as when it is fully justified and understood, because in this case it will be removed in the most radical manner from the sphere proper to art.

For this reason, criticism of art, when truly æsthetic or historical, becomes at the same time amplified into a criti-

cism of life, since it is not possible to judge—that is, to characterise—works of art without at the same time judging and characterising the works of the whole life: as we observe with the truly great critics, and above all with De Sanctis, in his "History of Italian Literature" and in his "Critical Essays," who is as profound a critic of art as of philosophy, morality, and politics; he is profound in the one because profound in the other, and inversely: the strength of his pure æsthetic consideration of art is the strength of his pure moral consideration of morality. Because the forms of the spirit, of which criticism avails itself as categories of judgment, although ideally distinguishable in unity, are not materially separable from one another and from unity, under penalty of seeing them vanish before us. We cannot, therefore, speak of a distinction of art from other criticism, save in an empirical manner, to indicate that the attention of the speaker or writer is directed to one rather than to another part of his indivisible argument. And the distinction is also empirical (I have hitherto preserved this here, in order to proceed with didactic clearness) between criticism and history of art: a distinction which has been specially determined by the fact that a polemical element prevails in the study of contemporary art and literature, which causes it to be more readily called "criticism," while in that of the art and literature of a more remote period prevails the narrative tone, and therefore it is more readily termed "history." In reality, true and complete criticism is the serene historical narration of what has happened; and history is the only true criticism that can be exercised upon the doings of humanity, which cannot be not-facts, since they have happened, and are not to be dominated by the spirit otherwise than by understanding them. And since the criticism of art has shewn itself inseparable from other criticism, so the history of art

can be separated from the complete history of human civilisation only for reasons of a literary nature, among which it certainly follows its own law, which is art, but from which it receives the historical movement, which belongs to the spirit as a whole, never to one form of the spirit separated from the others.

BENEDETTO CROCE.

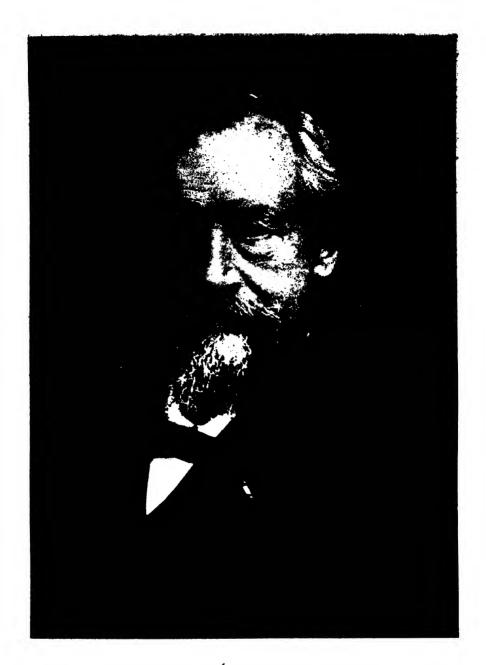
# MUTATIONS IN HEREDITY GEOGRAPHICAL BOTANY MODERN CYTOLOGICAL PROBLEMS THE IDEALS OF AN EXPERIMENT GARDEN<sup>1</sup>

# First Lecture MUTATIONS IN HEREDITY

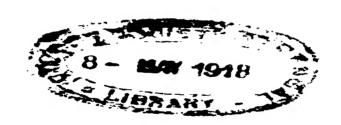
SINCE the publication of the two volumes of my "Mutation Theory" ten years have elapsed. At that time the prevailing opinion was that very small and often even invisible changes could gradually be increased and accumulated, and that this process could lead to specific differences, and even to the production of the characters of genera and larger groups. This conception was the principle of the theory of selection as proposed by Darwin, as well as the starting-point for the hypothesis of orthogenesis, of the direct influence of environment, and of many others. It was generally accepted in the teachings of plant improvement in agriculture, and, as a matter of fact, the origin of new varietics by leaps and bounds was a fact well known only to horticulturists.

In opposition to this conception, I tried to show that the origin of new forms complies, in nature as well as in agriculture, to the mode which was observed to be followed in horticulture, and that the whole evolution of the plant kingdom has been brought about by a long series of successive small leaps. The extraordinarily slow evolution which was

<sup>&</sup>lt;sup>1</sup> Four lectures presented at the inauguration of the Rice Institute, by Professor Hugo de Vries, Director of the Hortus Botanicus and Professor of Botany in the University of Amsterdam.



Ingo dervies



a necessary consequence of the then prevailing opinion required an almost unlimited duration of time; but the new principle of mutations reduced the biological time to the limits which had been determined by physicists and geologists for the duration of life on this earth. The starting-point for the new ideas was the distinction between two main types of variability: fluctuation and mutation. I had deduced this principle from my interpretation of Darwin's well known provisional hypothesis of pangenesis, and convinced myself of its truth by means of a series of experiments. On the basis of these theoretical considerations I proposed the mutation theory, which means that the characters of all organisms are built up of sharply distinguished units. These qualities may be combined into groups, and in allied species the same units and groups may be met with. They do not pass gradually into one another; transitions fail between them, although they may often be observed between the external forms of plants and animals.

The changes in the number and the position of these units, as well as those in their relative connections, constitute the domain of mutability. They are the causes of discontinuous variation, or of the sudden appearance of externally visible deviations. The steps are, as a rule, only small ones; but are inherited as such from the very beginning, without transitions. Apart from these, the different organs and qualities continually vary in number as well as in measure and weight. In doing so they are observed to follow the laws of probability and to be influenced by external factors; favorable conditions may increase them in one way, while unfavorable circumstances may determine their augmentation in the opposite direction. Such changes are described as fluctuations or as fluctuating variability. On the basis of the investigations of Quetelet, their laws have been very completely studied. All these phenomena are governed by internal as

well as by external causes. The internal ones are given by the hereditary units and determine the nature of the changes which may take place; while the external factors decide when and to what extent the deviations from the average will occur.

As well as fluctuations, mutations are induced by external and internal causes, as I have distinctly pointed out. The determination of these, however, is far more difficult than in the case of fluctuations. It is only in a general way that my experiments show that mutability may be increased by favorable conditions of life. In connection with this fact, we may assume that, in nature, the origin of new forms is not due to a hard struggle, but is promoted by a luxuriant environment and by easy conditions of development. It is true that a struggle for life must be; but this comes in after the new forms have already been produced, and, as it seems, often only after a considerable lapse of time. Such a struggle for life demands no greater sacrifices than those which are unavoidable, even under the common conditions of the field; while in the old selection theory the sacrificing of thousands of lives was required for every step in progressive development.

In the last ten years the principle of character units has gained a firm hold for itself in evolutionary science. It has transferred the problems from the domain of speculation to that of experiment, and has brought the teachings of Mendel (which had been disregarded up to that time) to universal acknowledgment. The generally accepted view of the continuous intergradation of characters into one another had for a long time been in the way of a broad appreciation of the merits of the principle of Mendel; but the theory of pangenesis has led me to experiments in hybridization which fully confirmed the results of Mendel, and clearly showed

their high importance. Moreover, the lines of research laid down by Mendel proved to be of easy application to an almost unlimited number of cases, and so the study of the last ten years has turned in the main to them, and thereby to a great extent neglected the direct investigation of the origin of new forms.

The theory of mutation is not intended to take the place of the theory of selection of Darwin. It is only one step further in the development of our appreciation of evolutionary phenomena. The problem of the theory of selection is the explanation of the overwhelming richness of living forms in nature. It has succeeded in bringing this under the grasp of our understanding; but it has the disadvantage of easily conducing to poetical speculations whenever one tries to apply the general views to single cases. In such cases many authors are content with hypothetical descriptions of what the relations of the phenomena may be supposed to be. Contrary to this method, the theory of mutation deals with the problem of the origin of the material from which natural At the time of Darwin the distinction selection chooses. between fluctuation and mutation had not yet been discovered; but as soon as this was the case it was clear that only the latter process could supply the material for further selection. This principle at once got rid of numerous difficulties which up to that period seemed to be inherent in the teachings of Darwin.

Among those who supported the new theory in its first years, although with some reserve, I cite in the first place Strasburger, who wrote as early as 1902 "that the formation of species does not start from fluctuating variability, but from mutations," and that especially "for the place of an organism in the natural system the degree of development reached by all the successive mutations is decisive." He

<sup>&</sup>lt;sup>1</sup> "Jahrb. f. wiss. Bot.," T. 37, 1902, p. 518.

was soon followed by the larger part of the botanists, although many among them took exception for the adaptation of species to their environment.

Among paleontologists, Charles A. White was the first to take publicly the side of the theory of mutation,1 and the most prominent representatives of this science soon adhered to his ideas. It might perhaps be said that in no other domain has the new principle been so rapidly and so generally acknowledged. Here numerous facts are in evident contradiction to the idea of an extremely slow evolution among fossil plants as well as animals. Other facts clearly show "that the degree of mutability of species has not always been the same during the geological periods of their existence, but is evidently subjected to changes" (p. 638). This sentence corresponds exactly to my conception of periods of mutability. Life before the Cambrian times is wholly unknown to us: but in this period all the main branches of the animal kingdom at once make their appearance, with the exception of the vertebrates only. Only by means of very complicated hypotheses could the old conception explain these broad facts. Among the floras of all times that of the Carboniferous period has without any doubt been by far the richest; it appeared suddenly, and afterward disappeared almost at once. Many types of organisms have escaped the changing influence of natural selection during a long succession of geological times, as, for instance, the genus Unio, which has come to us almost without any modification from the Mesozoic period. In the Tertiary layers of Florida, Dall has pointed out the occurrence of numerous forms which have come over from one period into the succeeding one, and which are still in part among living species. The evolution of the pedigree of the vertebrates during Tertiarv times has been an exceedingly rapid one—by far too fast to

<sup>&</sup>lt;sup>1</sup> Smithsonian Report for 1903, pp. 631-640.

be compatible with the old view of slow improvement. The same conclusion holds good for birds, for fishes, for phanerogamic plants, and for quite a number of smaller groups. All in all, the geological facts plead against a slow and for a relatively rapid evolution, thereby justifying the conception of modification by leaps. Such were the arguments of White, but it would take me too long to cite them in all their details.

In the domain of zoölogy the old and the new conception are still sharply opposed. The new ideas easily comply with the celebrated theory of Hubrecht concerning the evolution of the pedigree of the vertebrates, and the author of this view has more than once vigorously supported my ideas. On the other hand, Plate is still among the adherents of the validity of the unmodified theory of selection.

In the field of agriculture the new conceptions are found to be in full harmony with the experience of Hjalmar Nilsson, the director of the Swedish agricultural experiment station at Svalöf. By means of elaborate experiments this investigator has shown that a selection of fluctuating differences has no value at all for the improvement of agricultural plants, especially cereals; and that all breeding of new races must start from a careful choice of the best among the elementary races, which are found in the present cultivated varieties. The unexpectedly large results which this method has rapidly produced have gained for it a general acknowledgment in agricultural circles, and the principle of slow improvement of races has since been replaced almost wholly by that of the choice of single mother-plants ("enstaka moderplanterna") and of the cultivation of pure races from their seed.

But still there is always much discussion and much opposition, and therefore it may be useful to give a short review of the main arguments which seem to plead against the new

theory. Before doing so, I might, however, point out two volumes which, from different points of view, deal with almost all the questions which are still open in this field and give a fair appreciation of the arguments brought forward by different authors. One of them is a German treatise on "Abstammungslehre" by Buekers<sup>1</sup>; the other, a volume in French on "Transformations brusques des êtres vivants," written by L. Blaringhem.<sup>2</sup> The first of these two books deals mainly with the questions from a critical point of view, and is very exhaustive in this respect; while Blaringhem supports his opinion by a thorough study and accurate description of a number of new mutations which occurred in his cultures.

Some authors have asserted that the theory of mutation has been deduced from the doctrine of hybridism. Others have pretended that my experiments with the evening primrose of Lamarck were its starting-point. Both these opinions are erroneous from the historical point of view as well as from a logical one. The mutation theory originated from the hypothesis of pangenesis.<sup>3</sup> This hypothesis suggested to Darwin the principle of the units which he called gemmules. Every one of these represented, in his opinion, a visible part of the organism, even of a single cell.

According to my conception, the units correspond to the qualities by the coöperation of which the whole character of the organism is built up. Each of these units may express itself in different parts of the individual. It is from this conception, as stated above, that I derived the hypothesis of the two main types of variability. In order to control this deduction by means of experiments, I studied, on the one hand, variability itself; and, on the other, hybridism. The

<sup>&</sup>lt;sup>1</sup> Dr. P. G. Buekers, "Abstammungslehre," Leipzig, 1909, § 354. <sup>2</sup> "Bibliothèque de Philosophie scientifique," Paris, E. Flammarion, 1911.

<sup>&</sup>lt;sup>8</sup> See Λ. Λ. W. Hubrecht, in "Popular Science Monthly," July, 1904, p. 222, and V. Haecker, "Allgemeine Vererbungslehre," 2° Λufl., 1912, p. 287.

first of these two groups of experiments included over a hundred different species, some of which showed signs of mutability, while by far the larger number did not. A small degree of the propensity to produce new forms was observed in Linaria vulgaris, Dahlia variabilis, Chrysanthemum segetum and Dracocephalum moldavicum. Among all the species studied by me, I found, however, only a single one which showed the new quality in quite a large degree producing new types almost every year, and thereby stimulating to an extensive as well as intensive study. I supported this inquiry by a critical review of the numerous facts scattered through the literature in the fields of agriculture, horticulture, teratology and other sciences; and, almost at the same time, the whole range of observations which pleaded for a sudden origin of cultivated varieties was exhaustively collected by Korshinsky from the horticultural literature.

Another widely distributed error is the opinion that the theory of mutation is opposed to the principle of selection. It is even asserted sometimes that the theory of selection should have been replaced by it. I have already pointed out that the real service done by Darwin to evolutionary science lies in the proposition of his principle of explaining the development of the organisms from one another, in its main lines as well as in its details, on the basis of well ascertained facts only. His means to reach this aim were the struggle for life and the survival of the fittest-or, in one word, natural selection. The question whence the material for this selection was derived was of course duly and fully dealt with; but our knowledge of the phenomena of variability was at that time still in its infancy, and far from being adequate to the demands Darwin made upon it. This was the reason why he did not succeed in convincing his contemporaries. It is only on this weak point that the theory of

mutation has to come in. Its aim is not to be sought in the explanation of the different forms of life. It starts from the principle that the changes which find their expression in variability are intrinsically connected with the germ-plasm; that they are provoked within this substance before fecundation, either in one or both of the sexual elements, and come to light only afterward, during the development of the new individual. Although evidently dependent on external factors, such as nutrition, etc., they are not each related to these in such a manner that it would already be possible for us to explain this dependency in its details. The older and some of the still prevailing theories consider that the changes take place first in the growing or even in the adult organs, and are only transferred afterward to the sexual cells.

From a general point of view, the chances of a new idea finding adherence often depend in a great degree on its applicability to other fields of inquiry besides its own experimental domain. General considerations are often more decisive than pure facts. In this respect the mutation theory has the great advantage of easily complying with the most widely divergent conceptions of the phenomena of adaptation. It may be combined with these even more intimately than the older views, as I shall show later on.

The empirical basis of the new teachings is the distinction between fluctuation and mutation. The first is the ordinary form of variation, often called individual, gradual or continuous variation, and well known to Darwin himself. It is almost always and everywhere active in a lesser or in a greater degree. Mutation, on the other hand, is a rare and most sporadic phenomenon only rarely occurring in groups, but by means of it new types are seen to arise suddenly, sharply, although often not widely distinct from the parental type. With this proposition many authors have since ex-

pressed their agreement, and in one of the newest manuals Karsten summarizes the now prevailing conviction by saying, "Spontaneous variation or mutation is sharply distinct from fluctuation, since it proceeds by leaps and at once produces hereditary differences." And even the most ardent opponent of my view—Plate—in concluding his lecture on "Inheritance and the Theory of Descent," says that "phyletical evolution is discontinuous in the changes of the determinants, although ordinarily continuous in its external display"; and in doing so he evidently concedes the main point in discussion.

Fluctuations are quantitative variations, but mutations are of the qualitative kind. Under the influence of selection, the first do not produce constant races which become independent from that selection, while the products of mutation are at once of an hereditary nature and constant. This principle has brought the study of elementary species into the first rank of biological interest. The investigations of Jordan, de Bary and many others had not succeeded in convincing biologists and systematists of the truth that the species of Linnæus are in reality collective entities, and that the real units of nature are the so-called small species. It is quite evident that it is impossible to observe the origin of such a collective species, since the conception is partly, at least, of an artificial nature. But now the origin of the small species has become an object of direct inquiry. One of the oldest objections against the theory of descent has thereby been surmounted forever. Even in the field of pure description the new ideas have their influence. It is conceded that even the so-called type specimens might not be homogeneous

Leipzig, 1911, p. 295.

<sup>2</sup> L. Plate, "Festschrift zum sechzigsten Geburtstage Richard Hertwigs," Bd. II, 1910, p. 607.

<sup>1</sup> Nussbaum, Karsten und Weber, "Lehrbuch der Biologie für Hochschulen,"

if they are based on different individuals collected in the field. Small differences of the nature of those existing between elementary species might occur among them and sooner or later become the source of misunderstanding. A pure prototype can evidently not be secured in this way, or at least its purity cannot be guaranteed. Starting from these considerations, Kellerman and Swingle have lately pointed out the necessity of taking all the type specimens for one species from one single individual, and proposed to distinguish those which comply with this principle by the name of merotypes.<sup>1</sup>

One of the greatest difficulties of the theory of selection as worked out by Darwin is found in the fact that changes which after some degree of development may be advantageous to their possessors cannot be of any use to them at their first appearance as almost invisible deviations from the old type, and even during a long period afterward. Notwithstanding this, the theory requires their being selected from among the others, and this on the ground of their usefulness. This objection has been dealt with exhaustively by a large number of authors; but in the last ten years all of them agree in conceding that it has been successfully met with by the principle of mutation.

One of the main supports for the ideas of Darwin was a comparison of selection as used in agricultural practice with the corresponding phenomena in nature. Unfortunately, the descriptions of their procedures given by the leading agriculturists were far from adequate to the use Darwin wanted to make of them. On the one hand, he succeeded in proving the analogy between artificial and natural selection by heaping up an overpowering material of facts, and it seems to me that this proof has been one of the principal factors in the

<sup>&</sup>lt;sup>1</sup> Journal of the Washington Academy of Sciences, Vol. II, May, 1912, No. 9, p. 222.

victory which his theory has so completely gained. But the agriculturists themselves did not clearly understand their practice, and even partially explained it in an erroncous way, and these errors were transferred unobservedly to the theory of natural selection. It was only a critical study of the classical and thoroughly scientifically conducted cultures of Rimpau in selecting his rye which yielded a satisfactory, although belated, understanding of the whole phenomenon.1 I found out that Rimpau, although believing he was selecting only the richest ears from among a uniform race, in reality chose the best elementary species from a motley mixture of types. From the progeny of his handful of chosen ears he subsequently eliminated the minor ones, until by means of a selection of some ten to twenty ears he finally reached a pure race, which, according to our present conception, must have consisted only of the progeny of the very best one of the ears he chose in the beginning. Such a pure race was no longer exposed to reversions, and this has been thoroughly proved in the case of the rve of Rimpau by the cultures of Schribaux in northern France. At present the principle is universally recognized. We may safely transfer it to the comparison of artificial and natural selection as proposed by Darwin, and conclude from it that Nature herself does not select her new species from fluctuating variations, but from the existing small types, or, in the end, from mutations that occurred at a previous time.

I now come to a consideration of the two principal theories which have secured for themselves quite a number of adherents and are still defended by many authors as auxiliaries of the old theory of selection. I mean the principles of orthogenesis and neo-Lamarckism, or the theory of direct influences. The former of these refers to the main

<sup>1</sup> Proceedings, American Philosophical Society, Vol. XLV, 1906, pp. 149-156.

lines in the pedigree of the animal and vegetable kingdoms, the latter to the adaptations in the ultimate branches of these pedigree trees. In my opinion, neither of them is opposed to the teachings of the theory of mutation, especially since they are destined for quite another field of phenomena.

This is clearly shown by the curious circumstance that the adherents of orthogenesis recognize the validity of the new theory for the explanation of adaptations, while the neo-Lamarckists declare it to be valuable only for the origin of the larger branches of the system. All of them recognize the process of mutation as the normal mode of origin of species, and make an exception only for the field in which they are especially engaged.<sup>1</sup>

Before continuing this discussion, it is, however, necessary to deal with the distinction between characters of organization and those of adaptation as proposed by Nägeli. The former are the marks of natural families and of higher groups; they have been evolved in very old geological times, and our knowledge concerning the climate and the life conditions of those periods is necessarily limited to a general outline and does not justify us in making a distinct idea of the environmental conditions of the single species and of the claims made upon them by the struggle for life. Therefore it is hardly possible to deal with the causes of their evolution and of the origin of new types of life with any higher degree of probability than that of more or less poetical descriptions.2 These characters of organization are often supposed to have originated in a manner essentially different from that of the characters of adaptation. In the former case,

<sup>&</sup>lt;sup>1</sup> Von Wettstein, "Handbuch der systematischen Botanik," 1901, p. 36; Strasburger, "Jahrb. f. wiss. Bot.," 1902, T. 37, p. 518 u. A. <sup>2</sup> This expression is not meant to include the least reproach. On the con-

<sup>&</sup>lt;sup>2</sup> This expression is not meant to include the least reproach. On the contrary, I myself often prefer using such forms of speech, trusting that my readers will recognize them for what they are intended to be. Critics who failed to see this point more than once have given me great amusement.

internal causes are assumed to be the most essential factors; while in the latter this rôle is given to the external conditions.

Adaptation is limited to the very youngest qualities of animals and plants, and this is carefully pointed out by the most prominent among living neo-Lamarckists, von Wettstein. He says, "As far as experience goes, we may assert that by means of direct adaptation nothing absolutely new is produced but that its results are in the main directed to an augmentation or a diminution of properties already present." And to this sentence he joins another, which eliminates all possible doubt, and which says that, after long times of direct adaptation and after the disappearance of such transitional forms as it may have produced, "the impression of an essential deviation" may be made upon us.<sup>1</sup>

From this discussion it is clear that the characters of organization and adaptation do not cover the whole field of systematic differences. The former are limited to those between the larger groups; and it is characteristic of them that they do not show any relation to the struggle for life -at least not at the present time. The characters of adaptation, on the other hand, are the marks of the youngest of all the systematic groups, and arc, as a rule, limited to species and subspecies. Between the two divisions there is a wide gap; but this field includes, curiously enough, exactly those cases which are the most interesting ones for the great principle of evolution. Large, and therefore, at all events, not very young groups, like most of the cactaceous and euphorbiaceous plants, in many cases show the most beautiful and stringent arrangements for a life under strongly specialized conditions. These, however, are to be included neither with the marks of organization as described by Nägeli, nor with those of adaptation as proposed by von Wettstein. There-

fore it seems unavoidable to collect them into a new division, for which it seems practical to choose the name of characters of specialization. For the cases to be covered by this expression are taken from plants which show a high degree of differentiation on very special lines; and the question whether this is useful or only innoxious to them is one which it is at best hardly possible to decide on a purely empirical basis. But on the layman they make the impression of the most beautiful adaptations.

Warming has distinctly pointed out that the real nature of orthogenesis, as well as of direct influence, is not clear to us. The latter of the two principles assumes an intimate correlation between the external factors and the usefulness of the deviations produced by them, but in Warming's opinion this relation is "of obscure nature." Therefore it seems justifiable to assume that this direct influence is not a single mystic force of nature, but the result of the combination of a larger or lesser number of such forces. But in this case it must be possible to make an analysis of them, and it strikes me that the theory of mutation is capable of supplying us with precisely the necessary means for this purpose.

The same reasoning and conclusion hold good for the principle of orthogenesis. Concerning this Coulter says: "Long ago it seemed possible to consider it to be 'a mysterious principle inherent in organic life,' or as an internal force which determined the direction of variability; but in our time, since the rôle of environmental conditions and the whole group of external factors have come into the foreground of biological interest, this conception can no longer be considered as sufficient. But, at all events, we hardly know how these external factors really influence evolution,

<sup>1</sup> Warming, "Ecology of Plants," 1909, p. 370.

and which is their true nature." And in connection with this he remarks that natural selection, mutation and orthogenesis are far from excluding one another.

Let us now consider these three groups separately. the case of orthogenesis we may limit ourselves to two First, the suggestion that it is not variation, but selection, which has been working in the same direction during long geological times. And although, as already pointed out, we know very little about the factors of the struggle for life in those remote times, this question seems to claim full appreciation. If we decide for a continuity in the selection, variability may be assumed as occurring in indistinct directions, even at those times. If, however, we take the opposite point of view, it remains an open question whether the onesided variability which we must then assume was of the nature of fluctuation or of mutation. And since the former determines only an augmentation or a diminution of qualities already present, we should conclude with the conception of series of mutations taking place in an unchanged direction. This would bring us in line with the proposition of indistinct mutability, since evidently all mutations which would take place in divergent directions would sooner or later have to disappear. Be this as it may, my aim is only to show that, even in such an hypothetical field, the theory of mutation has the best chance of complying with our knowledge of the available facts, without the need of recurring to secondary hypotheses. I have already pointed out that the paleontologists are best prepared to recognize the principle of mutation for orthogenetic evolution. In concluding it seems to me that orthogenesis may best be explained as produced by successive mutations, which themselves have been conducted by orthogenetic selection.

<sup>&</sup>lt;sup>1</sup> Coulter, Barnes and Cowles, "Textbook of Botany," Vol. I, p. 290.

The conception that characters of adaptation cannot be due to selection, but must be induced by direct environmental influences, was formerly derived in the main from the fact that very small deviations from a given type can have hardly any advantage in the struggle for life, while the theory of natural selection must assume a distinct activity of its principle from the very beginning. As is well known, this difficulty is met with in the most satisfactory manner by the theory of mutation, and on this point almost all authors agree. Different propositions for reconciliation have been made. Thus, for example, the direct influence of the environment seems sufficient to von Wettstein, while Strasburger holds the opinion that it must always be accompanied by selection in order to take its effect. Evidently such an assumption would make the whole theory superfluous.

Two objections must still be considered. In the first place, it should be pointed out that the differential characters on which the diagnostic descriptions of species are based are rarely of the nature of adaptations. In the second place, a most common source of confusion is the lack of a sharp distinction between plasticity and phylogenetic adaptation.

If, in botanical excursions or in determining the identity of collected plants, we have an eye open for the question concerning the meaning of the distinguishing characters for the plants themselves, we must usually concede that they are in reality far from having any real usefulness, or that at least we cannot point out their use if we limit ourselves to purely empirical arguments; for example, Ranunculus bulbosus and R. Philonotis have the slips of their calyx turned downward; Myosotis versicolor opens its flowers before the corolla assumes the blue color; Viola arvensis has a calyx which is longer than the corolla; umbelliferous plants are often dis-

<sup>1</sup> Von Wettstein, "Handbuch," p. 39.

tinguished by the occurrence or the absence of a common or a partial involucrum; Spergula Morisonii has a narrow membranaceous ring around its seeds; the species of Taraxacum and other groups are often apogamous; and so on in an endless series of arguments. What is the use of such qualities? The answer is, as a rule, none at all, since the nearest allies are as successful in their struggle for life without them. This is true in a still higher degree for the distinguishing marks of elementary species, and, as has been pointed out by Willis. for those endemisms which are not relicts but are growing still in the midst of their presumed ancestors.1 Very often erroneous conceptions concerning the use of distinct qualities are seen to prevail. Thus the red color of many flowers is presumed to attract some species of insects and to find its use in this; but as a matter of fact it is often only a local expression of a quality which may be seen in activity in other parts of the organism as well. Many white varieties of red or blue species are weaker in the struggle for life than their ancestors, and this is the reason why they so regularly disappear very soon after making a local appearance. This struggle for life is not fought out by means of the flowers, but during the vegetative period, wholly independent of the visits of insects and the question of fecundation. This is best seen in perennial plants or in small shrubs where the red or blue flowering forms are often seen to hold their ground, while the white ones are incapable of doing so. I cite, for instance, a culture of Daphne Mezereum and of its white variety, both in a number of specimens. The white ones were weak and succumbed to our climate after some years; while the red ones were continually seen to thrive. Between Ranunculus bulbosus and R. Philonotis it is not those characters that may be studied on dried material which are

1 See F. Graebner, "Lehrbuch der Pflanzengeographie," 1910, p. 70.

decisive in the struggle for life. On the contrary, the result depends on the predilection of the former for a dry soil, and of the latter for moist meadow-land. Numerous instances of the same kind might be given, and it seems to me that they would exceed by far the number of those cases in which elementary characters, as opposed to compound ones, might be proved to be useful.

Those cases in which the production of new species has been ascribed to the direct influence of the environment may commonly be explained on the principle of mutations as easily as on that of the accumulation of very small and almost invisible deviations.

Seasonal dimorphism is one of the most widely known arguments of von Wettstein. Some plants of the alpine meadows occur in two elementary forms, one of which flowers and ripens its seeds before the summer period of mowing, while the other begins its vigorous growth only as soon as this period is over. At that time, when the great significance and the general occurrence of elementary species were not yet realized, it seemed allowable to assume for this case a special process of adaptation. But, in the light of our present knowledge, the other assumption is at least as fully justified—viz., that the mowing has simply selected, from among a group of preëxisting forms, those which did not suffer by it in the one way or in the other. Unconscious selection would then have acted here just in the same way as conscious artificial selection does elsewhere. In this way the proposition of direct environmental influences may be easily and advantageously combined with the principle of the origin of species by mutation. Species have the power of adapting themselves to the prevailing conditions of life, but they do this by means of the great number of elementary forms of which each of them consists.

The other objection was that connected with the phenomena of plasticity. By this term is meant the power of many species to live under quite different conditions. Not rarely this is associated with striking changes in their dimensions and in other characters; and these differences may be so large as to be taken for specific ones on first inspection. The best known example is that of Polygonum amphibium, which has a terrestrial form and a floating one. Some authors have described the former of these as another species and have given it the name of Polygonum Hartwrightii. But Massart has shown that by transferring one of the two forms to the life conditions of the other it is always possible to change it into the other type, and that even both types may be developed as branches from the same plant, provided this is growing just at the margin of the water. In the case of alpine plants Bonnier has shown that it suffices to transplant a part of a rhizome into the plain to make the new stems assume the type that is known to be characteristic for the new conditions. In almost all cases where plants may be multiplied in a vegetative way it is possible, in accordance with this principle, to show that their plasticity (which is often called their adaptability) is a latent quality capable of coming into action at once in response to changes in environmental conditions. From perennial species we may conclude that the same must hold good for annual ones too. The capability of many ordinary weeds, which like a soil rich in saltpeter or in other nitrogenous substances, to attain gigantic dimensions under such conditions while they remain dwarfish on poor or dry soil, even in such a degree as to conclude their growth after the production of a single fruit, as, for example, is seen in Datura Stramonium, is no doubt one of the most beautiful instances of adaptability; but it is

evident that it does not involve any change in the hereditary qualities.

In all such cases it is clear that the assumed analogy between these adaptations and the origin of new species is only apparent. Qualities may lie dormant for thousands of years, as in the case of the alpine plants, and under changed conditions come suddenly into activity again; they will evidently do this every time that the corresponding stimulus excites them. Without any doubt, many of these qualities are useful, but this does not prove that they have originated on account of their usefulness. The conception that they owe their existence to some mutation, and in this respect follow the general rule, has at least the same degree of probability. In this case no supplementary hypotheses would be necessary. The researches of Costantin, Goebel, Klebs, Stahl, Vöchting, Frank, Karsten and many others have brought our knowledge concerning the phenomena of plasticity to a high degree of development; everywhere it may be seen, however, that the resemblance to the processes of the origin of species has no real signification. Nature often gives us the impression of a most beautiful harmony between living organisms and their environment, and thereby between the latter and their ontogenetic evolution, and it is all too tempting to conclude from this that organisms as a rule have been adapted to their life conditions. This conclusion, however, is in many cases only a postulate and does not rest upon an empirical ground. It goes without saying that animals and plants cannot live under extremely unfavorable or injurious conditions, and that, for this reason, we must find everywhere better or less fitted forms. But, as a matter of fact, most plants are contented in nature with an environment which is far from being the best for them; and where the trade happens to bring their seeds to other countries, they are often seen to

thrive far better and to multiply with unexpected rapidity. It is a curious fact that they are best adapted to conditions which are quite new to them and which they never enjoyed before.

Our third division was that of the characters of specialization. It lies between that of the qualities of organization of Nägeli, which have no relation to the surrounding world, and the consequences of adaptation of von Wettstein, which do not produce any really new steps in the line of evolution. Every student must be struck by the fact that the most beautiful examples of so-called adaptations are found in the distinguishing marks, not of species but of genera and the larger groups, even of whole families. In a geological sense they are therefore so old that an appreciation of the single factors of the environmental conditions under which they have originated must necessarily be impossible. As a rule, such adaptations do not consist in a single quality, but in very complicated and highly developed arrangements, which can have been attained only by a series of successive changes. I refer to the flowers of the orchids, to insectivorous plants, to many cases of climbing species, to the tubers on the roots of the leguminous plants, to the Cactacea and Euphorbiacea of the desert, and so on. All of them are specialized in a very high degree, and this we assume to be of use to them, at least in many cases. But it seems to me that this usefulness is most liable to overestimation, and in reality consists mostly in a compensation of other hurtful qualities. Later investigations have shown, more than once, that the presumed use does not exist at all—in any case, not at the present time. For instance, let us take the heterostyly of the primroses, which according to Weiss, is more hurtful than useful; or the flowers of Orchis and Ophrys, which discourage insects rather than invite them to visit, as was discovered by Detto.

The anthers of Mimulus and Torenia, which contain a large supply of good pollen but never open, must be considered simply a useless waste of material. The Drosera has no apparent advantage at all in its ability to catch insects, when we compare its distribution and the rate of its multiplication with that of the species with which it lives; on a rich soil it thrives just as well without the food supplied to it by the insects. The species of Utricularia are adapted in the most beautiful manner to capture small animals, but that this should be advantageous to them in their struggle with neighboring plants nobody can prove.

If, however, we concede that they have originated as the result of their usefulness, we do not gain any real understanding of the different factors of these complicated qualities. Neither this assumption nor experience can decide whether the units out of which these qualities have been built up have had their origin in sudden leaps, or in the accumulation of slow and originally invisible changes. In other words, they may be due to mutations as well as to fluctuations, and to the activity of orthogenesis as well as to that of direct environmental influence. From this point of view, there is not the least justification in assuming special supplementary hypotheses for their explanation. The conception that these characters of specialization have originated in quite the same manner as any other distinguishing marks of species as well as of the larger groups has evidently the same right, and perhaps even a greater right, to our appreciation than any special assumption.

Leaving these considerations of a more general nature, we may now return to the experimental side of the question. Here two propositions demand a careful treatment. The first of them is the sentence that fluctuations cannot, by means of the coöperation of selection, lead to constant races,

which should be independent of a continuance of that same selection. The second proposition is the contention that mutations do produce such constant races.

In the realm of selection the first principle is to distinguish sharply between pure elementary species and the collective species of the systematists. On the one hand, we may try to improve the small species themselves by means of selection; on the other, we are concerned simply with a choice from among the mixed groups of already constant and hereditary Any doubt which may have existed concerning the reality of this distinction has of late been completely surmounted by the practical processes of breeding which have been introduced by Nilsson into agriculture, and which were founded on his deep scientific knowledge of the problems with which he had to deal. The pure races which he succeeded in isolating from the old mixed varieties of cereals may still produce deviations in the way of mutations or as a result of accidental crosses, but these changes always occur suddenly. It is not possible to improve his strains as such by means of continuous selection. The same is true of many of the older agricultural races which have been won by a more or less unconscious process of selection.

Within the elementary species, artificial selection in many cases may be conducive to real improvements which in a sense are hereditary. In others, however, this result seems not to be attainable. But in any case such races do not become independent of continued selection. Especially instructive in this respect is the history of the cultivated sugar-beets. From a broader point of view, our beets consist of a number of elementary species, and any large breeder has, as a rule, his own kind which he has purified by means of selection. The principle of the culture of separate families is followed. It starts from single mother-plants, and

every family is the progeny of only one such specimen. The possible, and often practically unavoidable, influence of foreign pollen must afterward be eliminated by means of well directed selection during some of the next generations. Such families are called élite races, and from them every year a branch is taken for the production of the seed needed for culture in the fields or for the trade. Accordingly it must be multiplied in a high degree, but this multiplication must always be accompanied by a continued selection on the basis of external characters as well as of the amount of sugar. One, or at best two, generations without such selection are allowed; but if a breeder should multiply his seed entirely without it, the value would soon sink far beneath the limit required in practice. There are no races rich in sugar which would maintain themselves without such continued help.

Next to the sugar-beets come a number of garden plants in their varieties with double flowers. Ordinarily, such a variety has originated only once, and is in this sense a true clementary species. But the breeder chooses his seeds from the best individuals only, in order to secure a high percentage of beautifully doubled flowers among the progeny. improvement of the seeds, however, is effective only for the very next generation, and therefore it is necessary to repeat the selection every year. Numerous instances could be given, and it seems that the rule prevails that the selected characters are in a high degree dependent upon the conditions of nourishment; or, in other words, that in reality the selection is only the choice of the best nourished individuals as seedbearers. Whenever this nourishment, by means of a fuller development of the seeds, is effective through some successive generations, the races are called high-bred, and are liable to decline in a few generations after the subsidence of the selection.

Excellent material for the study of such high-bred races is afforded by some deviations of seedling plants, and especially by tricotylous and syncotylous races. I have cultivated some of these races and found them, in the main, to contain about fifty per cent. of deviating individuals. By means of selection it was easy to bring this standard, in a few years, up to ninety per cent. and more, or down to ten per cent. and less. The selector, however, must not be content with choosing the most perfectly tricotylous or syncotylous seedlings for the continuance of the race. This external mark has only a very secondary value. We have to breed from those individuals whose progeny is the richest in the desired deviation, and therefore to determine this standard for a number of seed-bearers, in order to choose from among them the one with the highest percentage figure. It is a fact well worthy of notice that such breedings succeed almost as easily by insect fecundation as by means of artificially conducted pure self-pollination (Oenothera hirtella, Antirrhinum majus), the reason being that the obnoxious effects of foreign pollen are at once eliminated by the selection itself.

Among the most interesting of these cases are the middle races, or those which continually oscillate between two ideal types, without being able to transgress their limits and to change into one of those types. Tricotylous races, for instance, oscillate between pure dicotyly and pure tricotyly; and although the single individuals may apparently show both these types, the race never reaches the one end, to the complete exclusion of the other. In such cases the range of variation is evidently an exceptionally wide one, and therefore it must be easy for selection to encroach upon it. But we should always keep in mind that the basis of selection should never be sought in the externally visible qualities of single individuals, but only in the average amount of these

qualities among their progeny. In other cases the width of variation within the limits of an elementary species is much smaller, and wherever this condition prevails it is often difficult to reach any amount of durable improvement by means of selection. Iohannsen has studied such instances, and his results may be considered as one of the best supports of the theory of mutation. For it must be clear to every one that, when selection can bring no improvement at all, it cannot even be supposed to be conducive to the production of new species and varieties. In order to be wholly sure of the purity of his cultures, Johannsen has limited his experiments to such forms as are fertile with their own pollen; but, unfortunately, this condition is far from being the general rule in nature. Moreover, he starts from a single self-pollinated individual, and in this point he follows the principle laid down in my mutation theory and introduced by Nilsson into agricultural practice. To such cultures, derived from single selected mother-plants, he gives the name of "pure lines." His method soon found universal approval, and by this means strongly contributed to the spread of the new ideas. Within these pure lines and in the examples chosen by him, selection does not provoke any real changes. High-bred races do not occur in this field, and so there is also no chance of winning new and constant races by means of them. The significance of this principle seems to me to be a very large one, and to hold good for far longer periods than those of ordinary experiments. We may deduce this from the cultures of Bonnier with the alpine plants. In this case natural selection has been at work during centuries, and in many instances probably since the last of the glaciary periods. But this has been of no avail—at least, not in such a degree that alpine plants would have become purely and exclusively adapted to their environment. For, as is well known, they

have not at all lost the property of accommodating themselves to the conditions of a low country.

Our second proposition was that mutations can be the source of new and constant races which are independent of selection. In discussing this point, we must distinguish between those cases which have been observed only after the mutation took place, and those which have been controlled for several generations in advance. The former we shall call empirical and the latter pedigree mutations. The former group includes those numerous cases of the origin of well observed novelties either in horticulture or in the field: while the other class is concerned with mutations occurring in carefully guarded cultures in an experimental garden, after at least several generations of the old type have been controlled. In the case of empirical mutations we must base our conclusions concerning the forefathers of the new type on the basis of observations made in the moment of its discovery, and often this may give a very convincing degree of probability. In the other case, the ancestors, however, are empirically known. Moreover, it is only these latter cases which afford us the necessary material for a detailed experimental study of the conditions under which the mutation took place.

The oldest and best known example of an empirical mutation is the sudden origin of Chelidonium laciniatum. A compilation of a large number of other cases has been given in my mutation theory and a critical and historical review of the instances recorded in horticultural literature has been published by Korshinsky. To these instances Solms-Laubach has added his Capsella Heegeri, and Blaringhem his Capsella Viguieri and others; and the list of cases is increasing almost yearly. We may therefore state that the fact itself is now beyond all doubt. Instances of pedigree mutations

have been described in my mutation theory, especially in the case of *Linaria vulgaris peloria* and of the double *Chrysanthemum segetum*, or corn-marigold. Referring for the description of these experiments to the source I mentioned, I will point out here the critical and methodological side of the problems involved.

In the case of the peloric toad-flax, the aim of the experiment was to control experimentally the instances of this mutation which had so often been observed in nature; in other words, to watch the occurrence of such a change in a well guarded pedigree culture. The observations made by different authors clearly pointed to a sudden origin without transitions or visible preparing steps. No intermediates had ever been found. My aim, however, was to see the mutation taking place. Evidently, peloric flowers owe their particularity, in a morphological sense, to the loss or to the latency of the symmetrical structure of the flower; but by far the greatest number of instances of empirical mutations refer to such losses, and as yet there is no ground for supposing that progressive changes should behave differently in this respect. In my experiment the first completely peloric individual i.e., the plant which had all of its flowers, without exception, in this condition—was seen in the fourth generation. It was soon followed by others, and in a sufficient number to show that the mutation occurred in about one per cent. of all the individuals, and was repeated in succeeding generations. Unfortunately, these peloric plants were almost wholly sterile; and I contrived only with difficulty to raise about a hundred individuals from their seed. These, however, repeated the anomaly, although with a few exceptions, due probably to the extraordinarily difficult conditions of the artificial self-pollination of these almost sterile flowers. Intermediate forms did not occur, neither in the number of

peloric flowers per plant nor in that of the spurs per flower. It is hardly necessary to point out that a single experimental mutation of this kind provides a much deeper insight into the phenomena than all the empirical mutations of this species taken together.

In some respects the case of the double corn-marigold is different from this. The starting-point of this experiment was derived from two empirical theses. One of these is the proposition already mentioned, that within pure races selection is the choice of only the best nourished individuals. The other is the increment of chance of the occurrence of mutations caused by a high degree of nutrition. From a combination of these two empirical rules we may derive the belief that in experiments on mutation a most carefully conducted and luxurious culture is to be combined with sharp-eyed selection. Moreover, we may apply a third rule which deals with the sensitive periods in the development of variable It says that selection chooses especially those individuals which have been best nourished during this sensitive period of the character in question, and from this we may deduce that the chance of new mutations lies mainly in the direction of those characters which we choose for our selection, or of such as are most intimately connected with them.

The point of this discussion should always be kept in mind in the planning of new experiments on mutation, as we shall easily see by applying it to the case before us. It involves the principle that the chance of winning a double variety may be enhanced by selection in the direction of increasing the number of the ray florets in the flower-heads. This augmentation refers to the outer range of florets, while the doubling consists in the change of the florets of the disk into rays. The two phenomena are therefore essentially different, and

in my cultures they were observed to be not connected by transitional or intermediate forms.

By means of very careful selection I succeeded in bringing the average number of ray florets in the flower-heads from the ordinary type of twenty-one (the number of the variety I started from) up to thirty-four; and in doing so the extremes were seen to reach even sixty-six rays per head. After this was reached a change in the disk suddenly made its appearance, and this in one of the seed-bearers chosen for its extremely high number of rays. Next year the seeds of this one plant were sown separately, and at once they gave the expected double race in full display. It seems justifiable to assume that the numerous double varieties of species of the family of the composites have originated in the same way, in the field or in the garden, and in the latter case probably under the influence of unconscious selection.

After the same method I succeeded in producing a twisted race of Dracocephalum moldavicum by means of the selection of tricotylous individuals. To this experiment I was led by the description given by Morren of a most beautiful instance of spiral twisting in another species of the same genus. But from these instances we may not conclude that such pedigree experiments will always give the desired result. Thus, for instance, I have tried in vain to win a double petalomanous variety of Ranunculus bulbosus, although such a variety from time to time occurs in the field in Holland. Also, I did not succeed in producing a purely four- or five-leaved race of the red clover; nor even a constant sevenleaved form. But it is still possible that the difficulties in finding out the most favorable methods of growing these wild plants are at least partly to be considered as the causes of this lack of success.

Advance in the study of the question of mutation seems

now to depend mainly on the accumulation of 'numerous pedigree cultures of this kind. It should be possible to repeat the mutations which are known to have occurred in nature or in horticulture, in an experimental way, with either the same or with allied species. The conclusions which are now derived from empirical mutations should be based upon observations in the experimental garden. In working on this principle, not only more exact proofs may be reached. but we should come into possession of the material needed for a more thorough study of mutations and of their internal and external conditions. Here is the starting-point for the long path which must still be explored in trying to produce intentionally chosen novelties; but it does not seem at all impossible to surmount the difficulties, even in this field, and thereby to open new sources of artificial improvement for our crops.

In nature, probably, the production of new forms has taken place sometimes sporadically and sometimes in groups. It is chiefly a paleontological question which of these two processes has had the prevailing part in the evolution of the vegetable and animal kingdoms. Have the main branches of the pedigree been started from among those rich groups of species and varieties which constitute the so-called polymorphic types, or are these latter types only products of the lesser branches? As far as our present knowledge goes, both cases seem to occur. At the present time the polymorphous genera and species, the misty spots of the older systematists, and the groups of explosive changes of Standfuss, are evidently the consequences of such mutation periods. But most of them are already past that stage, and no traces of mutability have been preserved in them. Or, perhaps, this changeability is limited to a few of the numerous forms, and it has as yet not been possible to discern these among

them. On the other hand, it is clear that cases of the simultaneous production of groups of new forms provide a far more suitable material for experimental researches than do sporadic mutations. The former type may include the most divergent kinds of specific changes. It is on the ground of this conception that in the beginning of my experiments I sought a species which would be in such a condition of mutability. I tried more than a hundred species, mostly of wild plants of Holland; cultivated them for several years in my garden, and finally selected one from among them which seemed best suited for my purpose.

This one was the evening primrose of Lamarck, introduced long ago from America into Europe, and which has run wild in different spots. It was the Oenothera Lamarckiana. The processes of the mutation of this evening primrose have been observed of late by so many investigators that no traces of doubt concerning their reality any longer remain. It is true that the whole case is still an isolated one, but it is evident that a further search will sooner or later lead to the discovery of analogous instances. On the other hand, the question of the significance of these observations as typical for the theory of evolution, as well as that concerning the true nature of the mutations themselves, is a subject of much discussion. It is a struggle for and against the Oenotheras and their evolutionary value; but this struggle is concerned with the mutations themselves as they occur in our experiments, and not-or at least not directly—with that primary condition of them which I have called the premutations. This internal tendency to mutation is proved by the fact that the same new forms may arise yearly from the main stem of the cultures, and often in a relatively large number of individuals. Generation after generation, the same mutations are repeated, and this re-

veals to us an hereditary condition of the germ-plasm which must have remained unchanged during all the twenty-five years of my experiments and evidently during a much longer period of time. In what way, and when, these internal predispositions have been acquired, the visible consequences of which are the mutations, is a wholly different question, which has until now hardly offered itself to experimental treatment.

This question of the premutation, or of the internal preparation of the mutability, is most intimately connected with that of the duration of the whole period of mutations. In this respect the first proofs have referred to the introduction of the Oenothera Lamarckiana into Europe, and have shown that it must have been already in a mutable condition at that time, or about sixty years ago. This conclusion was derived from the fact that the different strains, issued from that introduction shortly after it had taken place, all showed the same phenomena of producing new forms.

Later observations, and a better appreciation of some older ones on the ground of them, then led to the view that the mutability must in this case be older than the species itself, and have developed gradually together with the specific differentiation within the group of the Onagras to which O. Lamarckiana belongs.

The main support of this view is the discovery of the fact that the European type of O. biennis has the same property of producing dwarfs which is so prominent in O. Lamarckiana. This has of late been observed by Stomps in his cultures of O. biennis cruciata, and it has occurred also in my experimental garden. The common view takes this O. biennis to be one of the forefathers of O. Lamarckiana, and therefore present indications assume that the property of producing dwarfs has been inherited by O. Lamarckiana

from such ancestors. This view is supported by that curious quality of the dwarfs of being sensitive in a high degree to the attacks of some kinds of bacteria of the soil; this sensitiveness and the changes in the structure which it produces being exactly the same in both of these kinds of dwarfs. I shall have to refer to this disease later on.

Elsewhere, also, among the nearest allies of the evening primrose of Lamarck, phenomena of mutability may be seen to occur. Oenothera cruciata has given in my cultures from their very beginning three types, which differed from each other especially in the form of the flower-buds; it has shown the same elementary forms in the cultures of MacDougal. This author studied also the Oenothera grandistora from Alabama, and the origin of new derivative forms from it, and stated that analogous deviating types are also met with in its original station near Tensaw. Moreover, the cultures of Davis have given evidence of a wide range of subordinate forms within the type of Oenothera grandistora. In the neighborhood of Courtney, Missouri, I observed, among numerous specimens of the ordinary American type of O. biennis, a deviating individual with narrow leaves. Analogous mutations have arisen from the seed collected in that station from normal plants and sown in my garden. One among them proved especially interesting in being of lower stature and of a more slender structure than its very stout ancestor.

According to some stray observations, mutability is not at all limited to these examples, but occurs in different allied species also. From all of these facts we may safely conclude that mutability is a wide-spread phenomenon in the group of the Onagras, and that it has not originated with the origin of O. Lamarckiana. This weighty conclusion has of late

found an unexpected support in the discovery of Stomps that the European O. biennis is liable to the production not only of dwarfs, but also to that of the main progressive type among my mutants, the gigas. As a matter of fact, it has not as yet been observed to throw off gigas-plants as such. But it has given a half-gigas—Oenothera biennis semigigas—characterized by intermediate marks between real gigas and ordinary O. biennis, and especially by having in its nuclei in the one half the fourteen chromosomes of the former, and in the other, the seven of the latter. In other words, it has twenty-one chromosomes, being in this respect wholly analogous to the triploid mutants of O. Lamarckiana so fully and ably described of late by Miss Anne M. Lutz.<sup>1</sup>

Returning to our general discussion, it is clear that the other species are only mutating in a lesser degree than O. Lamarckiana, and from this fact we conclude that the extent of this property must have increased gradually during the phyletic evolution of the group. Or, in other words, the present mutability of Oenothera Lamarckiana is built up by a number of factors, more than one of which have evidently originated already with its ancestors. It goes without saying that the single steps of this process must themselves be regarded, on the basis of our theory, as constituting each of them a special mutation.

By means of the facts which I have just described, many objections made by different authors may easily be surmounted. The question whether O. Lamarckiana has still one or more wild stations is no longer of interest, since most of the other mutating species are recognized to be good wild types. This is especially the case with O. grandiflora. From

<sup>&</sup>lt;sup>1</sup> Since the reading of this address numerous cases of mutability in allied species of *Oenothera* have been discovered by II. II. Bartlett. (Note of 1915.)

my discovery that O. Lamarckiana produces twin hybrids, O. lata and O. velutina, when it is crossed with certain of the older species, some authors have drawn the conclusion of a hybrid nature. But Davis has shown that O. grandiflora produces the same twins in analogous crosses; and in order to explain this fact by a hybrid condition the adversaries would either have to assume such a condition separately for the two species, or suppose a hybrid origin for their common ancestors. Both suppositions seem to lie far beyond the realm of credible scientific hypotheses.

Other grounds for assuming a hybrid nature for O. Lamarckiana must disappear before the same group of facts. As a matter of fact, it is generally conceded that in polymorphous groups of species some forms may have been the result of crosses between others. This opinion was held by Linnæus, and for the cereals it is evidently true, as is proved by the researches of Nilsson and others. Any one who has studied the species of Oenothera in botanic gardens must have been struck by the fact that they are very rich in constant hybrids. But all such observations are far from containing even a single trace of proof in favor of the assertion that mutations should be a consequence of previous crosses.

Some authors deal with the struggle against the Oenotheras in a rather inconsiderate way, especially among those who enjoy any argument pleading for "the possibility of the Mutation Theory being based on false premises." As an example, I may give the observation of Boulenger. He found a station for Oenothera Lamarckiana in Bretagne, not far from La Garde St. Cast (Côtes du Nord). Here it had started from the neighborhood of the hotel and spread

<sup>1</sup> G. A. Boulenger, in "Journal of Botany," October, 1907.

through the surrounding dunes during several years. He found that it showed a high degree of variability, especially in the direction of the characters of the European O. biennis, a long line of transitions and intermediate steps being clearly made out. He tried to recognize among them the types of my mutants, but they proved to be wholly of another nature. Neither did he succeed in determining a limit which would separate two groups, the one belonging to O. Lamarckiana and the other to O. biennis. From these facts he concluded that O. Lamarckiana may locally revert to some ancestral form which would have been very similar to, if not identical with, the O. biennis. Every botanist would of course have come to a different conclusion and assumed that O. biennis had already been present on that spot, being a common inhabitant of the dunes, and that it had readily crossed with the introduced Lamarckiana so as to produce quite a number of intermediates of hybrid origin. And even the pointing out of this possibility would have destroyed the whole basis on which Boulenger thought it safe to attack the new theory.

Moreover, it is rather easy to prove that the transitions of Boulenger must really have been such hybrids. In crossing the species in question, we come upon three clearly distinct types, two of which have been already dealt with. These are Oenothera hybrida læta and velutina. They result from the cross O. biennis × Lamarckiana and constitute its twin hybrids. The former has broader and flatter, the other narrow and rolled leaves; but aside from this mark, they differ in almost all their organs and qualities. The third hybrid results from the reciprocal cross, O. Lamarckiana × biennis; its characters are very similar to those of O. biennis, from which in some specimens it is often hardly discernible. To determine the limits of these five

types in a mixed group of plants may seem a difficult task even to an experienced eye; in any case, Boulenger failed to recognize them. O. biennis is one of the more common species in many parts of the dunes of Holland, where it is known to have grown already at the time of Linnæus. Not rarely O. Lamarckiana is sown on the same spots, being a favorite food for birds; in such cases the hybrids will arise by the natural processes of fecundation by insects. This of course also happens from time to time in the dunes of Holland, and I have observed it in an unusually broad area for the Oenotheras in the neighborhood of Zandvoort, where I studied it with special care in 1905 and 1910. In the spring of 1906 I sowed seeds of this motley group in my experiment garden; and in 1911 I introduced a set of rosettes and got them to flower. The hybrid types were easily recognized, although, on account of their transgressive variability, they seemed to constitute continuous lines of variation in many characters. In the dunes these differences are less evident than in the experiment garden, on account of the very different life conditions. In groups, however, it is easy to ascertain the types, but from such a station I would never use the seeds for any experiments in mutability. Every single individual must always be regarded with some doubt as to the purity of its origin.

In England, also, the two species often grow together. Charles Bailey has described such a station from the neighborhood of St. Anne's on the Sea, near Liverpool. Therefore I asked one of my friends to visit this station for me, and he informed me that in some of the valleys O. Lamarckiana was seen to be pure, while in others it was mixed with O. biennis and the hybrids.

In order to give a general review of the single mutants

which have arisen from the Oenothera Lamarckiana, we may bring them into certain groups:

- Λ. Progressive mutations: O. gigas.
- B. Retrogressive and degressive mutations:
  - 1. Mendel cases: O. brevistylis.
  - 2. Half Mendel cases: O. nanella, O. rubrinervis.
  - 3. Not Mendelizing mutants: O. lata, O. scintillans, O. oblonga, O. lævifolia.

Besides these, there is a long list of instances which have not as yet been studied by means of crosses, as, for example, O. albida, O. elliptica, O. leptocarpa, O. semilata, O. spathulata, O. sublinearis, O. subovata, and many others to which no names have been given on account of their sterility or of their excessive feebleness. Mutations have also been won by other investigators; among them the O. rubricalyx of Gates, the O. ammophila of Abromeit, and the O. blanda of Schouten must here be mentioned. During the last ten years I have not tried to increase the number of the mutants; but notwithstanding this, I have secured some interesting novelties. The fact that in this whole group only one species is of a progressive nature, while the large majority are either degressive or retrogressive, has had stress laid upon it by some authors as a strong objection, but it is just what we should expect on the ground of our knowledge of other polymorphous groups.

As is well known, a certain group of authors assert that all hybrids and all characters must necessarily follow the rules of Mendel. A criticism of this evidently one-sided conception would take me too far from my real subject. At the present moment I will therefore limit myself to the contention that conclusions drawn from immutable plants are

not a priori applicable to those which are in a condition of mutability. On the contrary, these latter behave in many respects differently, and it is only with them that I shall have to deal here.

Let us first look at the progressive mutations. According to our theoretical conceptions, they owe their origin to the appearance of a new kind of hereditary unit, or pangens, which must have been split off by some one of the previously existing units. This latter can be in a condition of premutability, and thereby able to repeat the same mutation from time to time. Whether this premutation is caused by its own condition, or is due to the influence of neighboring pangens, is a question which is not now in need of an answer. It is only a few progressive mutations that are of a phyletic nature—i.e., made for contributing to the building up of the pedigree of the whole system; by far the greatest number must, of course, be limited to ordinary specific differences.

In the foreground of our discussion of Oenothera gigas we may put the fact that it possesses, in its nuclei, a double number of chromosomes in comparison with the species from which it arose and with almost all of its other derivatives. O. gigas has twenty-eight instead of fourteen in the vegetative cells, or fourteen instead of seven in the generative elements. This important fact was discovered in 1907 by Miss Anne M. Lutz and corroborated shortly afterward by Gates, and later on by my pupils Geerts and Stomps. It has brought the new species to the foreground of cytological interest. Similar duplications of the set of chromosomes constitute important specific marks in other groups of plants; and in no single case are there arguments in favor of regarding it as a retrogressive change.

For the origin of a progressive mutant, in this case of a plant with a double number of chromosomes, it is obviously

necessary that two mutated sexual cells should combine, as was first pointed out by Stomps. This condition is not the same for retrogressive and digressive mutations, as we shall see later on. It is true that Gates has expressed a different opinion and asserted that the duplication takes place only after fecundation, not being a real mutation, but more in the nature of an accident.1 This, however, would bring the whole phenomenon into the class of acquired characters which are now generally considered as not hereditary. From this point of view, the conception is in evident contradiction to the facts, since the gigas has continued its existence already during several generations. In this connection I may point to the double-nucleated cells of Spirogyra in the experiments of Gerassimow, which retain this special mark during all the vegetative divisions, but lose it as soon as fecundation comes into play. Moreover, the facts since discovered fully disprove the view of Gates.

Oenothera gigas has been seen with sufficient evidence to arise only once in my cultures. This was in 1895, from pure seeds of 1891. It is only of this race that the chromosomes have been counted. In the beginning I believed that I saw it in other years also; but at that time I did not know the characters of the hybrid between it and Lamarckiana. Looking back to those cases, it now seems to me that they were only half mutants, produced by the conjugation of a mutated sexual cell with a normal one. In this case they should have had twenty-one chromosomes in their nuclei, but they have not been studied in this respect and did not bear any seed. Such supposed half mutants have since been seen to arise more than once, because it was now known that there are reasons for expecting them and looking for them. For one of these the chromosomes have been counted by Stomps,

<sup>1</sup> R. R. Gates, "Archiv für Zellforschung," 3 Bd., 4 Heft, 1909, p. 549.

who found the expected number of twenty-one. This observation proves, first, that the duplication takes place before fecundation, and secondly, that the mutation is not so rare in the germ-cells themselves that we should be justified in considering it as an accident. By means of a careful and extensive study, Miss Lutz has discovered the same fact. In her cultures she observed ten half-gigas mutants arising from O. Lamarckiana, and in counting the number of chromosomes for all of them, she found it, without exception, to be twenty-one.

It may here be mentioned that Heribert Nilsson discovered in 1907, in Sweden, a mutation of O. Lamarckiana in gigas.<sup>2</sup> It gave an hereditary race, but nothing has been published in regard to the nuclei. Another important fact is the discovery of Geerts, who met once, in his cytological studies of O. Lamarckiana, with a mother-cell of an embryo sac which showed in its division twenty-eight instead of four-teen chromosomes. Controlling these observations, I have accurately compared my half mutants with the artificial hybrids between O. gigas and O. Lamarckiana, and convinced myself of their external identity in all respects.

On the basis of these experiences it is possible to calculate the mutation coefficient for O. gigas. Most suitable for this purpose are crosses of O. Lamarckiana with such species as produce only, or almost only, yellow, very weak and soon dying hybrid germs. This is the case when O. Lamarckiana is pollinated with the pollen of O. cruciata, O. muricata or O. Millersi (nov. sp.). We have only to count the germinating seeds and to cultivate the few green ones among them. As in Lamarckiana, all of its derivatives give such yellow seedlings, the only exception being that of O. gigas.

<sup>&</sup>lt;sup>1</sup> Miss Anne M. Lutz, "Triploid mutants in Oenothera," Biok Centralbl., Bd. 32, July, 1912, p. 384.

<sup>2</sup> "Bot. Not.," 1909, pp. 97-99.

Those sexual cells which have been mutated into this form will therefore produce green seedlings, which it will then be easy to isolate from the rest. In growing up they may soon be recognized by their much stouter stature, and for this reason Stomps has proposed to call them Hero. In counting their chromosomes, he found them to be twenty-one in each nucleus, this number being the sum of seven chromosomes derived from the father (O. cruciata, etc.) and of fourteen derived from the mutated egg. This, of course, is a sufficient proof; but the Hero plants may afterward be easily recognized as such by their stout flower-buds and other characteristics.

Among fifteen thousand yellow seedlings, forty-five examples of *Hero* were counted, giving a percentage of 0.3. If now we assume that the mutations are as numerous in the male sexual cells, the chance of their meeting together and thereby producing a full *gigas* will obviously be equal to the quadrate of this number, or 0.0009,—say about 0.001%. In my mutation theory I had provisionally conjectured this number to be 0.01%.

The size of the cells and of some of the organs of O. gigas has increased in consequence of this doubling of the chromosome number and in accordance with the laws discovered by Boveri and Marchal. This fact was first pointed out by Gates. This author extended his conclusions to all the differences between O. gigas and O. Lamarckiana; but this has been shown by Stomps to be unjustifiable. Neither the biennial habit, nor the large seeds in the small capsules, nor the adhesion of the axillary buds to the stem above the leaf can be explained in this way. The same is the case with other marks. Here I might, however, lay stress on two points which can hardly be considered as consequences of a double set of chromosomes, but which have of old been con-

sidered as true specific characteristics as opposed to mere varietal marks. I am thinking of the strongly diminished fertility of almost all the crosses and hybrids of O. gigas; and, in the second place, of the fact that the hybrids are intermediates between their parents and constant as such in their progeny, whenever they have any.

O. Lamarckiana, as a rule, gives a normal harvest of seeds, after being crossed with allied species, amounting to about 0.3 cc. per capsule. O. qiqas, however, does not produce after the same crosses more than 0.01 to 0.02 cc. of seeds per capsule; and if sometimes the harvest is found to be larger, the seeds are, as a rule, not capable of germinating, although apparently of good structure. Often it is very difficult to win hybrid seeds at all; as, for instance, in the crosses with the European and the American species of O. biennis with O. strigosa, with O. Hookeri and even with O. Lamarckiana and the larger number of its derivatives. Moreover, the hybrids, if once produced, prove afterward to be almost, or wholly, sterile after self-fecundation, and the second generation often embraces only a very few individuals. Reciprocal hybrids are identical, provided the nature of the other parent permits it, and the externally visible qualities are apparently just the mean between the two parents.

On the ground of all these facts I take it for granted that O. gigas is a good species, arisen in a progressive way from its parent, although distinguished from this by only a single unit character. In all these respects it behaves differently from all the other mutants.

We now come to a discussion of O. brevistylis. It is distinguished from its parent form mainly by the partial loss of the epigynous condition of the flowers. Besides this, it is the only one among all the derivatives of O. Lamarckiana

that exactly follows the law of Mendel; and this in its crosses with the parental species as well as with its derivatives and with the older species. In some crosses it may be seen to split into the twin hybrids *læta* and *velutina* in the same manner as its ancestor, but then both of the twins will split in respect to the length of the style, according to Mendel's formulæ.

Of course the same splitting must occur in the field where it grows together with O. Lamarckiana. As a matter of fact, it is not possible to distinguish the hybrids from that species on first inspection; but in bringing numbers of rosettes of root-leaves to the garden from time to time a single plant may be met with, the progeny of which contains the short-styled individuals in the number required by Mendel's rule. Such a case I happened to find in my cultures in 1905. From this we may infer that the short-styled specimens (which almost every year are seen to grow in the field) may be offspring of such hybrids, and thus their existence is far from proving the presence of another source, such as a direct mutation from O. Lamarckiana. Moreover, it seems that this mutability is wholly exhausted, since the mutation has never repeated itself in my cultures.

If we try to penetrate into the mechanism of the original mutation to which my race owes its existence, we find that obviously the change of a single sexual cell must be considered as sufficient. Its fecundation by a normal cell will give rise to a hybrid, from the seeds of which the pure type of O. brevistylis will come into existence. The hybrid could not be recognized in the field, but the short-styled individuals at once strike the eye by wholly different qualities. These themselves produce no seed at all, or hardly any; but in fecundating the surrounding Lamarckianas they will give rise to hybrids, from which the pure type may once more be

produced. There can be no doubt that it is in this way that the O. brevistylis has kept its place in the field during the almost twenty years of my observations.

We may now turn our attention to those mutants which follow the laws of Mendel only half-way. They do not comply with these rules in their crosses with the parental form, nor with the majority of its derivatives. But in those crosses with other species which split them into twins the rule is that one of the twins follows these formulæ while the other does not. To this group we may bring O. nanella and O. rubrinervis.

Before detailing the results of the crosses of these two new species, I must call your attention to one of the most curious objections that have been made in the struggle of some authors against the Oenotheras. I mean the contention that the dwarfs should not be a pure hereditary race, but only diseased individuals of the ordinary Lamarckiana. Of course nobody who ever saw the two cultures side by side can hold such an opinion, since transitions are always absent. The dwarfs do not attain half the height of the parental form, and are almost all of the same stature. This is purely reproduced from seed, without exceptions or deviations. The contention I mentioned starts from a discovery made by Zevlstra. He observed a curious type of bacterium within the cells of the dwarfs, and showed that the presence of this parasite is the cause of some of their characters, formerly held for specific marks: thus, for instance, the broadened bases of the leaves, the brittleness of their stalks, the frequent curvature of the flower-buds, the failure of the style in some flowers, and others. But in opposition to these minor points, the stature of the dwarfs is neither caused nor sensibly affected by the parasite. This may be proved in an easy way by cultivating the dwarfs on a soil rich in phosphate

of lime and relatively poor in nitrogenous manure. Under such conditions the phenomena of the disease are seen to disappear completely, or almost so.¹ The leaves become narrow and stalked, the internodes longer, the brittleness is lost, the flower-buds are straight, and the flowers open in a normal way. Often one or the other leaf still shows signs of the disease, and so betrays the presence of bacteria in the cells. But the main point is that the stature remains the same; the dwarfs are still dwarfs, even when they are in the best of health. They constitute a distinct mutation, which, however, is distinguished from the parental type in two points—viz., the stature and the sensitiveness to certain kinds of bacteria of the soil. As already stated, the same holds good for the dwarfs of the Oenothera biennis.

From the crosses of O. nanella and O. rubrinervis with some of the older species the same twins arise as from the analogous crosses of O. Lamarckiana itself. They are the lata and velutina, of which I have already spoken more than once. In such cases dwarfs are lacking in the first generation; and from this we should expect a splitting in the second, according to Mendel's law. As a matter of fact, this splitting does occur, but only among the progeny of one of the twins. The other gives a constant race without dwarfs. And since the twins are usually produced in about equal numbers, it is one half of the progeny which complies with Mendel's law. Hence the name of "half-Mendel hybrids." As a rule, it is the velutina which produces the dwarfs, while the lata remains constant.<sup>2</sup>

It is evident that such splittings cannot occur in the field

<sup>&</sup>lt;sup>1</sup> "Science," N. S., Vol. XXXV, No. 906, pp. 753-754, May, 1912.

<sup>2</sup> For more details see my book, "Gruppenweise Artbildung," which is soon to be published. A modification of the process of splitting may be introduced into these experiments by the use of heterogamous species, as, for instance, O. muricata. See also "Ber. d. d. bot. Ges.," Bd. XXVI a. 1908, p. 667.

on spots where the Lamarckiana is free from the admixture of other species. In such cases we are concerned only with the crosses of the derivatives among themselves and with the parent type. From these crosses only the parental types are repeated, and, as a rule, to the exclusion of others. Fecundating themselves, they will prove constant. these experimentally ascertained facts we may conclude as to what must happen in the field. A mutation may keep its hold there in three different ways: first, by means of selffecundation; secondly, by means of intercrossing with the parental species; and thirdly, by being produced anew, from time to time, from the main stock. To which of the three processes a given individual owes its origin can of course not be seen in the field; and so there is almost never a direct proof of mutations occurring there, except in those cases where the mutants succumb in the struggle for life before opening their flowers. And this is not at all rare under the adverse conditions of the field at Hilversum.

The results of our crosses show that in many cases the coöperation of two mutated sexual cells is not a necessary condition for a mutation to be produced. It is often quite sufficient that the mutated cell be fecundated by an ordinary one. If this does not occur too rarely—as a rule, in one half of the instances—the mutation will be lost; while in the other half it will dominate and develop its qualities in the new individual. For this is the rule governing artificial crosses. In those cases where it is lost, the new individuals will be identical externally with the ordinary Lamarckiana; but it might be possible that such individuals should prove to possess a greater liability for mutating than do others. This point, however, has not as yet been investigated. It might be suggested that it is in just this way that mutability is maintained in the field; but the results of some artificial

crosses do not plead in favor of this opinion, since the *Lamarckiana* individuals produced from such crosses do not show any increase in their mutability.

The facts which we have now described could be used as a starting-point for answering the question concerning the nature of the process of premutation, or of the initial change which induces the condition of mutability. In doing so we should have to assume that originally some mutation had occurred in a sexual cell and that from the copulation of this with a normal plant no mutant, but a seemingly ordinary Lamarckiana, had arisen. Then we might assume that this copulation had induced a mutable condition, which must be supposed to have become hereditary and to have given rise to an hereditary race. If such a change had taken place in the lapse of time, first for the mutability into O. nanella, it could have been followed by a similar change for O. rubrinervis, then for O. lata and O. scintillans, and so on for the whole range of known and as yet unknown mutants.

But such speculations hardly throw any light on the real nature of the processes of premutation, nor on that of the premutated condition, nor on the power of mutating derived from it. I have only mentioned them in order to show that the hypothesis of Bateson concerning this process is as superfluous as it is erroneous. This author contended (1902) that mutability might be a result of crosses with other preexisting species, which would have been in the possession of the qualities afterward displayed by the mutants. In opposition to this supposition, many authors, and among them MacDougal, have pointed out that the species required for the justification of this view do not, as a matter of fact, occur. And if we review the qualities of the different new types produced by O. Lamarckiana as mutants, the number of which amounts to more than twenty, we shall soon be

convinced that the large majority of them are too weak in some respect or another to be able to exist in nature. They would have been crowded out almost as soon as they had arisen. The only way of escaping this difficulty would be to assume that those hypothetical species had possessed the desired qualities only in a latent condition. But this supposition would, in another respect, be contrary to the views of Bateson. Under these circumstances, I think it must be conceded to be a more simple supposition to leave out the conception of a long row of hypothetical ancestors, and only to assume a succession of those premutations the consequences of which may yearly be observed in the mutations they produce.

But still one could be inclined to consider the premutation as a consequence of the cross of a mutated sexual cell with an unchanged one. In order to produce the desired result, such crosses would have to occur more than once, since only half of them may be expected to produce mutable Lamarckiana plants; and the reason for such repetitions would then remain an obscure point in the discussion. But, as already stated, all these considerations do not bring us nearer to an understanding of the phenomena. Therefore I will limit myself to the citing of the extensive criticism of Blaringhem (l. c., pp. 173-186), and to pointing out the most important fact described by Geerts-namely, that the rudimentary condition of the pollen grain, which plays so large a part in those hypotheses which ascribe a hybrid nature to O. Lamarckiana, is not at all characteristic of this species and its nearest allies, but is seen to occur throughout almost the whole family of the Onagraceæ. It is evident from this that it cannot be considered as proof of a hybrid nature of any species of that family.

Moreover, I might once more lay stress on the assertion

that it is not permissible to apply conclusions drawn from immutable plants in an explication of the conditions of mutable ones. Such a process would be justifiable only in case it were experimentally shown to be possible to change the ordinary immutable types into the rare and so much desired mutable forms, only by means of artificial crosses. But as yet all experience is contrary to such a conclusion.

The last group we have to consider embraces those mutants which in no respect comply with the laws of Mendel. It may be sufficient to deal with them only very briefly here. Their first generation, after being crossed with the parental species, is as a rule a twofold one which only repeats both of the parental forms. In the case of O. lævifolia and O. oblonga these types are at once constant, while in that of O. lata and O. scintillans, which are inconstant types themselves, the form which externally corresponds to them does so in respect to its constancy also. Only the Lamarckiana individuals sprung from these crosses remain constant when self-fertilized.

It is clear that the discussion given above for the appearance of individuals deviating in the field, as well as that for the process of premutation, is directly applicable to this case too. It would be useless to repeat them. But the results of my crosses indicate a long range of possibilities, which it is as yet hardly possible to combine into a simple and clear scheme. They have only one feature in common, and this is the total absence of splittings conforming to Mendelian laws.

Of course it is not possible to review here all the objections made against the significance of the Oenotheras for the mutation principle. The theory does not stand or fall with the validity of a single example. It has been derived from general considerations, and is supported by a critical review of numerous facts taken from the most diverse fields of

natural science. It has found rapid recognition in almost all circles of biological inquiry, and has caused the principle of pangenesis, laid down by Darwin, to become the startingpoint for the theory of heredity. It is true that, as I have already pointed out in the introduction to my mutation theory (Vol. I, p. v), work on the basis of this principle is far more easy in the domain of hybridology than in that of pure heredity. The development of the experimental studies within the last ten or twelve years has fully justified this assertion. Hybridology, or at least that part of this science which deals with Mendelism, has developed to a bright and flourishing science, while only a few investigators have devoted their work to the study of pure descent. In the next few years the main interest will probably turn to the production of new species within pure and well-guarded strains,2 partly in order to get extensive proofs of the fact itself, and partly to find their explanation. Along these lines scientific research is gradually approaching its highest scope: the artificial production of new forms of life—forms planned beforehand.

<sup>2</sup> See L. Blaringhem, "Transformations brusques," l. c.

<sup>&</sup>lt;sup>1</sup> See C. Stuart Gager, "Intracellular Pangenesis," English edition (Chicago, The Open Court), 1911.

## Second Lecture GEOGRAPHICAL BOTANY

THE distribution of plants on the earth's surface is generally considered as the present condition and result of the geological evolution of the plant-kingdom. It may be considered as the image of a gigantic pedigree-tree, seen from above. The main branches of this tree determine the large geographical districts, which thus are shown to depend in a far higher degree on common descent than on climatic and other environmental conditions. The smaller branches intermingle, partly on the limits of the districts, partly on account of the wider distribution of special groups.

The causes of the geographical distribution are thus seen to be twofold. One part of this whole science describes the delimitation of the regions inhabited by organisms of various degrees of affinity. The other is concerned with the external causes which must have governed these groupings, brings them into relation with the geological changes of the surface of the earth, and inquires how these changes may have influenced the plants themselves.

This latter problem must be dealt with chiefly on the ground of the observation of actual migration, and it is this point which I wish to consider in the present lecture. Alphonse de Candolle has distinguished between migration on a broad and migration on a small scale. To this end he has divided the whole surface of the earth into a large number of districts, each of which presents a flora of a distinct type, different from that of neighboring districts and more or less clearly uniform throughout its own domain. In doing so, he found that about ninety per cent. of all described species

are limited to one single district, thus having undergone only a small degree of migration. In many families only one fifth per cent. of the species, in others ten fifteenths per cent., have been found in two districts, inhabiting mainly their adjoining parts.

From these facts we see that migration on a larger scale is to be regarded as relatively a very rare phenomenon. This impresses us even more when we consider the large number of instances where wide migration follows the paths of man and is evidently the effect of his mingling with the normal slow processes of nature. On the other hand, it is just such cases of wide distribution which have been most thoroughly observed and recorded, and in which the single facts are clear enough for a definite judgment. It is, therefore, from these that we have to start in our discussion.

In doing so, there is one main point which strikes us, in the first place. In almost all text-books and broad reviews it is theoretically assumed that migration produces specific changes, that plants change in traveling, and that this is one of the main sources of specific differences. In the same textbooks and reviews the facts dealt with plead, as a body, in favor of a contrary conclusion, every single case of migration which is described in some detail relating to a species which did not change in the process. The stability of the species is even seen to be the main argument in proving the migration. Even Warming's "Œcology of Plants," which deals with the actual migration of unchanging species in a more thorough manner than perhaps any other work on this subject, and which contains numerous most valuable proofs for the stability of species, turns at the end to the theoretical conception I have just quoted, thus suddenly and unexpectedly leaving its own solid basis of well observed facts.

Avoiding this unqualified conclusion, we must simply con-

cede that the whole domain of plant-migration affords no proofs of specific changes having been produced by this process. Of course, species have originated from others, and often may have done so during migrations, partly during those on a smaller scale, and partly, but probably more rarely, during the world-wide distribution of some mundane forms. But from this it does not follow that they were produced by the migration, that the migration was the cause, or even only the main cause, of the origin of species. This origin may have taken place wholly independently of any migration, and the effects would evidently be the same. In other words, the hypothesis of specific differentiation by means of migration is as superfluous as it is unsupported by actual facts.

In the current conception, the life-conditions are called upon to explain the beautiful adaptations of plants to their environment. We wish to admire the harmony which we imagine we see everywhere in nature. The ordinary fitness of plants for life under the conditions they are just enjoying, or rather enduring, must have its obvious cause, as well as the most complicated organizations which adapt plants to some very specialized environments. In all these considerations, however, there is one great error. We desire to explain the adaptations to the present life-conditions, and therefore tacitly assume that the species have originated under their influence. This, however, as a rule, is not so. Life-conditions, climatic as well as biological, are far more variable than are the species themselves. Numerous species are much older than their present environment. Many of our most common species are known to be older than the glacial periods, their fossil remains having been found in the upper tertiary deposits (e.g., Stratiotes aloides). How can we know under what conditions they have originated?

The common answer is that they are adapted to their present environment, and therefore must have been produced under like conditions. But if this argument is to explain their adaptation, it is evidently nothing else than an ordinary circulus vitiosus, the same thing being taken for cause and effect.

Exactly the same conclusion holds good when we consider the area of distribution of those species which are not confined to a single spot or a small country. They live, as a rule, under similar but not under exactly the same conditions everywhere. The more we compare the different habitats of the same species in different countries, the greater will appear the differences in their environment. First, climate and soil, then to a higher degree the plants with which it has to compete for its existence, and the useful and obnoxious animals it finds. Is the species considered equally adapted to all these environments? Evidently not, since under some it multiplies more rapidly than under others. But how can we decide under which of them it has originated? It would be most reasonable to assume that it originated under those which are the most favorable; but this, as is well known, would be erroneous, since numerous species thrive much better and multiply far more rapidly in countries into which they have recently been introduced than in their own original habitat.

Fitness for present life-conditions, therefore, can hardly be considered as a result of adaptation, and we have to recur to previous hypothetical environments to explain the much admired adjustments. All speculations of this kind are merely reduced to more or less plausible and more or less poetical considerations, which, however, as a rule, lack even the possibility of comparative or experimental evidence. The more we try to work out the principle of adaptation in

its details, the more we leave the field of actual inquiry and lose ourselves in the domain of pleasant speculations.

Plants are not, as a rule, most widely spread under just those environmental conditions that are actually best fitted for them. They simply multiply most abundantly in those localities where they do not happen to meet with other species which are better fitted. One of the best instances are the desert plants, which are so beautifully adapted to endure the hardness of such extreme circumstances. they are, as a rule, more fitted for better conditions, like them better, and thrive more abundantly under their influence. As an example, I choose the creosote-bush, Larrea tridentata or Covillea tridentata, the distribution of which I carefully studied in Arizona and California, and which has been so ably described by Spalding from observations made at the Desert Laboratory of the Carnegie Institution of Tucson in Arizona. I quote the following description from an article by Mr. Spalding in the "Botanical Gazette" of 1904:

"The creosote-bush is one of the most characteristic species of the deserts of Arizona and adjacent States. It is, perhaps, the one most constantly present and most firmly established. It occupies extended areas where its removal would leave a bare waste, but at the same time shares, on mesas and foot-hills, a great variety of soils and exposure with other species that exhibit far less capacity of accommodation than itself. This high degree of plasticity is particularly noticeable as regards water supply. On the low grounds near Fort Lowell the creosote-bush is seen to be far more vigorous than on the dry soil of the plains. Here and there around Tucson specimens are found in places where they are well watered, and correspondingly show a striking contrast with the usual form. Their leaves are deep green

and markedly larger in size, and the whole plant presents the appearance of robust health and remarkable vigor, very different from the poor specimens with narrow pale leaves and more or less defoliated branches. Plants that have been well watered for a period of years, moreover, are far more fruitful than their companions standing in dry ground near by. It is evident that, if there is one set of conditions to be assumed as normal for this species, it is exactly not that of the desert, where they are most abundant, often to the exclusion of all other kinds of shrubs, but their one lifecondition should be conceded to be, quite on the contrary, a large supply of water to their roots, since only here they reach their full development. Arid conditions are only tolerated, and this, no doubt, to a remarkable degree, but the plant is dwarfed and suffers in other ways while it endures them."

Spalding assumes that the creosote-bush has originated under conditions similar to those which are now most favorable to it, but has acquired habits that enable it to withstand excessive drought, without, however, losing its capacity of full development on better watered places.

Have those habits been acquired after the species was differentiated with all those marks which now constitute its specific character? Evidently not. They are simply part of it. The plant is adapted to two different sets of conditions, but must have developed these propensities at the same time, and therefore under the influence of the same life-conditions. So it is in numerous cases. Even the cactuses, which are seemingly so exclusively adapted to a life on arid plains, thrive better in moist soils and often in forests, where one would hardly expect them.

Some authors have asserted that migrations of plants are entirely ineffective without the adjustment of the species to

the new habitat. This assertion can hardly be doubted, but the adjustment may depend on a previously given property as well as on the direct influence of the new environment. Such properties may be well discussed under the name of plasticity, since this merely refers to the fact of easy accommodation, without inquiring into its cause. Now it is evident that wherever plants are brought under conditions which are not suited for them and for which they lack this power of accommodation, they simply die out, after a shorter or a longer lapse of time. New conditions are, as a rule, not capable of inducing corresponding changes; they simply kill the unfit and allow the fit to multiply and to gain a new territory. Numerous experiments have been made on this point,—and often on a very large scale,—some by mere chance, others with direct purpose. A few instances may suffice to prove this.

In the Bois de Boulogne near Paris have been sown seeds of many hundred different kinds. About a century ago, almost yearly all the superfluous seeds from the botanical garden were thrown away on such places, where they seemed to have the best chance of germinating and thriving. Many species succeeded, but only for a few years; then they disappeared. Only a single one has been able to keep its hold under the new conditions; this was the Potentilla pennsylvanica, which has become widely spread and is now almost indigenous. All around Montpellier, from 1770 to 1810, Nissole and Gouan sowed many hundreds of exotic species, always choosing such as would seem to be capable of adapting themselves to that region. At the end not a single one was left. Targioni Tozzetti had the same experience around Florence. Godron made a careful study of the plants grown from stray seeds which had been introduced with the wool at Montpellier. Numerous foreign plants appeared

every year around the places where the wool was teased. In his "Florula juvenalis" he enumerated three hundred and eighty-seven of these introduced species. All of them have since disappeared, with the exception of Onopordon virens and Jussiaa grandiflora, both of which have become very common and troublesome weeds. In the same way, Galinsoga parviflora and Corispermum Marshalli were accidentally introduced into Holland about a century ago, and they have multiplied in great numbers without, however, gaining more than a local habitat.

In all such cases, and numerous others also, a few species have proved well fitted to their new conditions; they multiply without changing themselves, while the great majority disappear on account of the impossibility of complying with the new environment. Of adaptations—i.e., of changes brought about by the new influences—never a trace has been observed.

From these causes we now return to those instances of high plasticity which enable species to live under two or more sets of different conditions. Polygonum amphibium may be adduced in the first place. It is adapted to life in ponds, producing long-stalked, glabrous, shiny, floating leaves and long flexible stems. It is found also on land, with crect stems and hairy leaves of a more pale green and almost unstalked. The same plant may, as shown in the experiments of Massart, when it grows just on the edge of the water, make both kinds of stems. It thus affords a most beautiful instance of double adaptation. According to the experiments of Bonnier, numerous species of the high alpine regions comply with the same principle. Cutting the rhizomes in two halves and planting the one in the plain and the other on the mountain, he saw one half retaining its alpine character, while the other half adapted itself to the

lowland conditions, losing its short stems and dense foliage and all those smaller marks which constitute the typical character of the alpine flora along the limits of the eternal snow regions.

Analogous phenomena have been observed by Holtermann in Ceylon. He directed his attention especially upon those tissues which have the function of storing great quantities of water and which thereby enable the plants to keep fresh during the long periods of drought in summer. In the leaves of Cyanotis Zeylanica, for example, the water-tissue covers about four fifths of the whole volume of the leaf, but only so where the species grows on dry soil. Specimens collected in moister conditions have a water-tissue which reaches hardly one tenth of the whole volume. I Ioltermann showed that this is only a question of a high degree of plasticity, since by transplanting a specimen from one place to another, the new leaves which are produced show at once the influence of the new environment. So it is in other cases -for instance, with Rhizophora mucronata, Lumnitzera racemosa, Bruquiera gymnorhiza, and others.

It is clear that we may call all these changes adaptations to new conditions. But then we must concede that these adaptations depend upon characters which were inherent in the species before it arrived in the new environment. The characters themselves are not the effect of the external influences considered; it is only that the opportunity of displaying properties which were previously latent has been afforded by the new habitat.

It is quite a common experience that many plants, when introduced into a new country, may at once prove to be better fitted for it than are its own inhabitants. A rapid multiplication is the consequence, and this is especially well known in those cases where the immigrants not only conquer

the indigenous forms, but do this to such a degree that large parts of the original flora are threatened with total extirpation. Here it is evident that the conditions in the new country cannot afford a basis for the explanation of the properties of the immigrants; on the contrary, the given characters of these species have to be considered as the causes of their rapid multiplication. If, however, this is so in cases where the introduction is historically known, how can we decide whether the same thing does not prevail in other instances, where the introduction is simply older than our historical records? At all events, the comparison of such cases would warn us against attributing locally observed instances of fitness to given life-conditions on the ground of the assumption of the direct influence of the latter. Even the most beautiful and most specialized cases of mutual adaptation between plants and their environment do not escape this objection. Insectivorous plants may have originated in localities where they gained an essential part of their food by catching insects, but they may as well have acquired this propensity where it was perhaps wholly useless to them.

From this point of view, it may be interesting to recall some of the well known instances of modern migration among plants. The water-pest, Elodea canadensis, was introduced from America into Europe about seventy years ago, and has so rapidly multiplied, in almost all countries, as to deserve well its name of pest. It is, however, only female, no male individuals having been introduced. It is thereby clearly incapable of changing into a new form, no bud-variation having ever been observed; and although it has adapted itself perfectly well to all its new conditions, as the phrase goes, this adaptation has proceeded without the least internal change of its characters. The same thing occurs with Acorus Calamus, one of the most common plants

throughout a large part of Europe, covering the edges of miles of canals and pools. It was introduced from Asia some centuries ago, but never produces any good seed with us. It multiplies only by its rhizomes, and evidently without any specific change.

Graebner, in his admirable text-book of general plantgeography, gives a detailed discussion of plant-migrations. He distinguishes between three main groups of naturalized species, which he calls the casuals, the aliens, and the deni-The first group embraces such cases as seeds and other vegetable organs introduced as weeds by means of trade. Scaports and railroads afford the common instances. An instance of the dispersion of weeds by the wool-trade of Montpellier has already been given, and numerous species are known to travel along railroads, in Europe as well as in America. One of the most dreadful examples is perhaps the Russian thistle, which in its dispersion all over the Dakotas and the adjoining States has clearly been shown to follow this way and to enter different parts of the country mainly by starting from the railroad stations. In these rare cases the casuals become aliens, but by far the larger part of them disappear after some time. Amarantus retroflexus was introduced accidentally from America into the country around Venice in 1733. Impatiens parviflora escaped in Geneva from the botanical garden about 1830, and afterward from sundry other gardens. Both of them are now common weeds in many places, and it would be impossible to distinguish them from the original flora, should their introduction have escaped observation. Erigeron canadense was sent from Canada to the botanical garden of Paris about a century before the time of Linnæus, who knew it as an extremely common weed throughout England, France, Italy, Holland, and Germany. Since his time it has spread

far into Russia. Numerous European weeds have been introduced by ships into St. Helena, and they have almost exterminated its original flora. Their multiplication has been astonishingly rapid. The Chenopodium ambrosioides was sown on that isle by Burchell in 1845, and became a common weed all over the whole island within only four St. Helena had, altogether, only fifty-two indigenous species, while seven hundred and forty-six have been introduced by ships, most of which have since widely spread. Analogous facts are given by numerous other isolated oceanic islands, showing that, as a rule, the flora of such a region does not consist of the plants best suited for it. In hardly any case have the life-conditions of such isolated places seemed to have been able to lead the indigenous species to such a degree of adaptation that they could withstand the invasion of forms which had originated under different conditions.

In Norway Dryas octopetala has spread in many places. Matricaria discoidea is still rapidly gaining ground in this country as well as in many others, even in California. The Napa-thistle (Centaurea Melitensis) is one of the most common weeds along the roads in California, having only recently been introduced from Europe. Parish has studied the invasion of thirty or more foreign species into the southern parts of California and described how the native flora slowly but continually retreats before them. Warming accurately studied the flora of the Faroe off the coast of England. They constitute an isolated group of oceanic islands and have a very young flora, since they must have been covered wholly by the ice at the time of the glacial period. All of its species must, therefore, have been introduced in recent times. If we except the very polymorphous genus Hieracium, there are no endemic species. The whole flora consists

of two hundred and eighty-five forms, most of which have been introduced from England, and partly, no doubt, by man. This latter group is estimated by Warming to embrace about forty per cent. of all the forms, the remaining forms having evidently been introduced previously by the winds or by sea currents. Such a new flora must, evidently, constitute an absolutely new biological environment for the first immigrants as well as for the newcomers. Notwithstanding this, and notwithstanding the fact that the length of time from the glacial period till the occupation of the island by man would seem to be quite sufficient for the production of new species, no such events have occurred, with the only exception already quoted. It is difficult to conceive of clearer opportunities for the origin of species than those afforded on the Faroe. If new environments should ever be able to determine such changes, they must surely have done so here. If not, it is hardly allowable to assume a direct influence of the environment upon the degree of adaptation of any given form. Warming gives further proof for the same conclusion where he deals with analogous facts in his "Œcology of Plants." He says (p. 364): "Numerous facts have proved that many species are still migrating and have not yet attained the distribution that soil, climate, their means of traveling and other relations would permit. Such species are able to emerge triumphant from struggles in many communities, without requiring the aid of any change in the inanimate surroundings. Senecio vernalis, in northern Germany, has spread toward the west as a pestilent weed, within a period of scarcely more than twice a man's life. Several hundreds of foreign species have reached New Zealand, where some of them defeat the native vegetation." And in conclusion from these and many similar broad groups of facts, he adds: "It is essential that climate and soil shall

suit immigrant plants; otherwise they fail to gain an entrance." It looks to me like a simple translation of this sentence into the terms of another theory, when we say climate and soil and the whole physical and biological circumstances of a country allow of the entrance of immigrants only on the distinct condition of a previously acquired fitness for the new environments; otherwise, they fail to gain such an entrance. No direct influence or adaptation is possible.

Salicornia herbacea is perhaps one of the best instances. It is common in Furope all along the coast, and it occurs in millions of individuals on the shallow shores of large parts of Holland. It has been introduced into America, where it has traveled from lake to lake, wherever the saltness of the water afforded it a natural habitat. In this way it has reached the shores of even the Great Salt Lake in Utah, where it grows in no less quantities than in Holland, although the climate is wholly different and the degree of saltness (about twenty per cent. NaCl) is many times higher than that of the sea. Here it has evidently met with quite new conditions of life; it has proved fit for them, and no difference between the form growing in Utah and that of Holland has as yet been observed.

Such species, even when they have originally been introduced by man into their new country, evidently now have become independent of his help. They constitute the group described by Graebner as that of the aliens. Numerous instances could be given, as it is perhaps the richest group among all the cases of migration. Colonists and denizens also owe their dispersion to man, the former occurring only along the roads, in the cultivated fields, or in waste places in their neighborhood, and in many cases depending for their continuous occurrence in such places on often renewed introduction. Such, for instance, are many of the most com-

mon weeds in our grain-fields—Centaurea Cyanus, Aquilegia vulgaris, Datura Stramonium, etc. Denizens are those species which have left these lines and mingled with the native flora to such a degree that their foreign origin becomes concealed, or is even in some cases doubtful. Artemisia absinthium, Aristolochia Clematitis, and Vinca minor afford instances of this condition.

The peopling of new soil is an interesting instance of mi-In Holland it is frequently observed when lakes are drained and changed into polders. Aster Tripolium and Cineraria palustris are among the first immigrants, occupying the area in millions of individuals, the first preferring a salty, the second a fresh-water region. Other species follow more slowly, but soon conquer the first, of which the Aster ordinarily keeps its hold, while the Cineraria is often crowded out very soon. In our dunes the first immigrants -- among which the Erigeron canadense is one of the most frequent—may thrive through centuries. But, in the end, when the calcareous parts of the sand become washed out by the rains, most of those species, requiring a certain amount of lime, will disappear and be succeeded by ordinary heath, Calluna vulgaris, and the whole association of species which prefer soils that are poor in chalk. Warming cites the description, given by Beck, of the kinds of vegetation that succeed one another on the sand-banks cast up by high water on the Danube. First, on the bare moist sand are found some herbs, including species of Polygonum and Chenopodium, among which seeds of Salix, Populus, Alnus, and Myricaria germanica may germinate. The next colonists are a number of other herbs, belonging particularly to species with long creeping rhizomes; some settle upon moister spots, others upon drier, and more or less rapidly increase the vegetation of the bank. Then the herbs be-

come suppressed by the shade of the trees; but in the end the struggle among the willows, alders, and poplars themselves ends in favor of the latter, to which then elms are seen to add themselves. All over the world struggles of this kind may be seen. Some species are crowded out, others conquer lesser or larger parts of the soil; but none of them is ever seen to become changed in the process. It is true that, in many cases, the duration of the struggle is too short to allow of any chance of specific adaptation. But then there are many other cases, as, for instance, our Dutch dunes, where the process has taken at least six or seven centuries, and where nothing has ever been seen in the way of direct influence or slow adjustment to the changing conditions.

The first invaders are often seen to be supplanted by others. The first are those which occur in the vicinity and possess the best means of dispersal by wind or by birds. Their number may steadily increase, but, sooner or later, other forms may come in—perhaps from distant regions—which prove to be better fitted for the conditions of that locality. Then the struggle for life becomes more intensive, until gradually an increasing number will be crowded out and the flora will become poorer and more uniform. As a rule, the more highly specialized forms will then prove to be the least fit, while coarser types, with less obvious adaptations, will comply more easily with the prevailing conditions and so become the ultimate conquerors of the soil.

Harshberger describes the flora of the Rocky Mountains, giving lists of their plants grouped according to their probable origins. The whole flora is a young one: some species invade the region from the east, others from the west, but all of them without showing visible changes in the way of adaptations to their new environment. There are, how-

ever, many plants restricted to this region which, in all probability, have been differentiated since the close of the glacial period, the territory having been glaciated at that time. How this differentiation was brought about, and under what conditions, it is of course impossible to tell; but the assumption that the life-conditions were then the same as they are now seems to me the least probable one could propose.

I conclude this enumeration of well known cases of migration without visible changes by alluding to the case of waterplants. As a rule, they have a wide dispersion—far wider ordinarily than their congeners that live on the land. The most curious instance is, perhaps, the carnivorous species Aldrovandia vesiculosa, which is highly adapted to the catching of insects, small crustaceans, and other small swimming animals of our pools and ponds by means of its leaves. It seems to show no relation whatever to its environment in these structures, being in no degree better fitted for life in water than all the other species with which it is found growing together and which lack this presumed weapon in the struggle for life. Its area necessarily consists of isolated spots, such as lakes, moors, and pools. Notwithstanding the great difficulties of transportation, which would seem to be in the way of its distribution, it is found all over Europe, in Germany, France, Italy, Hungary, and Russia. Moreover, it is observed in eastern Asia, the Indian Archipelago, in Australia, and even in the central parts of Africa, almost every single locality lying at enormous distances from all the others. It is exposed to great differences of climate and soil, and especially in its biological surroundings and competitors. But all these influences have not been able to change it in the least. Everywhere it is simply the same highly specialized form.

In all these cases we clearly see that the capacity of accommodation to a new environment does not depend upon the possibility of assuming new characters under the influence of the new factors, but solely on the degree of plasticity. This, however, is a latent quality which was already inherent in the species long before it was brought under the new influences, and which, therefore, must have been acquired independently of them. Although it may be regarded as a quality by itself, it has evidently nothing to do with the question of the production of new characters under the direct influence of environment,-I mean, with the origination of such qualities, in response to the requirements of this environment, as would fit the plants under consideration to it. How the plasticity has been brought about is another question, which has to be considered by itself, without mixing it up with that of its usefulness long after its origination.

Migration and rapid dispersion without changes of specific characters are perhaps most clearly illustrated by those fungus-pests which have come either from America to Europe, or from Europe to the new continent. dreaded diseases of cultivated plants afford instances. Among them, those of the potato and the grape-vine, Phytophthora infestans and Oidium Tuckeri. Among insects, the Phylloxera is perhaps the best known instance, while the Colorado beetle does not seem to be well suited for European orchards. Few migrating plants have been so closely followed in their movements and so thoroughly studied in all their physiological and morphological properties, in order to find the means of successfully combating them, as these pests. Any slight change in their specific characters, any production of new races especially suited for the new conditions, would surely have been discovered and widely studied and described. Nothing of the kind has occurred,

however, and no real adaptation has taken place. The rapid spreading has been the result of previously existing characters, but it has not had any relation to the origination of new forms.

As an instance of the rapid spreading of a fungus-disease I choose the rust of our Malva and the other genera of the same family, as studied by Eriksson (Puccinia Malvacearum). It came from South America and reached Spain in 1869, this being the first invasion of Europe. years later it was observed near St. Armand, in a northern department of France, and in the next year-1873-it was found spreading all over that country, in England, and also in Germany. In the next two or three years the number of its stations rapidly increased, and it migrated to Switzerland, Austria, Hungary, Finland, and Greece (1876-1890). Australia was among the first countries to be infected; Africa and North America, among the last (1885). Wherever it has penetrated, it has soon become a dreaded pest, impeding the culture of malvaceous plants in a most troublesome degree.

Once more, these instances show that migrations are not, as a rule, accompanied by specific changes. Such may occur during the traveling-period of a species, quite as well as during any other times of its existence; but then there is no single reason to consider them as the consequence of the changed conditions of life. The same conclusion will be forced upon us, now, as we come to the consideration of those cases where the climate and other environmental conditions must have changed without, or almost without, corresponding migrations.

Battandier describes the probable origin of the present flora of the Sahara desert. Originally, this region must have had an ordinary degree of rainfall and moisture, and

constituted as fertile a country as any of the surrounding parts of northern Africa. Then, for some reason or other, the rainfall must slowly have diminished, taking centuries, or even the larger part of the quaternary period, to reach the conditions which now prevail. The consequent changes in this flora must have been correspondingly slow, and must have consisted mainly in the disappearing of the larger part of the species: first of those which were dependent on the higher degree of moisture; then of others, until at the present time only the most drought-resisting forms are spared.

Battandier sees no reason for assuming that any specific changes were brought about by this great process; on the contrary, he points out the fact that a large number of the species of this arid region are what we call monotypic genera, each genus consisting of a single species. If there had been any degree of adaptation during this whole period of increasing dryness, new species would have been produced, most likely, from those forms which, by their own inherent capacities, would be the very last to be threatened with extermination. Those genera would, therefore, have produced quite a number of smaller or even of larger species, adapting themselves more and more to the changing conditions and stocking the desert, in the same way as other deserts have been stocked, from adjoining countries. They have not done so, and from this we may conclude that the single species, of which each of the genera consists, have not undergone any change in the direction of drought resistance. but have simply been those which happened to be the best fitted for the life in the desert. A thick epidermis, a small display of leaves, long and deep roots, were the main qualifications for this choice. All species which were not so endowed must have disappeared; for only those which enjoved these properties could resist, in the long run.

The swamp cypress, Taxodium distichum, Cercis siliquastrum, and many others, have been found fossil in the upper tertiary deposits. So far as their remains admit of a conclusion, they have not undergone any specific change during this long period of their existence. Climatic conditions have, however, very much changed, including, perhaps, the greatest differences in temperature that can ever have exerted an influence upon the vegetable inhabitants of this world. The biological environment has changed in about the same measure, since most of the species with which they had to compete in the beginning have now disappeared and been supplanted by others. In this case it is once more clear that environmental changes do not necessarily change specific characters. And from this we may conclude that either adaptations have wholly different causes, or at least that there is only a fortuitous, and no real, causal connection between the two large groups of phenomena. Darwin's proposition that the changes took place independently of the question of their being useful or not, and that the external influences simply furthered the first and thereby extirpated the useless, seems still to be the best and most natural explanation of the great phenomena of biological evolution.

Local varieties and geographical races are often adduced as proofs of the direct influence of external factors. A certain number of species, growing in Europe as well as in America, show small differences which hardly reach the degree of ordinary varieties. Hairiness, size and form of the leaves, and other minor points constitute the differentiating marks (Veronica scutellata, Circæa lutetiana, etc.). Many varieties are distinguished as australis, arctica, borealis, or as var. montana, alpestris, pyrenaica, and so on. Often such varieties show beautiful adaptations to the local conditions under which they grow; but in no case is it pos-

sible to tell whether they have acquired these during their migration or during their stay in the new environment, or perhaps previous to their being subjected to the influences in question. In reality, such cases have no value at all as proofs; they may be explained as easily in one way as in the other.

A most interesting line of research is suggested by these considerations. It is to bring the descendants of the most extreme migrations of one and the same species together and to cultivate them under the same climate and, if possible, in the same biological environment. Three cases are possible. In the first one we may happen to choose plastic species, the individuals of which may live under very different conditions and do well. Brought together, they will lose their differences and assume the same form and structure. Many of them will do so even if only rhizomes or cuttings are transferred; others from seed, in the very first generation; and only a few, as it seems, will need one or two generations before the temporary influence of the locality from which they were taken will be wholly lost. The second case refers to those species which, through their coarse organization, hardly need any plasticity to comply with the most diverse conditions. Such seems to be the nettle (Urtica dioica), which follows man on his travels all over the earth, and which has often indicated to explorers of new countries the spots which had already previously been visited by others.

Our third case is that of the local varieties and geographical races. They must be expected to keep up their differences, at least in the beginning. But by continuing the experiment it is probable that some of them will yield valuable facts for a decision between the opposing theories. If the external conditions have a direct influence on specific or varietal characters, changing these in a gradual way so as

to make them fit for their environment, it must be expected that the differences beween our local varieties will slowly but surely disappear by cultivating them on one and the same spot. If such direct influence does not exist, two results may be expected: either the varieties will keep their differences indefinitely, or they may show atavistic changes which will reduce them to one and the same type. Such changes will then, however, be sudden and without a visible relation to the environment. Of course they must have external causes as well as internal, the external determining the moment at which the event will happen, and probably consisting in a combination of quite a number of factors. The factors being the ordinary ones, the combination may be temporarily a new one and thereby produce an effect not previously seen.

However, it is so often fallacious to indicate probable results of biological experiments that it seems better not to extend this discussion. Its only aim is to show an easy way in which, in my opinion, experimental proofs concerning the production of new forms of plants may certainly be hoped for.

Local varieties and endemic species are not necessarily distinguished from their nearest allies by characters that bear the stamp of an adjustment to the special environment. They may have originated quite independently of any adaptation. This important fact has been pointed out by Willis on the ground of his observations on the flora of Ceylon. On this island about one third of all vegetable species are peculiar to it, not being found anywhere else. They are endemic, and, at least for the majority of them, we must assume that they have originated on the very spots where they are now found, and probably not sufficiently long ago to allow us to assume that climatic and biological conditions were at that time different from what they are now. So here

we have a most desirable opportunity of studying specific characters in relation to the environment under which they did originate. Moreover, the endemic species of Ceylon belong to genera which are represented in its flora by one or more common species widely spread on this island and in neighboring countries, and which, therefore, may in many cases be presumed to be the ancestors from which the endemic types of to-day have sprung.

The result of this inquiry has been illustrated by a minute study of two species of Coleus. C. barbatus is a quite common type on Ceylon, but C. elongatus, which is nearly related to it, is found only on Mount Ritigale, a mountain which is relatively rich in species which do not occur anywhere else on Ceylon or outside of it. Willis enumerates the differentiating marks of the two forms in a table, and clearly shows that they can have no imaginable relation to the differences in environment. Thus the marks which separated the new C. elongatus from the old C. barbatus are not the effect of any adjustment to its new habitat. They are quite independent of any such process. The same holds good for numerous other species belonging to the most widely divergent genera and growing on different isolated mountains. Nowhere could he discover any proof that the special characters of the endemic types could have been brought about in response to the demands of the local surroundings.

Cockayne, in his studies of the endemic species and varieties of New Zealand, comes to the same conclusion; and lately Gerbault discovered two striking mutations of Viola scotophylla on the territory of Saint-Ouen-de-Mimbré in the department of Sarthe in France. Both of them occurred in a small number of individuals among the normal specimens of the species, and their particular marks showed no relation whatever to their environment. Many other authors

have adduced observations of the same kind, showing that, as a fact, new forms may well arise without any response to external factors.

Briefly summing up the results of this discussion, we see, in the first place, that migration is far too rare a phenomenon to account for the evolution of the vegetable kingdom, and that where it occurs, it proceeds without visibly changing the migrating forms. In the same way, geological changes of climate may have been accompanied by the production of new forms; but there is no evidence that this has occurred in such a way as to provoke directly useful changes. On the other hand, the characters of local and endemic types do not betray any definite relation to their special environment,—at least in the best studied instances. All in all, the facts which are at present available plead against the hypothesis of a direct adjusting influence of environment upon plants, and comply with the proposition of changes brought about by other causes and afterward subjected to natural selection.

Personally, I assume that the species-making changes occur by leaps and bounds, however small; but this point has not been referred to in the discussions of this lecture.

HUGO DE VRIES.

# Third Lecture MODERN CYTOLOGICAL PROBLEMS

THE modern study of the structure of living cells and of their different parts and organs marks a definite period in the history of biological science. An increasing number of students are turning their efforts to these questions, and the methods of research are continually developing themselves. Foremost among botanists are Strasburger and Grégoire; among zoölogists, Wilson and Boveri; but many other celebrated names would have to be added. This whole line of thought has come under the influence of the idea of Roux, which states a distinct parallelism between the life-history of growing and dividing cells and the phenomena of heredity. A large part of the work now being done in the field of cytology goes to support these views of Roux, and to show the exact coincidence, even in minute details, of the facts observed in cells and of the processes we would expect to find in them on the ground of this hypothesis. Text-books and reviews give adequate information on these subjects, and the extensive material of facts is, in all its details, easily available to the student.

In this work the attention is focused on the questions concerned with the nucleus, with its structure and the process of its division. Under this influence, the study of the outer parts of the protoplasm has been somewhat neglected. They ask for different methods; fixed and stained material is hardly suitable for them. The arbitrary division of the whole protoplasm into nucleus and cytoplasm, although very easy in the study of the former, is only too liable to diminish the interest of the latter.

For this reason, I shall try to give here a short survey of

cytological questions not directly connected with the present main lines of study. On most of these points the available facts are still insufficient for a definite judgment, and authors differ in their appreciation of the facts, according to their own partial experience or to the text-books they are using.

The great principle of all natural science is that of generalization. Without it, all our knowledge would be only imperfect and partial; in fact, how small is the number of cases studied, in comparison with the almost unlimited array of instances really occurring in nature! Generalization is at once a right and a duty; without it, the applicability of well proved facts would be so limited as to be hardly of any use. It is the best guide in almost all special researches, and if Darwin's theory of evolution were measured only by the number of new facts to the discovery of which it has shown the way, it would still occupy a foremost place in the history of scientific investigation.

From this point of view, the student who is not contented with following the acknowledged lines of work, but wishes to enlarge the field of his investigations, has to start from the well established facts brought forward in the best studied parts of his field of research, to use them as the basis for broad generalizations, and then to control the results these last will yield when applied to special cases.

In this lecture, therefore, I will try to indicate some of these broad generalizations and show which fields of inquiry they open and to which suggestions they lead, hoping in this way to direct the interest to some points which have been wholly neglected and to others which are misunderstood by lack of the right guiding principles. Many valuable suggestions may be derived from the work of older investigators; these have lain dormant for long periods of years, have been lost from view, but have not, therefore, lost their usefulness.

My first and main point is the hypothesis of panmerism. The different parts of the living protoplasm multiply themselves by division, and in the large number of well ascertained cases this is their only way of originating. The one does not produce the other; each organ reproduces only itself. For the nucleus this fact is now one of the best established of the whole field; experience and theory agree in leaving not the least doubt concerning its general validity. Boveri's great discovery of the individuality of chromosomes shows its validity for these bodies especially; historically considered, it shows, at the same time, the difficulty in gaining a general conviction even for such a clear and simple conception. The golden rule simplex sigillum veri is generally slow in its working!

In the second place, we have the work of Schmitz, Schimper, and Arthur Meyer on the great group of the leucoplasts and their derivatives. The amyloplasts which produce the starch-grains from sugars, the chlorophyll bodies into which they may change by assuming a green color, the chromoplasts which in so many cases are clearly derived from these, follow the rule established for the nuclei. They multiply by division, are in most cases easily seen to do so, and no other way of their originating has as yet been demonstrated beyond a doubt. The number of well studied cases is so large that the exceptions, if such there are, may well be regarded as only apparent and in need of a careful reinvestigation.

From these two cases we may turn our attention to the ectoplasm, or ectoplast, as it should rather be called. It is multiplied in the division of cells, the larger part of the ectoplasm of the two daughter-cells being simply the two halves of the same organ of the mother-cell. Doubts exist only in regard to the origin of the new parts lying along the

division-plane. This origin is best known from the study of quite a number of investigators in the case of Spirogyra. Here the ectoplasm is folded inward, the fold going all around the cell and steadily increasing toward its center until the division is complete. The new parts clearly originate from the old one, no new ectoplasm being independently produced. If we now take this case as a prototype and try to apply it to all other cases of ordinary cell-division in plants, no serious obstacle is encountered. The division always starts from the old cell-wall; sometimes from all sides, at other times beginning at one point and slowly extending from this. Evidently this is a difference of only secondary importance. Even in embryo-sacs, which divide their nuclei a great many times before cell-division begins, the nuclei are known to distribute themselves in a single layer along the ectoplast, and the division takes its first start from this, proceeding inward toward the central vacuole.

The only well ascertained exception to the general applicability of the principle of panmerism to the ectoplasts is in the case of the origin of the ascospores as studied by Harper. We should expect the original nucleus, from which all the nuclei of the spores in the same ascus are ultimately derived, to be surrounded by its own protoplasm, having its own ectoplast, which would be derived in the ordinary way of cell-division from the ectoplast of the ascus itself. No such structures have as yet been described, although the observed facts do not exclude their possibility. The case may be the same as that of the spermatozooids of phanerogams, which were for many years taken to be nuclei only, until Guignard discovered their thin layer of outer protoplasm.

The most difficult case seems to be that of the vacuoles. Since Went first showed them to exist in meristematic and in sexual cells, which were formerly held to consist of solid pro-

toplasm only, their general occurrence in young cells is almost universally conceded and the net-structure observed in these elements is recognized to be due to their presence. They are well known to divide and also to combine into larger vacuoles with the definite differentiation of the growing cell. Many authors, however, assume that, besides multiplying by division, they may also originate directly from the ordinary cytoplasm. It is easy to see them dividing in living cells, but to observe their independent origin might be extremely difficult, and convincing proofs have not as yet been given. Some authors have supported their opinion from observations on the origin of vacuoles in the Myxomycetes; but then they have confounded the real water-vacuoles with the so-called food-vacuoles, which are parts of the ectoplast pushed inward with the food particles and surrounding these.

The least known part of the cell is, beyond doubt, the granular plasm, which in so many plant cells is seen to flow along the cell-wall. Before discussing this point, however, I wish to consider the current conception concerning the way in which the nucleus exercises its influence on the surrounding parts of the protoplast. From numerous observations it is evident that such relations must exist. Tangl and Nestler studied the movements of the nuclei in response to wounds, and showed that they precede and regulate the cell-divisions which lead to the production of a new layer of cork, shutting off the injured parts of the tissues. Many other similar cases have been described by Haberlandt, but the most interesting are the experimental researches of Gerassimow on Spirogyra. This author discovered that by means of sudden refrigeration with ether or chloroform dividing cells of this alga may be induced to contract the connecting fibers of their nuclei in such a way as to bring both of the nuclei into one cell. The division of the cell itself is not hindered by this

process, and at the end of it we have one cell with two nuclei and one without such an organ. Both may be kept alive for weeks, but their functions are seen to be different. The nucleated cells grow and divide almost in the same way as normal ones, but the enucleated elements lack this property. They continue to produce organic food from the carbon dioxid of the surrounding water, heap it on in their chloroplasts, and increase their osmotic pressure accordingly; bur there is hardly any sign of their being able to use this food for further growth and differentiation. From these facts we conclude that the ectoplast, in order to maintain the extension and growth of the cell-wall, must derive something from the nucleus. If this latter be cut off from a cell by a cellwall, no such derivation is any longer possible. The inducement derived from the nucleus may continue to work for a short time, as in the experiments of Klebs on plasmolytic cells of Spirogyra. Here the protoplast may be divided into two parts; the one containing the nucleus will make a new cell wall and continue to grow, while the other half may surround itself by a thin layer of cellulose, but soon must stop its production. No cell-division occurs in the cells without a nucleus, and their further behavior shows that probably all their functions may last only a limited time. At the end deterioration and death are the result of the impossibility of being affected by the nucleus.

Continuance and regulation of the functions of the outer organs of the protoplasts thus depend on the activity of the nucleus. Something is given off which stimulates and directs the work of the other organs. It is possible, however, in very rare cases to observe this influence directly. The best instance is that of the origin of the blepharoplasts in the spermatozooids of the common liverwort, Marchantia polymorpha, studied by Ikeno. In the mother-cells of these

organs he saw, shortly before the last divisions, small separate granular bodies lying in the nuclei. Such bodies are lacking in the nuclei of ordinary vegetative cells, and also during the divisions that lead to the production of the mothercells. After appearing in these, these granular bodies soon leave the nuclei and take their places at the poles of the nuclear spindle. Here they stay until the divisions have finally led to the formation of the spermatozooids, and in this last phase they are moved toward one of the ends of the cell, where they combine with the ectoplast and grow along with this, producing that part of this organ from which the cilia will be protruded. They then take the name of blepharoplasts. Thus we see that a main part, at least, of these organs is directly derived from the nucleus, and we may confidently assume that the ectoplasm, without this acquisition, would not of itself be able to build up the cilia.

From these and numerous other facts we may derive the conclusion that the means by which the nuclei stimulate and direct the functions of the other organs of the protoplast consist in the giving off of material particles which combine with those organs, multiply themselves within them, and thus determine their functions. It is probable that the larger part of them is given off during the resting stage, and not during mitosis. Many authors—and among them, in the first place, Conklin—have observed the excretion of material particles from the nucleus. They are often stained in the same way as the chromatin, and not rarely exceed by far the quantity of these substances found at the same time within the nuclei. This shows that before leaving them they are produced in such quantities as may well support the view of their great importance in the regulation of hereditary characters.

Leaving the study of the many possibilities concerning the

properties of these material bearers of characters to those interested in the hypothesis of intracellular pangenesis, we have here only to consider the question as to how these particles may be conveyed from the nucleus toward the organs they are destined to supply with the means of further activity. Here we are struck by the fact that in so many cells the nucleus is the center of the flowing movements of the granular protoplasm. The hairs of Cucurbita and those on the stamens of Tradescantia are the best known instances: the currents are seen to radiate from the nucleus in almost all directions. In other cases there is only one rotating current, and it goes along the nucleus or even may carry this around the cell, as in Vallisneria. From such cases we may derive the supposition that these currents must be the ways, and even the means, for the transportation of the material bearers of the hereditary characters. From their place of origin they may reach any point of the living part of the cell, every single leucoplast or chloroplast and every more or less differentiated part of the ectoplast. It is a curious fact that in the large cells of Spirogyra the starch-producing parts of the spiral bands of chlorophyll are often directly combined by fine threads with the central nucleus. Their special differentiation, part of which is directed toward the accumulation of albuminous substances, would lead us to expect such a connection.

Beyond all doubt, the transportation of these pangens is not the only function of the flowing protoplasm. In many cases it is evident that it serves for the transportation of nutrient substances, and in one of the best known instances—that of *Chara* and *Nitella*—it would seem obvious that this is their main function. The big and beautiful starch grains which these plants heap up during the summer in the lower parts of their stems—often concealed in the mud of the

ponds, and serving as reserve material for the winter—are evidently derived from the activity of the upper parts and branches of the stems. The only means for this extensive transportation of nutritive material is evidently given in the currents of the protoplasm, and these flow exactly in the direction which this conception would lead us to expect. The rapidity of these currents is such that if there were no cell walls across the tube, the visible particles would be transported at a speed of about one meter per twenty-four hours. This would amply suffice to bring the products of the activity of the chlorophyll to the bottom of the pond.

The movement of the protoplasm in Chara and Nitella, just quoted, was formerly considered as one of nature's greatest curiosities. It is more in the line of modern research to consider it as an extreme instance of a general rule. Everywhere in the plant kingdom where vascular organs for transportation are absent, these currents assume this function. This fact is most evident in hairs in general, and especially in the root-hairs. In the latter, as was shown by Jönsson and others, the granular protoplasm is almost always seen to flow from the top of the hair toward its base and backward, thus affording a tangible conveyance for all the substances absorbed by the hairs. In many tissues these movements may also be seen taking place easily in watery parts, but with some difficulty in drier organs, such as the bark of woody species. Even in the meristematic condition the protoplasm seems never to be at rest, at least under favorable conditions, but always more or less clearly flowing. Some authors, it is true, have not succeeded in controlling these facts, and have even been led to consider the movements in such tissues as due to accidental causes, as, for example, the injuring of the cells in the making of the microscopical preparations. This, however, must be distinctly

considered as erroneous, the phenomena being far too general for such an explanation and requiring a too special differentiation of the protoplasm to allow the suggestion of an accident.

Outside of the vascular tissues, the flowing of the protoplasm is the only intensive means of transportation of nutrient material. Diffusion is too slow, by far, for this end. From the celebrated experiments of Graham on liquid diffusion, Stephan has calculated the time one milligram of common salt (NaCl) would require to ascend from a solution of ten per cent., through a vertical column of water, to a height of one meter. He found that it would take three hundred and nineteen days.

Cane-sugar is much slower and would need two years and seven months for the same height, under the same conditions; while with albuminous substances the experiment would last about fourteen years. Such velocities are evidently inadequate for the movements of soluble substances in plants; moreover, the differences in concentration are almost always much smaller than in Stephan's examples. If one takes a glass tube of over one meter in length, filled with water and initially containing some crystals of a colored salt (e.g., sulphate of copper) at its base, it is easy to show that it takes more than a year for the salt to reach the upper parts in a visible quantity.

The experiments of Janse with Caulerpa have shown the great importance and high degree of differentiation of the protoplasmic currents in these big unicellular algæ. Pollentubes show the same phenomenon in their living parts, and the same may be seen everywhere else. In young roots there is an almost continual circulation of the protoplasm in the cells of the cortical tissues, conveying the absorbed substances from the root-hairs, through the endodermis, toward

the vascular bundles. In the endodermis these substances are taken up and brought under that osmotic pressure which will bring them upward by means of the great current of fluid in the vessels of the xylem.

It is commonly assumed that the flowing protoplasm is a living part of the cell and produces its movement through its own organization. Hofmeister was among the first to work out this idea, but it is to Engelmann that a definite theory of this action is due. He compared the flowing protoplasm with the movements of muscles, assuming contractile elements in it which would be analogous to the visible sarcous elements of the muscles. In changing their capability for imbibition of water in response to stimuli, these contractile elements would increase in breadth, but decrease in length. Such a polarity might also be deduced from other observations, as, for example, those of the changes of the refraction of light in some particular instances. The explanation of the circulating and rotating movements of the granular protoplasm, based on this principle, requires the assumption of a regular periodic contraction of these particles. But when we try to apply it to specially observed instances, great difficulties are met with and new hypotheses almost always must be sought in order to surmount them. A really satisfactory conception of the whole mechanism can hardly be reached on the basis of this principle.

Leaving it, we come to the opposite extreme and must assume that the flowing protoplasm is not a living part of the cell, but only a more or less viscous fluid. The source of its movements must then be looked for outside of it, partly along the ectoplast, partly along the tonoplasts or walls of the vacuoles, where the currents pass between these. We are led to the hypothesis of invisible tracks on which the impulse for these currents must be produced. This hy-

pothesis seems not as yet to have attracted much attention, but it has the advantage of explaining the observed facts on the basis of analogous observations, without requiring for them a new fundamental theory. It simply brings the flowing of the inner granular protoplasm in line with other cases of protoplasmic movement.

In order to elucidate this conception we may take as an example the observations of Max Schulze on the movements of diatoms. He studied them in water to which a small amount of finely divided carmin was added. The most striking case is that of a diatom lying on one of its flat sides and turning the other upward. When particles of the carmin, in sinking, come in contact with the central line of this side, they are seen to move along it until they reach the end of the cell, then turn backward and proceed along the same Other particles may sink, but not touch this central line directly beside them; these will show no movement. From these and other observations, Schulze decided that there must exist a narrow band of outer protoplasm, which, although as limpid as water and thereby invisible, would be the pushing force and actually carry the particles that fell on it. This same track of protoplasm would suffice to explain the ordinary movements of the diatoms when they slide along larger algæ, or along one another (as in the case of Bacillaria), or upon the glass slides in microscopical preparations. It is well known that they can move only when they touch other objects by one of their faces, and that they are always at rest when lying on their side. The energy developed in these movements is sufficiently judged of by the size of the cell and the rapidity of their gliding, but Schulze showed that they are even capable of carrying much larger weights with them.

The assuming of analogous tracks of active protoplasm

in ordinary cells would seem to give a sufficient basis for the explanation of the phenomena of circulation and rotation. The tracks would be active, the currents passive and pushed by them. It would be an easy matter to explain the undulating movements on the tracks by comparing them with muscles, cilia, and other widely studied objects.

If the tracks are active and the flowing parts passive, we should expect the velocity to be the greatest for those parts which are directly in contact with the tracks and to diminish with increasing distance. Such differences in velocity are well known in many cases, but have been studied most accurately by a great number of investigators in the case of Nitella. According to Nägeli, Dutrochet, Göppert, Cohn, and others, the outer layers of the mighty current in the large cells of this alga are seen to be the fastest in their movements, the velocity decreasing toward the central vacuole as well as toward the limits of the current, which are indicated by the absence of one of the longitudinal rows of chlorophyll bodies. With decreasing vitality the current stops first along these sides, thereby becoming narrower, and the central parts, which ordinarily are the quickest, are also the last to retain their movement. It is clear that the whole of the protoplasmic fluid is pushed by the activity of a stratum of outer protoplasm clothing the layer of the chlorophyll bodies on the inside and following the direction of the spirally ascending lines of these organs. Similar observations have been made by Vesque for the root-hairs of Hydrocharis, and by Heidenhain and Jürgensen in the leaves of Vallisneria.

These hypothetical pushing tracks would have to be considered as living organs of the cell with the same right as chloroplasts and other visible parts. They may be assumed to be morphologically constant, but very variable in their degree of activity, and changing the direction of their move-

ment, from time to time, as in the case of the diatoms just quoted. One of the best instances for their study is afforded by the tentacles of *Drosera* at the time of digesting insects or other albuminous food. Then the large vacuole, with its deebly stained red contents, is seen to become excessively contracted and at the same time divided into numerous smaller ones. A great space is produced between the ectoplast and the tonoplasts—a space which is probably used for The rapid transportation of large quantities of albumen. Freed from the greater vacuoles, the currents of the flowing protoplasm become more easily visible and small vacuoles of different forms and sizes adhere to them like drops of a red liquor and are clearly moved along them. Here the conception of semi-solid pushing tracks sticking to the ectoplast at once suggests itself; it affords a simple and easy explanation of all these most curious phenomena which make the study of these tentacles a very attractive one.

Semi-solid tracks of the kind described seem to play a large part in the differentiation of cells, and especially in the production of their ultimate form and of the structure of their walls. This principle is most beautifully illustrated by the description, given by Dippel, of the evolution of the spiral threads in the elaters of the liverworts. He studied especially the cases of Marchantia polymorpha and Fegatella conica. These elaters are long and narrow cells with a double spiral which suddenly extends on the opening of the fruit, and thereby flings out the numerous spores lying between them. In the young fruits the elaters are still small, filled with protoplasm, and with a smooth cell wall. Numerous vacuoles are seen within the granular substance. Gradually these arrange themselves along the ectoplast, taking definite positions and leaving between them tracks of the granular protoplasm which combine together to constitute

a double spiral band. Soon after, this band becomes the prototype of the ultimate spiral of the elater, exactly indicating the line along which this latter will be produced. But it still consists only of protoplasm. This is easily seen when the contents are isolated from the cell-wall by means of plasmolytic contraction; for the inside of the wall is still wholly smooth, without the least indication of a spiral structure. Then the deposition of cellulose begins along the outside of the tracks, while the currents of the fluid plasm follow these on their inside. In this way the final spiral is laid down against the wall, and after this is completed the protoplasm will be disorganized and ultimately disappear. Thus the structure of the wall may be considered simply as a copy of the corresponding structure of the protoplast. Analogous phenomena have been observed in the evolution of the netcoverings of the inside of many vessels, and in other cases.

This intimate connection of the ectoplast and its conducting tracks of flowing plasm with the differentiation of the cell-wall leads us to consider this organ also as a living part of the whole protoplast. Unfortunately it is almost always so very thin that no definite structure can be seen, but in the rare cases of greater thickness such a structure becomes evident and is well known. The best instances are the Myxomycetes, where Strasburger and others have studied it, and the swarm-spores of some algae, where a connection of the structure of the ectoplast with the bases of the cilia may be observed.

The question of the semi-permeability of the ectoplasm is not directly connected with that of its living condition. The task of regulating the diffusion of soluble substances into the protoplast, and from this outward, need not necessarily be confided to the whole ectoplasm, since an extremely thin outer layer would be quite sufficient for it. We may even

suppose the ectoplast clothed with a kind of precipitation membrane, but it is an open question whether such a supposition would suffice to explain the phenomena of permeability. I can only allude to Overton's theory, which assumes the outer ayer of the ectoplast to be impregnated by a mixture of cholesterin, lecithin, and allied substances, and the processes of permeation to be regulated by the solubility of the different substances in this mixture. The facts given by Küster and others agree in the main with this idea, but show deviations in detail which, however, may be due to a lack of sufficient knowledge of all the chemical compounds which really constitute the cholesterin-lecithin layer.

A chief function of the ectoplast is the lengthening of the cell wall during the period of growth. The stretching force, of course, is given by the osmotic pressure or turgor of the cell sap, but it is the cell wall that regulates the extension in so far as it makes some parts extensible and others not. This problem has been most thoroughly studied by Errera in the case of a mold, Phycomyces nitens. Here it is clear that in the same cell the young growing parts are extensible, while the older ones are not. Extensibility depends mainly upon the presence of colloidal pectinous substances in the cell-wall, and may be increased locally and temporarily by the changing of these into soluble compounds. This inversion is ascribed to the intervention of enzymes, which, in their turn, must be exuded in distinct places and at the proper times by the ectoplast, thereby indicating a differentiation of this organ which may be considered as wholly analogous to that described in the example of the elaters of liverworts.

I must now return to a consideration of the tonoplasts or walls of the vacuoles. As already pictured, these also must be considered as living parts of the cell, as organs whose

main function is the accumulation of soluble matter in the cell sap, partly as food material and partly as the source of osmotic pressure. These tonoplasts may be clothed on their inner side, towards the cell sap, by a semi-permeable layer analogous to that on the outside of the ectoplast; but facts which would allow of a discussion of this hypothesis are for the present hardly available.

The function of the tonoplasts is a double one. One relates to the exosmosis, the other to the endosmosis, of the constituents of the cell sap. In regard to the first process, they behave, so far as we now know, according to the laws governing diffusion through semi-permeable walls. processes seem to be wholly of a physico-chemical nature. Far different from this is their behavior as affecting endosmosis. In this case the soluble substances are taken from dilute solutions and heaped up, the concentration steadily increasing until it exceeds that of the surrounding fluids sufficiently to conquer the resistance of the cell wall and extend it. The source of osmotic force thus really lies in the vital activity of the tonoplasts which produce the required concentration; the solution within the vacuole is only the means of transferring this force upon the processes of turgidity.

Pulsating vacuoles, especially those of Euglena, are perhaps the most demonstrative instances of the living condition of the tonoplasts, and numerous authors have thoroughly studied their movements.

The life-history of these tonoplasts has gained a new and most attractive chapter by the discovery, made recently by Stomps, of the part they take in the mechanism of nuclear division. Until a short time ago, the most fantastic views concerning this mechanism prevailed, but Stomps has shown that almost the whole process may be easily explained by

simply assuming that the same properties and forces are at work in the nuclei that we know to be active in the tonoplasts. At the time when division is almost terminated and the daughter-nuclei pass over into the resting condition, the chromosomes themselves are changed into a reticular condition. This change is prefaced by the appearance of small vacuoles within them. They may be seen lying in a longitudinal line in the midst of the chromosome. In increasing, they push the chromatic substances asunder until they are reduced to knots and points in the angles between the vacuoles, thus producing the foamy or reticular condition. Every single chromosome is thus changed into a net, and the whole nucleus is only the combination of the sundry nets, the net being nothing else than the optical section of the foam. The outer parts of the outer layer of all these small vacuoles surround the mass of chromatic substances and are continuous. By their confluence the membrane of the nucleus, the origin of which was hitherto wholly unknown, is produced; it disappears when the vacuoles afterward contract and resume their central position.

Almost all of the points reviewed in this lecture are thoroughly in need of renewed investigation. Many conclusions rest on too small a number of well observed facts. Quite a large number of phenomena, until this time studied in only a few plants, may be taken up in other species, thus giving the expectation that new sides of the problems will come into consideration. Points which it is difficult to elucidate for some forms may easily yield to examination in others. All such work should, however, be guided by broad considerations, starting from the principle that the main task of living protoplasm is to change one form of energy into another. The initial forces should in every case be compared with

the resulting ones, and the mechanism by which the changes are produced should be clearly explained. This mechanism is governed by the hereditary characters of the species in question, and, from this point of view, will escape our analysis for a long time to come. But, apart from this highest of all problems, so much remains to be done, that the study of the cytoplasm outside of the nucleus well deserves to claim the interest of a great number of investigators.

#### Fourth Lecture

THE IDEALS OF AN EXPERIMENT GARDEN<sup>1</sup>

THE future of the human race depends, in a large measure, on the improvement of our food-stuffs. Our present crops do not produce what they might if better cared for, nor what they will obviously be required to do a relatively short time hence. The population of the whole world is rapidly increasing—far more rapidly than the production of even the first material necessities of life. Ten years ago Sir William Crookes pointed out that the production of wheat, although regularly increasing, gradually falls back when compared with the fast-growing demand of the continually augmenting population of the earth. After half a century, perhaps earlier, it will not be possible to supply the necessary food with our present agricultural plants and our now prevailing methods of culture. Life will become difficult, the struggle for life will become more and more intense.

Of late, Herbert J. Webber, in a strongly convincing article, has emphasized the same argument. Although there is no immediate concern, since the world will comfortably support a much larger population, the future of our race is in obvious danger from the wasteful methods now employed in the utilization of the world's resources. Forests and mineral deposits are slowly disappearing. The easily available coal-beds are almost exhausted; more coal will have to be sought for at far greater depths, or perhaps under the bottom of the ocean. All sources of supply are rapidly diminishing. Of all these, the main ones are our agricul-

<sup>&</sup>lt;sup>1</sup> A popular lecture, illustrated with many lantern slides, delivered under the auspices of the Rice Institute, at the Majestic Theater, Houston, as a part of the programme of the opening festival.

tural plants, and these, fortunately, are capable of a high degree of improvement which may enable them to answer to the increasing demands for a very long time.

In the United States one half of the soil is now being cultivated, and only half of this is really improved land. It is true that the aridity of the other parts and the difficulty of irrigation are the main causes of this condition; but if we could produce a sufficient number of varieties that would yield paying crops in the arid regions, a most desirable progress would be secured.

Considerations of this kind are going far to place the improvement of agricultural plants in the foreground of public interest. The aims and methods of this improvement are, however, twofold. The first necessity is to make better yielding or more resistant or specially adapted races out of our present crops, and to increase their production by this means, year by year. This is the task of the agricultural experiment stations, and good work is being done on a large scale in this line all over the cultivated world.

This plant improvement is based on two generally recognized principles. One is the selection of elementary species, and the other is hybridizing. All of the great crops, such as corn, cotton, tobacco, wheat, rice, and almost all others, consist of impure races, of mixtures of better and minor varieties, of sharply distinguishable forms, some of which are far more promising than others as to yield, degree of resistance, adaptation to different soils and climates, etc. Selection consists in the choice and isolation of such types, in the estimation of their worth under given conditions, and in the multiplying of their seed in order to produce new and valuable commercial races.

The elementary forms are very numerous in almost all of the old races, often coming up to a hundred and more.

But it is obvious that their number is limited, and that as soon as all of the best ones have been isolated, this source of improvement must become exhausted.

Exactly the same thing is the case with hybridizing. It consists in the combination of the valuable characters of different varieties and species into one and the same plant. If properly conducted, constant commercial races may be derived from such crosses, and some of our best wheats, many kinds of grapes, and a large number of other agricultural plants owe their origin to the application of this principle.

But it is evident that here also, some day, a limit must be reached. It may last fifty years, perhaps a century or more, but in the end all or almost all of the valuable hybrids which can be made will have been produced. Nothing really new is acquired: it is only new combinations of given qualitics, and to such work there must always come an end.

Resuming this discussion, we may say that selection and hybridizing, which are our present means of improving agricultural plants, are both, from their very nature, limited methods. They find their ultimate barrier in the given qualities of the existing races: on the one hand, in the limited number of useful types in their mixtures; on the other, in the limited number of possible combinations of the now existing qualities in allied forms.

Or, in one word, they are both based on the principle of exhausting the present possibilities.

What are our descendants to do when the end has been reached? The demands of the population will in all probability go on increasing rapidly, and the agriculturist will find it impossible to keep pace with them. No doubt most of you will answer that the present work of improvement may last for one or two centuries, or even more, and that therefore the question is of no immediate concern.

Doubtless this is true so far as practice goes. Practice may leave this side of the problem to science, but the duty of science is to foresee the coming necessities and to develop its methods in such a way as to be able to provide the means of answering them as soon as they are felt in practical life.

Opposed to the necessities of the present time, which only ask for the exhausting of present possibilities, science has to provide the means of answering to the future necessities of mankind, or, shortly, to prepare future possibilities.

Here we must direct our eyes to the instruction which nature is giving us. Plants and animals have not always been the same on this earth. From the lowest forms they have gradually developed to their present high condition. We must try to learn to imitate the work of nature on this line of improvement. It is definitely without limits. It has proceeded without interruption through millions of years, and has not yet come to rest. We must study the laws of this great process in order to find the means of guiding it in the directions which will answer the future demands of humanity. By doing so we may discover how to produce still more useful races when selection and hybridizing shall have been exhausted.

Such are, to my mind, the high ideals of a scientific experiment garden; and it is among the duties of our universities and scientific institutes to erect and maintain such gardens for the enlargement of our present field of knowledge and the future benefit of practical agriculture and industry.

In producing new forms, nature proceeds by small leaps and bounds, or mutations, as they are now usually called. Many of these, of course, are required in order to produce something strikingly new; but a few, and even often only one of them, may be sufficient to secure an appreciable amelioration in our agricultural crops. Therefore we wish

to know, in the first place, how these leaps and bounds are being produced, and, in the second instance, how we may govern and guide them.

Thus the first aim of the work is only to repeat these mutations. Such a repetition will unveil to us the laws of the process and be our guide in all future work. This first step has now been made, although provisionally only and in a small number of cases. But in all such lines of work the beginning is the most difficult step, and so we may confidently hope that other steps will follow, and that the end may be reached before the time when practical life will urgently ask for new methods of producing the necessary supply of foodstuffs.

For about twenty-five years these ideals have guided the work in the experimental garden of the University of Amsterdam. I shall now try to show you, by means of lantern slides, first, the arrangement of this garden and its different tools for the work; secondly, those plants in which I have succeeded in repeating such mutations as they themselves, or their nearest allies, are known to produce in nature; thirdly, some others which are among the next to be tried in this respect; and, in the last place, a group of plants which produce mutations every year and in a relatively large number, thereby supplying us with most desirable material for continued study in this most interesting field of scientific inquiry.

HUGO DE VRIES.

## PHILOSOPHICAL LANDMARKS

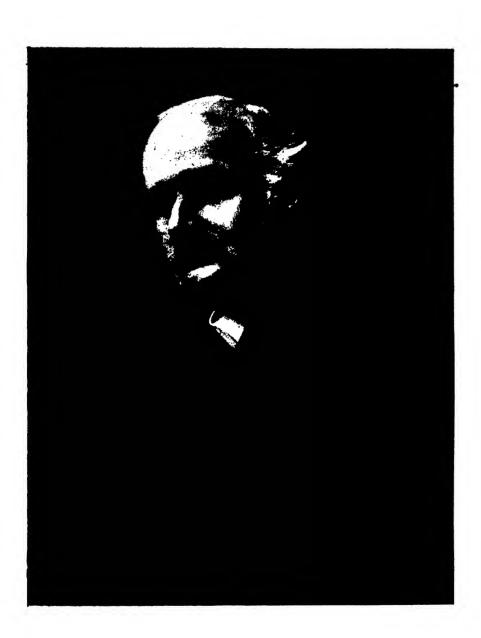
# BEING A SURVEY OF THE RECENT GAINS AND THE PRESENT PROBLEMS OF REFLECTIVE THOUGHT1

#### First Lecture

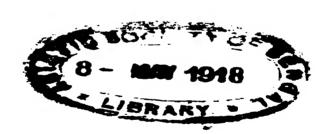
THEN John Milton wrote the "Areopagitica" and predicted the future greatness of the English people, that people had staked its life upon its liberty, and was in danger of losing it. It was in the midst of the unspeakable disasters of civil war. During the centuries which have succeeded Milton's day the English nation has never ceased to struggle against obstructions without and obstacles within. It is a nation tried to its uttermost. But, on the whole, and to an extent which is rare in human affairs, its history has verified the vision of the poet. Its prosperity in all matters of lasting worth has been very great. It has borne well the weight of its responsibilities, and, in spite of imperfections, it has so fulfilled its mission to mankind that though England, like Israel, Greece, and Rome, were now to perish, it would, like them, remain for the human race a precious possession forever.

It may be profitable for you, whose nationality has also "been welded not in peace but in the storm of battle," to inquire what was the ground of the poet's assured confidence in his country. What evidence lay there and then before him which would justify his trust in the destiny of his people? In its circumstances there was none, for these were

<sup>&</sup>lt;sup>1</sup> Three lectures presented at the inauguration of the Rice Institute, by Sir Henry Jones, Professor of Moral Philosophy in the University of Glasgow, and Hibbert Lecturer on Metaphysics at Manchester College, Oxford.



they fores



untoward to the last degree. It had neither wealth of material resources, nor greatness of population, nor weight of armaments, nor vast extent of territory. It was a small and a poor people, without great traditions or high rank among the nations, and inhabiting a portion of a little island. Yet, with rarely paralleled political pride, Milton called upon "the Lords and Commons of England to consider what Nation it was whereof they were, and whereof they were governors," so that they might match the greatness of their trust. In doing so, he referred solely to the intrinsic character of the people, and indeed to one element therein. He found them "a nation pliant and prone to knowledge." They "prized the liberty to know, to utter and to argue freely according to conscience, above all liberties." It was only on this ground that the nation seemed to the poet to be "like an eagle renewing her mighty youth." In his sight she was first among the nations of his time, because she was first in her love of truth; therefore was "she destined to be great and honourable in these later ages."

From one point of view we may say that there was nothing new in Milton's attitude. The truth to which he gave such stately expression is, in fact, a truism. It is as old as man's first reflection upon his own destiny. Homer teaches it when he makes the Greeks advance to battle in ordered and silent ranks, under wise commanders inspired by Athena, while the Trojans stream out in a confused and shouting mob, driven forward by Ares, the god who is the embodiment of animal ferocity and passion. This is the conviction of the wise in "all generations": that if there be any law in human affairs or any continuity in their confused history, it is that which dwells in man's own soul and secures the victory of the ordering intelligence and the disciplined will over the blind forces that operate in his world.

But from another point of view the attitude of Milton may be called unique and even surprising. Stern moralist as he was, and a spirit which was devoted to the service of the Highest, we should have expected him to dwell first upon the ethical or the religious conditions of a nation's welfare. But it is its "proneness and pliancy to knowledge," and the store it set upon the liberty to know, to which he assigns the highest value and the first importance.

Had he lived in our day, we should have reduced the significance of his mission and called him an "intellectualist"; for we are prone to prize faith in some domains, and practice in others, above knowledge, and to regard "truth" as mere means to a further good. I believe, however, that Milton spoke well and wisely. "The liberty to know" is in fact greater than all other liberties; for it is their condition. Man cannot enter into his inheritance, whether that inheritance be natural or spiritual, except through this door. As the beauty of the natural scene is there only to the seeing eye, so the utilities of Nature's forces and the treasury of her resources are open only to him who can comprehend them; and the obligations which are also the opportunities of man's moral achievement exist only for him who adopts them as the convictions of his own mind and the purposes of his own Efficient practice, whether on the minutest or on the will. widest scale, rests upon clear and relevant knowledge. It is as necessary to the artisan in handling his tools as it is to a statesman guiding the affairs of a nation. The fact which is not comprehended is an outer necessity which limits man's freedom, frustrating his intelligence and obstructing his will. The discoveries and inventions of modern science in all their wide range, and man's whole progress in civilization, bear witness to this truth: it is the intelligence of man which alone

can emancipate him. His charter of freedom is inscribed in her own soul.

Now it is the main characteristic of our time that it has, at least in one great department, laid this lesson well to heart. We consider no labor too severe or continued, no equipment too costly, which promises, by means of the natural sciences, to secure more intimate communion between the reason of man and the reason which is embedded in the physical order It is only in this way that we can bring its powers to our will. We have learned that the iron-hearted mechanism of nature, which were it not for man's rational endowment would entangle him in its vast scheme, can by means of his understanding of it be changed into the rich possession of his mind and the instrument of his will. Its unchangeable and inexorable laws, seized by way of their meaning, are made to minister to his purposes and to express his spontaneity. means of knowledge man stands a sovereign among the natural powers, and he is free, not in their despite, but by their help, for they enlarge the scope of his effective will.

This, indeed, is the ultimate and by far the most significant consequence of man's intelligent converse with the outer world, the greatest of all the gifts of the natural sciences to mankind. But it is not that which has attracted our attention. As a rule, we trace the influence of the theoretical discoveries of science no further than the practical inventions in which they result; and if we discern, we do not reflectively consider, the manner in which they recoil upon man himself. The achievement upon which in this age we justly pride our selves is the interpretation of Nature's laws, and our consequent sway over her energies. We seek little more, and we look no further, as a rule. We forget that it is the indirect, the remote, the unexpected and unsought consequences of man's actions which mean most. It is a law of his life, and a

symbol of the generosity of the scheme within which he lives, that he always builds more wisely than he knows. He is guided unconsciously as by an architectonic mind, which comprehends him and his environment, and whose purposes he cannot guess until he beholds them accomplished.

It is my purpose to call your attention to this aspect of the scientific enterprise which you are so auspiciously inaugurating here to-day. I would fain indicate the manner in which the natural sciences, for which you are making your most generous provision, must not only extend your mastery over the outer world, but reverberate within your inner selves, enriching and enlarging the powers of your rational nature.

When man's thought sets free the forces of the open world, these take up his deeds and carry them forward to issues which he cannot clearly foresee, and yet which he dare not leave unconsidered. For these also yield their best gifts only to the spirit which can at once obey and control them; and neither the obedience nor the control is possible except in the measure in which they are comprehended.

This consequence is seen to follow the moment we discern what takes place when man acquires knowledge of any object. It is that the nature of mind is itself exhibited in the process. He cannot enter into closer communion with the natural world by means of the sciences without at the same time both manifesting and realizing the powers of his own soul. Mind, like every other form of energy, natural and spiritual, shows what it is in what it does. It exhibits itself in its operations. It is by matching his intellectual power against the world and forcing its obdurate facts to yield their meaning that he reveals the splendor of his rational endowment. Could we have known the potencies which slumber within him, if we could have known his mind and his ways of life when the phenomena of nature, instead of being open to

his thought and subservient to his will, were nothing more than objects of fear and wonder? Or is it not true, rather, that the process by which he has gradually withdrawn the veil from the face of nature and brought to light order among its contingencies, is the same in its other and great aspect as the process of the self-revelation of his own spirit? For knowledge comes neither from mind nor from its object. but from both. It is neither a posteriori nor a priori, because it is both the one and the other, and that always. Truth is neither unveiled by man, nor is it given to him ready-made. It is, in every item of it, the result of the interaction of mind and its object. Light springs from the impact of spirit and nature. Nay, as we shall see more fully hereafter, these imply each other, they are elements in one scheme, opposed but complementary aspects of the one reality. And it is only in their unity that they have significance, value, or use.

I do not anticipate any contradiction when I say that the greatest and by far the most significant of all the consequences of man's triumphant progress in his comprehension of the physical cosmos is the light which that process has thrown upon man himself. But its full meaning can be seen only when we consider another and a still remoter conscquence. Man's more intimate communion with nature by means of natural science has brought him into closer communion with his fellows. Seeking no such end, the sciences have made men, throughout the civilized world, members of one another. They have broken down man's isolation, refuted his egoism even when it leaves him selfish, made him independent whether they will or no, welded their interests together, and constituted them into organs of a vast whole to which they give and from which they borrow all the elements of their larger life. Within it they find their in-

dividual functions; and, seeking their own ends, they nevertheless constitute a vast, complex, and single whole whose elements collaborate even when they conflict, and whose power for all human purposes no man can measure.

The first revelation of the potencies which slumbered in man's spirit was made when the reason within him succeeded in holding rational communion with the reason that is embedded in the physical cosmos. But this second revelation is greater. We can see his powers in the fullness of their might when he is thus united in one scheme with his fellows, and spirit communes face to face with spirit. Then is the range of his personality in truth extended, and the reach of his mind and will. The blacksmith at his forge, like the thinker in his study, is seen to serve and to be served by the interchanging enterprises of the general mind of his times. For it is no flight of rhetoric, but the simple truth, to say that our interests now are cosmopolitan. This is illustrated in the common ways of our daily life: in the food we eat, the clothes we wear, and the tools we use. The same change which has passed over the face of nature has passed over the spirit of man. Science is translating facts into instances of universal laws. It is tearing facts out of their seeming isolation. It is revealing them as temporary resting-places of unresting energies, momentary combinations of forces which have come from the beginning of things and are moving onward on an endless way. Nature is no longer an aggregate of disconnected facts, or the scene of contingent happenings. It is the realm of concrete universal laws. These have not supplanted the facts, it is true, nor arrested the happenings; but they have illumined them, showing that they are the mere foci of the world's unresting energies.

But the universal in nature is at once the offspring and the parent of the universal in man; so that he too, by the indirect

influence of the sciences, is being reinterpreted and regenerated. Man remains, it is true, and must remain, a unique personality. To the end he will maintain his subjective integrity and inviolable privacy; he will look upon the wide world through his own most individual thought, and act upon it from the secret depths of his own most exclusive will. But the thought and the will which are his own and exclusive are" capable of a wide comprehension. He is also being revealed as an individuated organ of a vast whole. He is the intense because the self-conscious focus of the meaning and the use of the world. He is a pulse-throb of a universal mind which sustains the natural order, and operates in him, through him, by him, and, I believe, for him. And this discovery, it seems to me, is the crowning achievement of the modern age. Its interest in the meaning of the outer world, and the consequent conversion of its forces into man's ministrants, have, without man's knowledge or purposed secking, begun the integration of humanity, and set it forth on an adventure more generous in its promise than he can compass by his freest thoughts.

Now it has seemed to me that if a votary of philosophy has any mission among you to-day, it is to invite your attention for a little to this vaster and remoter realm of the consequences of devoting your thoughts in this institution to the discovery of nature's secrets. For every truth attained breaks out into a new problem demanding a new solution; every practical achievement brings into it a new task; and every goal of spirit is a point of departure on new adventures. And it is the peculiar task of philosophy to suggest to the minds of men the regions not yet conquered and the inheritance not yet gained and secured.

The main outlines of our next adventure are becoming obvious. It is to comprehend the laws according to which

this new world of the interconnected wills of men must operate. The demand for knowledge—for knowledge that is systematic, tried, and secure—of this world of man is already felt to be urgent in some directions. I presume that there is no maker or seller of material things among you who does not know that if he is to secure his own economical well-being, he must know something of the world's mind and be able to interpret and anticipate its wants. This problem is infinitely more complex, and the risks of error are incalculably greater than they were when human society consisted of small, isolated, simple, self-centered and self-supporting units. His success or his failure in his business enterprises comes upon him from the ends of the world, and he must widen the range of his purposes.

But what applies to the economic phase of our modern life applies in like manner to all its elements. Control can come only by the way of comprehension, and forces which we do not understand are inexhaustible sources of risks and surprises. And who comprehends the social forces of these times? All the civilized nations of the world exhibit the same phenomena. We have emancipated the people; we have awakened their sense of their rights; we have multiplied their wants and extended the range of their desires; and, in one word, we have ushered in what we can hardly do more than name and fear-namely, Democracy. It is a thing which is to be its own law; it is to walk in the light of its own convictions; it is to map out the lines of its own welfare; it is to repudiate every authority, political, moral, or religious, which wears a despotic face; it must issue its own imperatives, and every appeal is to itself alone.

The greatest discovery ever made by man was made by the Greeks when, cutting themselves free from the traditions of the ancient world, they alighted upon the conception of a

civil state where citizens should be free. The most momentous experiment of mankind is that of carrying out their conception to its ultimate consequence in a true democracy. But that experiment, conducted among the elemental powers of man's world and involving all the major issues of his welfare, is carried on in the bewildering twilight of mere opin-First, appearances are taken for facts; there is little inquiry, and there is less logic or method. The democracies of the world, guided by no prophetic seer and possessing little light of their own, are stumbling along an untried and unknown way to an unimagined goal. They are convinced of their illusions only by suffering their consequences, and they discover the truth only by exhausting the possibilities of error. It is a costly method and an insecure one. Universal unrest verging constantly toward conflict characterizes all their ways.

I do not think that we can trust this method much longer. The need for self-comprehension is becoming urgent. The risks of ignoring the problems of the general life of man are growing greater as the democracies wax in magnitude and strength, assert themselves with less and less reserve, and are less and less patient of restraint. And, moreover, a fundamental discrepancy has arisen between the inner or selfconscious life of recent times and its outward circumstances. Man's knowledge and control of himself have fallen out of step with his knowledge and control of his physical environment. In the case of the latter the boundaries of the nations are overleapt and the exclusiveness of their individualism is multiplied. Scientific knowledge and inventions and the vast economic resources which issue from them are objects of cosmopolitan interchange. But our ethical temperament has received no such enlargement or emancipation, and is still narrow and class-tainted and parochial. And this dis-

crepancy will bring its penalties. Have you ever known any instance of incongruence between the inner and outer conditions of a nation's life which has not been fraught with peril? It is this cause which divides a nation against itself and constrains it to have recourse to the violent remedy of revolution. A reinterpreted world is a reconstructed world. It propounds new problems for man. And they are like the riddles of the Sphinx: they must be answered on pain of death; they have no answer except Man himself.

Surveying the modern situation as a whole, what is it, then, that we see? It is the vast extent of the domain which the physical sciences have conquered within so brief a period of the history of the human race that it seems but the hour of the dawn; the great army of explorers in every civilized land, equipped with every instrument which can aid their search, who are year by year and almost day by day pressing its boundaries further: the growing marvel of the practical inventions which follow hard upon the theoretic discoveries; the utilities, latent from the beginning of time in the structure of the physical world, which these inventions are setting free; and, on the other hand, the inexhaustible variety and unconfined range of man's wants and desires which all these things have called into existence, and which are clamorous for satisfaction; the complex, restless, tumultuous, and yet unruled world of industry and commerce which has been welded together and is designed to meet these wants; the consequent integration of mankind into organized communities; the rise of the great order of national, political states which are themselves but organs of a still wider humanity, all of them from time to time disturbed and occasionally well-nigh distraught by the economic and social collisions of their elements. Such are the results which we must attribute mainly to the devotion and the triumphant progress of modern-

science. Guided and inspired by them, the multitudinous activities of individual minds and wills, each of them permanently set upon its own personal ends, have put together a vast social structure with almost as little conscious purpose as that which guides the coral insects building their reefs amid the ocean's waves. That structure has its own laws of being and ways of operating, and these are as remorseless as the laws of the physical cosmos. But I believe that they are as beneficent, too, provided they are understood. then, can we doubt that man must fit himself for this new world which he has called into being, or that in order to do so he must go forth on a new adventure? It is not only that of comprehending the physical world and employing its energies, but of comprehending the master-power which is the cause of the great change. Side by side with the sciences of Nature, the sciences of man must arise. Man must come back to himself, contemplating the mystery of his own spirit, for in it is the key of the final enigma of the world.

But this is the specific venture of Philosophy, and Philosophy has fallen into disrepute. So scanty has been the harvest of her long toil, as compared with that which the natural sciences have brought triumphantly home, that the general mind of the modern age would turn away from her. Philosophy, the mother of all the sciences, has now to plead, and even at times to plead in vain, for permission to erect a humble lodge among the mansions of her daughters. We would prize her gifts beyond all others, could she but bring them within our reach. But we despair of her powers. Even the incomplete, tentative, errant, but slowly progressive interpretation which man alone can give of any object, seems to be impossible for us when our problem is Man. An obstacle lies across the very threshold of this, the most urgent as well

as the greatest of man's spiritual enterprises: it is his diffidence when face to face with the mystery of his own being.

And, in truth, the mystery is very great. Even his physical structure is revealed by science to be the consummation and the most complex epitome of the cosmic scheme, and all its problems converge in him. And his soul, his mind, his spirit is the self-conscious counterpart of all his world. He is its expression, in him brute force emerges into meaning, and its reality takes upon itself the form of truth. The complexity of the problem is infinite, and the consciousness of its magnitude paralyzes the inquiry of philosophy.

Moreover, when we are dealing with spirit and its manifestations in any one of the arts or sciences, or in the most complex social world in which all these are sustained, the method which has been so successful in the investigation of the facts of the outer world cannot be employed, except at the greatest risk and under constant correction. The natural sciences can, without much violence to their object, distinguish and even isolate its aspects and deal with them separately. But when we leave the physical sphere, where relations are relatively external and contingent, and ascend stage by stage along the internal relations of organic life to the intense unity of self-consciousness, in which all differences are at once sustained and overcome, abstraction becomes more and more misleading. There every element depends for its being, function, and meaning upon the whole system of which it is a part. The problem of the whole comes upon us everywhere, and it seems impossible to attain any truth without grasping it in its totality.

It follows that philosophy has no more right to be abstract than a work of art, or to be fragmentary than religious faith. Even the pragmatist, whose main mission seems to be to maintain that the world is, at least in part, the

playground of contingencies, must make the apparently preposterous claim of pronouncing upon its final nature and grasping it as a whole. He also is "a spectator of all time and all existence," and its condemning judge.

And it follows that even as an outline the philosopher's version of the universe of reality must fail, and fail in every way. Its principles are mere hypotheses, and nothing is fully demonstrated. The application of the hypotheses to facts is incomplete on every side; they retain their secrets, remain enigmatic, and they seem to conflict with one another and with the system as a whole. And the failure of philosophy, which we might well prognosticate from the magnitude of its task, seems to be more than indorsed by its troubled and apparently futile history. We are driven to think that the enterprise exceeds our powers, that there is no resource in reason, and that the philosopher must take his seat among humble men, and say, like them,

I stretch lame hands of faith and grope, And gather dust and chaff, and call To what I feel is I.ord of all, And faintly trust the larger hope.

And man cannot set aside the enigma. He must persist in the attempt. But the question arises, Why do men persist in the attempt? And the wisest of men, why do they not turn aside from the vast inquiry and "cultivate their gardens"? Can it be that it is impossible for them to do so without violating their own rational nature? Is there some necessity either in man himself, or in the nature of things, or in both, which he cannot escape, but which constrains him to confront the mystery? Can he not take refuge in his own limitations?

Just when we are safest, there's a sunset-touch, A fancy from a flower-bell, some one's death, A chorus-ending from Euripides,—
And that's enough for fifty hopes and fears, As old and new at once as Nature's self,
To rap and knock and enter in our soul,
Take hands and dance there, a fantastic ring,
Round the ancient idol, on his base again,—
The grand Perhaps!

This fact, sustained by the experience of mankind always and in all ages when it is at its best, sustained by its despair no less than by its hopes, by its agnosticism and skepticism no less than by its faith, leads us to look again at the adventure of philosophy and its assumed failure. What does it mean?

In the first place, it throws a fresh light upon the nature of man. It shows that he cannot escape the sense of his infinite environment. To shut it out of his mind were to rend his own spirit in twain, for it enters within. The infinite is part of the furniture of his soul. He is like a dweller on a little island in the midst of the open ocean, everywhere within the sound of the thunder of the breakers. If he endeavors to satisfy himself with a narrow scheme of life, he finds that he is at war both with himself and with the nature of things. He may seek satisfaction, as Carlyle and many others have advised, by lowering his demands and limiting his outlook. His first crude expositions of himself reveal within nothing but animal wants on a large scale, and he may neither see nor desire to find in the world around anything except that which promises to stay their hunger. But reflection enters if the process of his own rational life is not arrested within him, and reflection breaks down his com-

placency and dispels the fake show of first appearances. His spirit is launched forth on its endless task.

And this is philosophy. It is not the quaint guest of starstruck souls which have forgotten their finitude and are dooned to range along the horizon of existence, peering into the darkness beyond and asking questions of its emptiness. Philosophy is the process whereby man, driven by the necessities of his rational nature, corrects the abstractions of his first sense-steeped experience, and endeavors, little by little, to bring to light and power the real—that is, the spiritual meaning of his structure and of the world in which he lives. I cannot believe in a destiny so cruel as to condemn man to seek and to return home empty. I even venture to say that the quest is never vain.

It is true that philosophy does not reach its goal, if that goal is a full and flawless and final scheme. But is it? Which of the enterprises of the human spirit either has, or ought to have, such a consummation? Not the sciences, not any one of the arts, not any form of man's practical activities. There is, with regard to every aim which he has sought to attain, the same incompleteness, imperfection, and lack of finality, and the same ground for skepticism to seize upon and condemn it.

But, in the next place, the skepticism which distrusts philosophy is itself philosophy, and a philosophy which has not been careful to examine its own assumptions. Let me indicate a few of these as we pass on our way.

In the first place, it is evident that skepticism cannot condemn except by reference to a standard or criterion, and that standard must itself be capable of justification, whether through carrying it within itself or as a means to that which does so. It must itself, in fact, assume an Absolute, and a knowledge of it. That which pretends to be true, even

though it be negative, bears within it a reference to a final end, and in its own place and context to embody it. Hence skepticism cannot condemn a conception which it must assume and use in its condemnation.

In the second place, the criterion set up by skepticism is not valid. Skepticism places a static goal for a nature which is through and through dynamic. It demands that mind should come to rest in a knowledge that is final. But selfconsciousness is a process. To arrest its activity is to extinguish it. It is active no less in possessing than it is in achieving knowledge. For knowledge or goodness to be, is to be in process of being maintained by the active powers of the intelligence and will: in other words, the moment that men cease to think and to will, these cease to exist. They are in process of being continually produced. The whole world of mind, like the physical cosmos, is the scene of the play of energies which never rest. Its existence is its becoming: it continues through continuous regeneration, and is ever new as well as always old. Both beginnings and endings are fictions. Man's mind lives and moves within a self-inclosed system for which to be is to change, and probably also to evolve, radiating forever into new splendors. And for man to live as spirit is to partake in the process. It is in some other world than that of man's experience that the skeptic should seek a reality that is fixed or a perfection that is static.

In the third place, skepticism has not only assumed for mind an end which contradicts its nature, and is on that account alone irrational as well as impossible: it has also misconstrued the process of knowing. It is represented as selfdefeating. Instead of revealing the nature of things as they are, it exhibits them only in their relation to man's means of knowing them, or as they are reflected in the medium of his

consciousness. This is held to distort them; so that in strictness man does not know real things, but phenomenal objects. Mind cannot get into actual touch with reality. It is shut up within a world of appearances; consciousness can deal with its own contents and see only the pictures on its own walls. And, further, every attempt which philosophy has ever made to establish a relation between ideas and facts, or phenomena and real objects, has failed. And its failure is necessary and inevitable, for it is manifestly impossible for reason to establish any relation between what is and what cannot be in consciousness. This suspicion of thought, "this disease of subjectivity," has penetrated deeply into the modern mind, and skepticism has assumed many forms. It is at times the positivism which affirms necessary ignorance of final causes; it is at others an agnosticism which endeavors to stop short of both affirmation and negation; it is at other times an intuitionism which on occasions and for rare moments comes into touch with reality in a way that is inexplicable and miraculous; it is at other times a dogmatism of either the intelligence or of the will that is a resolve to affirm when we cannot know, a pragmatism or a pluralism. In all cases it relegates those things which man most desires to know into a region which lies beyond the reach of his intelligence, or it attributes to subconsciousness, or to mere feeling, or to mysticism and intuition, what it denies to the use of man's rational faculties.

To deal with these skeptical assumptions with any fullness lies beyond my immediate purpose. But we may observe in passing, what is obvious, namely, that the skeptic cannot condemn all human knowledge without condemning his own. His pronouncement on the nature of mind, the relativity of its processes, the phenomenal character of its objects, the

unknowable nature of reality, must share the fate of all other knowledge.

He must choose between denying the validity of all knowledge and affirming his own, and in both cases alike his conclusion is self-contradictory.

But, in the next place, his attitude is exposed to other ways of refutation than that of a mere argumentum ad hominem or a tu quoque. The skeptic converts the condition which is necessary to knowledge against the possibility of knowledge, as if that which constitutes it could also destroy it. No doubt knowledge is relative; that is to say, it depends upon the nature of mind as well as upon the nature of things. But is its relativity a defect? What would the skeptic have? Is it a mind which has no affinity with the world of objects, or a world which is divorced from, and independent of, the intelligence? The relation of things to mind and of mind to things may be an indication of the fundamental character of Indeed, there is no attribute of the real so indisputable as that by which it interacts with mind, and through and by and only during that interaction exhibits and even realizes its fullness of being. Knowledge, or rather knowing -- for there is no such ambiguous reality as "a world of knowledge" supposed to intervene between consciousness and the facts with which it deals—is the interaction of mind and things, and a living intercourse. And that intercourse is direct and immediate even when we form erroneous opinions. Error is the pathological activity of undeveloped minds. We borrow the whole contents of our intelligence from the world in which we live, even our illusions, and we can create neither truth nor falsehood out of the emptiness of an isolated and self-closed mind. On the other hand, the world owes to reason alone the evidence of its existence and the expression of its order and meaning. But we recognize

neither that which we borrow nor that which we lend, and we speak of parts of our knowledge as a priori and of parts as a posteriori, as if some truths were fabricated by ourselves without the aid of the world, and others were emitted by the world without the use of mind. Knowing is a joint enterprise in which both are involved.

There is, perhaps, no phenomenon of modern thought which demands a closer diagnosis than this "disease of subjectivity," which is not only a cause of the distrust of philosophy, but which would paralyze the enterprise of reason in all other directions, if in our practice, which is wider than our theories, we did not set it at naught. It seems to me to rest, in the last resort, like all the forms of modern skepticism, upon unjustifiable dualisms. For we have been separating when we ought only to have distinguished, and converting differences into contradictions. And, on the other hand, we have been assuming that to reconcile differences is to remove them, leaving nothing but flat and stale sameness. We have not distinguished between sameness and identity, nor realized that identity can—and, I believe, must—express itself in change and maintain itself thereby.

The assumptions arise from the fact that we naturally carry over into our philosophical research the conceptions which we have found useful in our physical inquiries, and endeavor to interpret the phenomena of mind in the same way as objects in the outer world. As in space every part excludes every other, and its continuity allows no diversity: thus only, it is presupposed, can the reality of all objects, including minds, be maintained. They must, we assume, be kept in isolation. Their relations to one another must be treated as contingent addenda: things into which they may enten and out of which they may live again, without any change in their real being. To be real, they exclude one an-

other. Interpenetration, the being of one object through and by reason of the being of other objects, is held not to constitute but to destroy. The finite and the infinite must stand apart. The will of man, if it is to be free—that is, if it is to be a will—must shut out the world. The subject must have only a negative attitude to objects; nature and spirit, mind and matter, must be absolute opposites.

When I endeavor to catch a glimpse of the trend of the thought of the present times, and to define, however generally, the problems in which it finds itself entangled and which it must try to solve. I find that it is occupied with some one or other of these dualisms. The tissue of reality has been torn asunder; and if there be any movement which above all others is indicative of the special mission of the times which are coming, and are already at the door, it is that of healing the rent and of finally refuting all notions of the primacy either of the whole over its elements, or of the elements over the whole. We must find room for the freedom of both mind and the world in knowledge; for both spiritual freedom and natural necessity in our practice; for both God and man in religion; for both individualism and socialism in our politics; for both the one and the many, the universal and the particular, everywhere; and we must view them as interpenetrable; for there is but one reality, and without its cooperation with its elements nothing exists or happens.

#### Second Lecture

E concluded the last lecture by showing that both in our thoughts and in our actions we first distinguish and then tend to sunder the contents of reality: our thoughts are always to some degree abstract and our practical pur poses one-sided. Reality, even at its simplest, has more aspects than we can either recognize or use: it takes all the sciences, each of them taking up its own set of relations, to explain the qualities of a lump of iron ore; and most, if not all of our industries to extract its uses. All thoughts and all ends are abstract.

But, among the conditions under which man lives, we must return as one of the most beneficent that he cannot be satisfied with abstractions. Both his own nature and the nature of things conspire together to secure him against narrowing the interests of his life. The reflected elements of reality press for recognition; and the elements which are recognized refuse to yield either their truth or their use, except in their context. They even refute themselves: one-sided truths become mistading errors, and one-sided purposes refuse to work. They call forth their opposites, and demand to be complemented and corrected by them and harmonized with them. The world resists being shredded into parts, and persistently maintains its concrete totality.

On the other hand, man's own nature also constrains him to move and to cooperate with the trend toward unity. Abstract experience is a mind divided against itself: it cannot stand. Man must either widen his outlook and extend the range of his purposes in response to the call of circumstance, or else do violence to his own rational nature by be-

coming the bondsman of habit and an automaton. And in either case he makes for some kind of completeness-either the completeness which shuts out or that which lavs hands upon and utilizes the environment; and the process of experience always changes him. The final effect of the deeds of his intelligence and will does not lie in the truth attained, or in the purpose realized, but in the recoil of these deeds upon himself. He rises from his acts either with hardened habits and strengthened prejudices, or else with a mind enriched with new ideas and a more effective will. Nor by any means can he return to his past. Strictly speaking, spirit has no past: for it always incorporates it with the present. Man gathers his experience into himself; carries it along with him, as an element in his mental structure, assimilated by his living personality. He can sometimes unravel his past out of his present by conscious memory directly demonstrating its presence within him; and even if he cannot give this direct proof of the existence of the past in the present, he gives indirect evidence of it either in the automatization of his life and the fixity and reiteration of his mental operations, or else in the added skill and compass of his thoughts and purposes. This arrestment of the past and its conversion into a living element in the moving life is the mark and marvel of the rational nature of man, distinguishing him above all other things from other beings, as the condition of his progress.

Moreover, it is in this way that he maintains his personal identity. For that consists not in any immutable sameness such as we attribute, rightly or wrongly, to material existence. The self-extenuating space, the succession of the contents of time, each supplanting its predecessor, must be overcome and its flow arrested if personal identity is maintained. And this is not possible except by the activity of a

self-consciousness which retains the past by waking it into the present. Even the sameness or permanence of the outer order implies, as Kant has shown, the reintegrating activity of self-consciousness. Reason in man thus becomes ever more concrete, systematizes ever more fully both its own life-content and its outer world. Its war with abstractions is perpetual: to lay down its arms is to yield its life.

It is not a defect of human reason that it must reach the concrete by way of abstractions: it is its nature. Error does not consist in merely entertaining abstractions, but in treating the abstractions as representative of the concrete whole of reality. It arises when man endeavors to fix the abstractions, or to employ them as final characterizations of reality. There is a true sense in which human knowledge may be said to begin with the particular and the simple, and to make its way toward the universal and concrete-to start from "the Many" and to seek "the One." But there is also a true sense in which knowledge may be said to begin with the indefinite "an undistinguished continuum," and to proceed to articulate and define its contents—to start from "the One" and to seek "the Many." From the first point of view, our experience is at first a sensuous manifold which has to be connected first into perceptions, then into conceptions, and finally into the organic and hyperorganic ideas of reason. And, pari passu, the object of experience, nature, at first appears to be the scene of disconnected happenings and to be a loose aggregate of unrelated facts, and eventually to appear as a universal cosmos. From the second point of view. our experience is at first a confused mass of sensations pressing into us through the pores of sense, and perceptions arise by distinguishing and articulating. And the object of experience, the world, changes its character in a corresponding way. Now error arises when either of these views is

adopted against the other, or as the whole truth, and made the basis of a philosophical account of the real. And that it is an error is shown by the necessity of correcting the original hypothesis by means of its opposite. For whichever presupposition we assume at the beginning is nothing but a startingpoint from which its complementary opposite must be reached. If the pluralist begins with the Many, particulars he must confessedly synthesize and unite; if the absolutist begins with the One, the indefinite whole he must analyze and articulate. Philosophers may differ as to the nature of reality, and their doctrines may range between an absolutism or pantheism that engulfs the many and deletes all differences, and a pluralism or monadism. It is true that neither on the side of its difference nor of its unity is human knowledge complete—that is to say, the distinctions which are made are not clear, differences escape our observation; and, on the other hand, the unity in which they are comprised may have both little compass and little significance. But pure difference and pure sameness baffle the intelligence by their meaninglessness; indeed, neither can be affirmed or denied except in relation to its opposite. Every judgment, every opinion, false or true, wide or narrow its influence, implies differences within a unity, and is always a system. The assumption of pure particularity which the pluralist makes, and of pure unity or sameness which the absolutist makes, is not valid of the object of knowledge at any stage, from the crudest ordinary consciousness to the completest constructive height of the speculative philosopher. The problem of passing either from the Many to the One, or from the One to the Many, is insoluble; but it is also a problem that the human mind is not obliged to ask. It is a problem asked neither by the nature of things nor by the nature of reason. It is as unnecessary and as insoluble as the problem of proving that

 $2 \times 2 = 91$ . And the way to deal with such a problem is not to ask it. The several philosophies which ask the question are the ordines of abstraction, and their error is revealed whenever the abstractions are faithfully pressed home. They will then be seen not only to call forth, but to pass into, their opposites, and thus to refute their own starting-point.

A general survey of the reflective thought of the present day will prove, I believe, that it is engaged upon this tark; and its main province lies in the explicitness of the assumptions and the rigor with which they are being followed to their conclusions. At no previous time were the advocates of the Many and of the One so frankly opposed or so evenly balanced, nor their contradiction more direct and full. Except in one or two instances, pluralism exists in order to complete absolutism, and means to have no mission except to maintain the existence of contingency and multiplicity, and it must itself perish in the hour of its victory. But the pluralism which aims at being constructive is an unusually interesting phenomenon, and much more characteristic of the times than the absolutism which it would refute and supplant.

As a matter of fact, the absolutism which is supposed to begin with a bare "universal" or "One," and to proceed to evolve the varied contents of experience from that "One," employing an a priori method of mere analysis, need not detain us. Such a method may have been employed by the Eleatics, and can be attributed, not without justice, to Spinoza. It is also supposed by critics to be employed by Hegel and his followers. But it does not concern us at present to determine by whom the theory is or has been maintained, nor under what great names it may shelter itself; for we are not engaged with the history of philosophy. We need not seek to ascertain whether the Absolute of Hegel stood for an empty One, or for the whole of reality as it is in all its con-

creteness in itself and for itself. Only the first, as the abstract Absolute, engages the attention of the pluralist and concerns us.

But it concerns us only to be dismissed. I admit at once, and without any reservation, that philosophy cannot begin from such an Absolute; that if it could begin, it could find no way from it to the rich complexity of real being; and that the method of mere analysis and a priori deduction can elicit nothing out of its emptiness. No doubt the psychological history of man's mind may give evidence of a process by which the indefinite mass of its original sensuous consciousness is distinguished into elements and sights and sounds, and even the Ego and the non-Ego are practically defined and their differences made explicit. But absolutists are held to be guilty of neglect, or even contempt, of psychological evidence rather than of converting psychology into a metaphysical absolutism, though I should find great difficulty in admitting its existence elsewhere than in the minds of its critics.

But it is not so with the opposite theory, which professes to start with "the Many" and to seek "the One"; which maintains that particulars are given and universals are found; that experience proceeds from discrete sensations to perceptions, and from perceptions to more general conceptions, and from those to the still wider "ideas of reason"; and that the object of experience, the whole region of ordered facts, presents itself at first as the scene of separate, individual occurrences, and an aggregate of things real in their independence of one another, each of them isolated, impervious, exclusive, an object of simple apprehension. The pluralists maintain, in so far as they are logically faithful to their fundamental hypothesis, that such is the true or final character of reality. If we affirm its unity as a whole, or the harmony of its elements in virtue of any universal

principle or law of being, we go beyond our evidence: we even flout the facts. All the objects of man's thought are finite; even God is one among, or, what comes to the same thing, one above and over above, other beings. Real existence implies singularity. A thing, in order to be, must be itself, must carry within it a private core, which is its own true being, and which remains its very self, whatever relations it may enter into or come out of.

All realities are particulars, we are told. Nothing exists beside particulars. There is "no unity" or common element, no real or existential universals, which exist or subsist in addition to the particulars. There are no things-in-general, and no events-in-general. Nothing exists which corresponds to such a general conception as "animal" or "tree" or "man"; but only this or that animal or tree or man. Nor is there any universal substrate which constitutes them into a class. A class is due to our classification: it is an idea, not a thing. We may, and do, find similarity between different objects: but each of them exists in and by itself, and the similarity is an idea which we form by comparing them with one another. Anything that destroys their intrinsic singularity or uniqueness destroys them: for them to be is to be each its own unique self.

How, then, do we account for law and order? It is simply and purely the outcome of intelligence. Everything that exists is its own law, an active essence, or character, behaving in its own particular way. There are, therefore, no repetitions in the realm of the real, any more than there are similarities, and no absolute fixity. Repetition, enumeration, measurement, mathematics are not possible except by abstraction, and are not true of any real existences. "All our assertions of identity among reals are at bottom negative, amount simply to saying that we discern no difference."

But what comes of this view of the universal laws, which

science seeks to establish, and the uniformity of nature which they postulate? Does not this doctrine "let contingency into the very heart of things"? Must not a perfectly discrete world be in every part of it unintelligible? The consistent pluralist answers these questions in the affirmative. So far as science deals in universals, it does not touch the reality of things. Thought must start from the particular, but it cannot return to it. Thought gives us only the universal, and universals are only hypostasized epistemological entities. Facts and universals, in short, belong to different orders: the former to the world of objective reality, the latter to the objective world of knowledge. Moreover, they do not even correspond. The universals are not true—that is, they indicate no existing realities, as perceptions may do. The socalled laws, and the universal and necessary causes of which natural science speaks, correspond to nothing that exists in reality. There are no laws or necessities or uniformities of nature. These are mere results of our own thinking, conceptions fabricated by our minds through observing, selecting, summarizing and generalizing the multitudinous, particular occurrences which really take place. "In the real world we can nowhere find that exact similarity which the mathematician can readily conceive, and the contention is that it nowhere exists." "There are never two beings which are perfectly alike, and in which it is not possible to find an internal difference"; and, a fortiori, no two events or occurrences or activities can be identical. There is, to our loose and general observation, an apparent repetition of events, of acts in the world, and we speak of "same causes" and "same effects"; but sameness and uniformity, together with the continuity and necessity which are assumed to spring from them, are mere thoughts. There are no natural laws, nor any real being corresponding to any concepts the physicist can find it

convenient to frame regarding the ultimate constituents of matter. Continuity must destroy particularity. Each real thing has its own unique constitution. Pluralism thus does not hesitate "to let contingency into the very heart of things." "I not only admit it," says Dr. Ward, "but contend that any other world would be meaningless."

But there is another application of this pluralism to which I must briefly refer. It is its application to the subjects of knowledge. The particularity, uniqueness, and exclusiveness which is the essential character or true being and essence of natural things, is attributed to minds, and to their experiences. Every mind exists, and for itself. There is no continuity between or in them, and each is absolutely impervious. Every mind maintains the absolute isolation of its own being. And the same holds of their experiences—or the same would hold if any general affirmation could be true. The presentations of one man cannot become the presentations of another. Every mind is the exclusive owner or retainer of its own truths and its own errors. To every self its own world, to every Ego its own non-Ego. Above all else, we must not play fast and loose with the uniqueness and isolation—with the being in itself and for itself-of personality, or of its experience.

How, then, can they agree? How can they disagree? How is any communication between them possible? Not by changing places, not in such a way that "the presentations of one could become accessible to the others." "This is just the most impossible thing in the world. Individuality consists precisely in this impossibility." There is no element common to the several experiences. Each monad mirrors its world "from a unique standpoint of its own." Universal truth, in the sense of a truth that is possessed or attainable by all minds, has to go the way of all other universals; and

if general conceptions are still possible, they are possible only in the sense that every mind has its own private stock of them. There are thus as many experiences as there are persons, and as many sciences as there are scientific men—probably more. And they are all interpretations, equally true or equally false—if, indeed, either falsity or truth can appertain to different worlds where every mind has its own object. Pluralism implies solipsism. "So far as reality consists in particulars, so far it pertains to each experience for itself alone; and so far the solipsist in theory, and the egoist, a solipsist in conduct, are logically unassailable, even though the proper place to put them be, as Schopenhauer said, the madhouse."

But we have just seen that on the pluralistic theory reality consists exclusively of particulars. What, then, can be the meaning of introducing the qualifying phrase "so far"? It is necessary in order to escape from solipsism, and, in other words, to enable the several persons to communicate with one another—communication consisting "in establishing relations between these primary realia." There must be a medium for mutual understanding, and by means of it they must arrive at common knowledge.

But what can "common knowledge" mean for the pluralist? Evidently not that the knowledge which L has is also possessed by M and N. They "cannot change places so that the presentations of one become accessible in their actual entirety to the others." "This is just the most impossible thing in the world. Individuality consists precisely in this impossibility." The knowledge of L, M and N may conceivably agree, but no part or element of the knowledge of L can be the knowledge of M or N. Each of them "mirrors the universe from a unique standpoint of his own." Every Ego has its own non-Ego. "Thus, when in place of the Ego

L we have M or N, so too in place of the non-Ego non-L we have non-M or non-N." The mutual independence and isolation of the subjects of knowledge thus carries with it the isolation and mutual exclusion of the objects of their knowledge. All experience, to begin with, is, we are told, "individual." It is the private knowledge of each person, and it is a knowledge of different objects. When ten men lock at the sun or moon, "each of these persons sees a different object." How, then, and in what sense do the ten come to know that the actual object of each is the same individual object for all? How can they hold any communication with one another so as to agree, or even disagree? "Except on the basis of individual experience, communication is impossible," for it is evident that, first of all, each must have something which he wishes to communicate. The difficulty would seem to be insuperable.

It is overcome, however, by one author in a very simple way. He assumes just the least possible "common knowledge"! "The most that L can indicate or communicate to M of any part of his own experience is so much of it as is common to the experience of both." We may be sure that the earliest intercourse is very slight: just simple indications, a mere pointing to a particular thing as this or that. But once it is begun, the process goes on successfully. "We point to other particulars resembling it, other shining, moving, round objects, and so, by suggesting its likeness to these, take the chance that parallel relations or comparisons will be verified by our fellow-men."

Criticism of this view seems to me to be superfluous. It is directly self-contradictory; and the contradiction is not in the least removed by admitting as little common knowledge to begin with as possible. For "common knowledge" or "common" anything is just what pluralism denies.

Nor does practice come to the help of theory, as we are asked to believe. I do not doubt in the least that "the case of ten hungry men and a loaf would be an impressive object-lesson"; and it ought to be specially impressive to the pluralist. For he would find it difficult to live up to his theory were he one of the ten. To do so, having his own unique experience of his own unique loaf, he should not object to any of the others eating their own unique loaves—supposing, indeed, he could be aware of their loaves. A pluralism that is consistent is certainly not supported by practical experience, and there is absolutely no transition possible from individual experience, such as it is represented by the pluralist, to that experience which is universal in the sense that different men understand one another and mean the same things by the same things.

It would be interesting to observe the manner in which the pluralist repeats, in his final philosophical account of reality as a whole, the same contradictory process as he employs in order to enable his theory to start on its way. For we find that the deity is introduced as a background of unity, or as some kind of substrate, or is even spoken of as "immanent." It is admitted, however, that such a conception of the unity of the whole cannot be "empirically verified." "The pluralist halts at the Many and their interaction; he declines to go further because he finds no warrant for so doing." But if it is objected that the hypothesis of unity is of no use unless it can be verified, we are reminded that philosophy is not sci-Science must verify empirically. The facts with which science deals "fall within experience, and this is sure, therefore, sooner or later to furnish a crucial test of the validity" (of its hypotheses). But philosophy cannot justify its ideas in this way. It employs another method. It justifies its "ideas" by appealing to "experience as a concrete

whole"; "and they are justified in proportion as they enable us to conceive this whole as a complete and systematic unity." But, we ask, is not the conception of the whole as "a complete and systematic unity" precisely what the pluralist cannot have? For, as we are told in the next sentence, "the pluralist halts at the Many and their interaction; he declines to go further because he finds no direct warrant for doing so." He gets his indirect warrant by an appeal to theism—that is, by an appeal to that which cannot be included in his theory because it contradicts it. The pluralist, being also a theist, admits a unity for which he has no warrant in experience, and with which the facts which are held to be given in experience, being a "Many," are directly inconsistent. Pluralism begins and ends with a contradiction.

The failure of pluralism in its application to the objects of knowledge is not less evident than it is in its application to the subjects of knowledge. The relation of the former to one another is as unintelligible and impossible as intercommunication between the latter. In fact, the problem in both cases is the same; for all objects of knowledge turn out to be in the last resort all subjects of knowledge, and all "things" are held to be persons. "The only things of which we have positive knowledge are subjects with intrinsic qualities, things that are something in themselves and something for themselves."

The pluralist admits relations between objects, as he admits the intercommunication of subjects and an experience which is universal. But they are not relations between things, in the sense of existing over and above that which they relate. There are not things here and relations there; in other words, there are no existential universals.

What, then, are relations? They are the activities of particulars, "the intercourse, the coöperation or conflict, actual

or possible, of the individuals themselves." "The passion and action of things must take the place of relation. . . . There are no objective relations other than this living action and passion." But we know nothing that is active or passive except minds, and nothing else can be for itself. Hence "the only causes of which we have positive knowledge are minds: these have a nature of their own, and hence can interact, determine and be determined." Pluralism ends in panpsychism. "The attractions and repulsions of which the physicist speaks only metaphorically, are to be taken literally -that is, as implying impulses initiated and determined by feeling." "For modern pluralism the universe is the totality of monads really interacting." The "Many of pluralism constitutes the class of entelechies or persons in the widest sense—beings, that is to say, who are something for themselves, conative and cognitive individuals bent on self-conservation and seeking the good." "They are severally related by their mutual interaction. . . . We have not two distinct and separable facts-first, the Many, existing in isolation, and then their interaction." "The universe is the totality of monads really interacting, and this is one fact." "The plurality implies the unity, and this unity implies the plurality a fact which is an inexhaustible wonder."

Now it is evident that the crucial question for this doctrine is the possibility of the interaction of the monads, or the cognitive and conative persons into which all reality, including so-called material reality, has been resolved. But we have found already that this is impossible, and I shall add only one consideration to those I have already advanced.

Let it be assumed that the monad or personality A knows and wills, and also that for it to be is to know and will. Let it be admitted, further, that monads B, C and D do and are the same. It is plain that the action and passion of A are

exclusively its own; so also are the actions of B, C and D. Is it less plain that in that case the relation or interaction of these several experiences, supposing it does result, is no part of the action or passion of any one of them? The assumption that the actions and passions do interact, and that they are experienced as interacting, may be quite true: but for the pluralist it must not only be made gratuitously and dogmatically, but in flat contradiction of the fundamental hypothesis of the particularity and exclusive individuality of every item of the "Many."

Moreover, I must ask one more question of the pluralist. Can any particle, monad, person or subject either be active or passive purely from within itself? The pluralist finds his clue to the nature of all reality in his own mind. Has he known his own mind, either mind or will, entirely apart from the universe in which it exists? Is action or passion in vacuo possible? And is not a mind out of all relation to the world, a self which has no not-self, a vacuum and pure fiction? To will, think, or even feel nothing is neither to think nor will nor feel; and a mind without any "content" is a nonentity.

On the other hand, if it has a content, that content, for all the purposes of "conation and cognition," is an object and a non-Ego. But an Ego which has its non-Ego or world as its content or object of experience is not the "particular," exclusive Ego of the pluralist. It at least implicitly contains its world! The Ego, instead of being exclusive and particular, turns out to be at least potentially all-comprehensive. The individual mind is the subjective expression and the spiritual focus of the universe. It is a Many in One; and to explain how this can be is the paramount problem of philosophy.

It is an old problem, this of the relation of the One and the Many; and I agree entirely with Dr. Ward when he says that "the solution is not to be obtained by passing over the

Many at the outset, trusting to deduce them afterward from an absolute One that is reached a priori"; and that "this method has proved itself illusory; the seeming attainment of the One has meant the disappearance of the Many." If, as he avers, Fichte, Schelling, Hegel, Schopenhauer, and others less distinguished verily held such an "absolutism or singularism,"-a question which I do not raise at present,-their recent thought does well in recoiling from their doctrines. I can only say that I have not understood them in this and that way. On the other hand, I find that Dr. Ward admits that pluralism has also "failed to reach a satisfactory solution of the problem of the One and the Many"; he allows "that no philosophy has ever managed to reconcile these two notions of an infinite power and an infinite variety of limited, individualized expressions of that power." But I would apply to pluralism, mutatis mutandis, precisely what he says of absolutism or singularism. The solution is not to be obtained by passing over the [One] [Many] at the outset, trusting to deduce [it] afterward from the Absolute [Many]. For the Many is not "given." The pure Many is as much an a priori construct as the "Absolute One," and as little given in experience. And as it is admitted that "Pluralism fails or has so far failed to account for the unity that it in fact involves," then the right and the duty of recoiling from the doctrine is as absolute and imperative as the right and duty of recoiling from its opposite.

Indeed, the promise as well as the problem of the philosophic thought of the twentieth century arises from the exposure of the impossibility of both of these abstract theories, and its rejection all along the line, from the most elementary perception to the most comprehensive reflective knowledge of the premises and the methods of both.

#### Third Lecture

O theory can be satisfactory if it is inconsistent with itself; and none can be satisfactory if it attains selfconsistency by merely ignoring or abolishing differences. Pluralism cannot afford to be self-contradictory, and singularism or absolutism cannot afford to affirm empty sameness. These rival schools, starting from opposite poles and employing opposite methods, would arrive at the same goal. They would admit in their scheme both unity and diversity, and they would reconcile these notions. And reconciliation would, for both alike, mean more than the admission of unity and diversity side by side. The One must be explicable only through the Many, and the Many only through the One. Such is the acknowledged condition and criterion of philosophic truth: it cannot contain ultimate incongruities nor be incomplete; it must be a system which is all-comprehensive, and in which all the elements have their own place and function.

It ought, it seems to me, to be obvious that the condition and criterion of reality must in these respects be the same for the real. To maintain a different criterion of truth and reality is not possible with establishing a fundamental discrepancy between them at all points. Reality can as well contain ultimate contingencies as truth can contain ultimate contradictions. Pluralism must as a philosophical theory be a doctrine of the universe as a whole, and if its doctrine must be self-consistent its universe must be one. And absolutism, if its "One" is to have meaning, must affirm the real diversity of the real. In a word, on any theory, the destiny of reality must be the same as that of truth. Epistemology and ontol-

ogy, even for those who recoil from saying that "reality is experience," must be two names for one doctrine. For the real gains no expression except in knowledge, and knowledge must have the real for its content.

No one will affirm that the concrete truth of the concrete real either has been or can be attained by human knowledge. In that sense no philosophy has ever pretended to be "absolute." But we found in the last lecture that such a truth cannot be approached, and that not even the first step can be taken toward it by a philosophy which omits either the One or the Many from its original premises. There is no way either from differences to unity or from unity to difference. Indeed, it might be shown that both pure difference and pure unity are confused and contradictory notions. To endeavor to start from either the one or the other is to start from the abstract and the meaningless.

What alternative remains for philosophy? Evidently to start from unity as expressing itself in diversity, or as already concrete. Knowledge must exhibit at every stage even the first—the essential characteristics of a system. Every object, whether it be that of immediate perception or that of philosophic reflection, whether it be a so-called simple fact or the universe in its totality, must have the character of individuality. This means that it must consist of parts or elements between which there are real differences; but, at the same time, the differences must so complement and sustain one another as to constitute one reality. And that reality is not the mere sum of the parts or elements, nor is it anything superimposed upon them by way of a containing supplement or envelope. For the one can neither be indifferent to the elements nor independent of them; nor are they, on their part, indifferent to or independent of one another or of the whole. The One and the Many must

derive their intrinsic character and their very being and function from each other. They must be distinguishable, for they are different; but they must not be separable, for they constitute a unity. On the other hand, they must be One, for they are forms of one reality; but they must not be fused into sameness, for they are different. But this means that individuality belongs both to the whole and also to every real element of the whole or instance of the Many. To deny the individuality of the whole is to disintegrate it into inexplicable and unreal differences, every one of which "is a surd for thought"; and to deny the individuality of the parts or elements is to reduce unity to emptiness and to make it meaningless. Hence, further, the One and the Many must be both dependent upon and independent of each other. They must exist in themselves, and nevertheless exist only in virtue of their relation to each other in a whole which is at once constituted by them and constitutive of them.

But, it may well be asked, does this not also imply that philosophy starts from and deals with a self-contradiction? It depends, I shall try to show, on the meaning of "individuality," of dependence and independence, of real being and of relation. In all cases it is the problem of philosophy to explain this apparent enigma. It is not to show that this view of the individuality and reality of the whole and of all its elements is true. We have seen that philosophy postulates this view of truth and reality in attempting to be a coherent or systematic doctrine. Nor is the postulate a mere a priori assumption, unsustained by experience. contrary, there is no department of experience which does not contain, or rather consist of, instances of the unity of the diverse, and of the diversity or complexity of the One. The problem confronts ordinary thought on every side, only it ignores it, and it is presented in every one of the arts and

sciences. Let me exemplify this fact by citing one or two examples. When four voices sing together the notes CEGC', or GCD, or DFA, harmony ensues. Now harmony is not mere unison, nor is it mere multiplicity. It is a single effect in which all the voices are fused into unity, but the fusion does not annul the differences nor destroy the individuality of the voice. The individual harmony consists of individual voices each of which is enriched by its relations and intensified in its beauty.

It is evident that the same holds of a piece of music as a whole. It consists of sequent movements, the first of which passes away to make room for its successor, and yet the character of the movements which come last depends upon—that is, somehow carries within—what went before, and continuity—nay, unity—remains by means of the succession.

Every work of art exhibits the same character of being a One in value of the Many, and presents the same problem. A turret depends for its artistic value upon the place it occupies in the edifice; and so does the artistic value of the edifice. Each gives and borrows its significance and worth from the other, and yet each has its own meaning. So it is also with a picture or a poem. Both the parts and the whole have their individual being and value, and yet these depend on their relation to one another in the whole.

When we turn from the arts to the sciences and to philosophy—to systematized knowledge—the same truth holds. The meaning of a statement depends upon its context and all its cognitive value. A statement may be rendered meaningless by changing its context; and truth itself becomes error when it is placed out of "the appropriate universe of discourse."

Nevertheless the unity of the systematic truth is not obtained by mere fusion. Every element in it retains its own

value, and makes its own contribution to the whole. When the mathematician, for instance, proves a theorem in geometry he is engaged in demonstrating one, and only one, truth: e.g., that the angles of a triangle are together equal to two right angles. But the single proof of a single truth somehow consists of many truths, and these are at once independent and interdependent. They are independent in that they cannot be done without, and nothing can replace them or perform their function in the proof; they are interdependent in so far as none of them has either significance or value except by reference to one another and to the single truth they subserve.

In short, the testimony of rational experience to the reality and the interdependence, to the individuality and to the essential and even constitutive interrelation of the Many and the One, is universal. The mere Many of the pluralist and the mere One of the absolutist are alike nothing more nor less than fictions. Experience gives no example of them. They are the results of the abstract treatment of experience.

It follows, therefore, that the interpretation of experience, which philosophy is, must accept this apparent enigma. Its problem is not to show whether, but how, this can be possible—to maintain the reality both of the One and the Many, and to reconcile in its theory what is already reconciled in reality.

But to maintain this view of philosophy, and to carry it out into its results, is to challenge a formidable array of abstractions. For, as we have already seen, the tissue of reality is torn by human knowledge and its seamless raiment rent asunder. We convert differences into contradictions, and isolate and fix our distinctions; and, in consequence, we find the differences irreconcilable. The reality and independence of the Many is assumed to imply that they are exclu-

sive; and any degree of community of existence is held, as a matter of course, to destroy their individuality. The sway of abstractions is very wide.

Nevertheless I believe, as I have said, that if there be any movement of thought in this twentieth century which specially characterizes its mission and promises significant results, it is that of first exposing and then rejecting these abstract opposites. It is, in one word, to repudiate the categories-what Kant by a new abstraction called the Categories of the Understanding, which are the categories of external and of both contingent and necessary relation. It is to reject in toto the view that the reality or individuality of anything can consist in or depend upon its isolation. It is to discover that to negate is not to contradict, and that to affirm is not to reduce into mere sameness. On the other hand, it is not to say that reality consists of relations; but it is to say that it is not independent of relations, and that if relations are abolished nothing whatsoever remains. It is to hold steadfast to the truth so plainly illustrated in every work of art, which consists at all times of individual parts every one of which has its own character and function, and which nevertheless is dependent for both its character and its function upon the work of art as a whole. For, whether we can explain it or not, a piece of music does consist of individual notes, and not of mere relations; and yet if the relations between the several notes be annulled they are changed, and no music remains. And whether we can explain it or not, every rational judgment, true or false, makes one affirmation, and that affirmation contains a diversity of elements.

But if this be the special mission of the philosophy of the twentieth century, it must be admitted that the promise of its fulfilment is, so far, faint. Its exposure of the necessary failure of the one-sided assumptions of both pluralism and

abstract absolutism is incomplete. It has not taken to heart that experience furnishes no example of either mere unity or mere diversity, and that these rival theories have pure fictions for their premises. Hence it has not repudiated either the method or the aim of these abstract doctrines. It is continuing the attempt to bring the One and the Many together, instead of proceeding from the presupposition that they always are together. Its process is either synthetic or analytic; synthetic in so far as it seeks to proceed from the mere Many; analytic in so far as it seeks to proceed from the mere One. It does not begin with the conception of system, of reality as a concrete element, nor proceed to observe its growth or evolution, by which unity becomes more deep and significant and the diversity of the parts more clear.

Let me illustrate this truth in the first place with regard to knowledge. The subject of knowledge-namely, the finite, rational self—is still regarded as a res completa; and the object which the subject seeks to know is regarded as another res completa. The problem of knowledge, therefore, assumes the form of showing how they can be brought together. And, further, it is assumed, though with a confidence sharply shaken, that the way of bringing them together is to resolve the one into the other, or, in other words, to abolish the difference between them. And if we have despaired of resolving the subject into its object by the way of materialism, we have, on the other hand, not repudiated the opposite method of resolving the world into the subjective experience of one or more subjects. Subjective idealism is still in vogue, for we say that reality is experience, and in panpsychism the monadism of Leibnitz is being resuscitated, so that all reality is made to consist of what one may call spiritual points, which have only intensive magnitude and no "body" except their own activities.

It is true that philosophers now speak of subject-object, and will even admit that spirit and nature are somehow correlates; but only the most limited use is made of the conception. And when it is affirmed that reality is experience, "experience" is allowed to remain utterly ambiguous so as to carry either an objective or a subjective reference at will. Or when it is explained, as for instance by Mr. Bradley, experience, and therefore reality, is said to consist of feelings, thought and volitions, and subjective idealism reappears.

That little use is made of the conception subject-object beyond the admission that reality is somehow spiritual, is evident from the fact that the psychologist, and also the epistemologist, not only distinguish but separate the functions of mind and things. The world of reality presents the data for mind, and mind then makes the knowledge. But the world cannot give until the mind takes, and the mind cannot take until the world gives; and there is no priority of any kind, either temporal or logical. The statement that reality is experience is meant to convey their intrinsic correlation. But the statement is allowed to remain vague; and experience is, after all, made to belong exclusively to the subject. It is his living conation and cognition, and the object world is its product; and the idealism which practically all philosophers now profess becomes a doctrine which reduces reality either into phenomena of consciousness, such as thoughts, feelings and volitions, or into spiritual monads, more or less confused personalities.

But consciousness cannot be active—that is to say, it cannot be consciousness—except in relation to objects, and the data of knowledge cannot be the results of knowing. Hence the function of the real in the act of knowing must be restored, and consciousness, with all its activities, must be its activity as consciousness, and as a consciousness which is in-

dividual. We must make room for the function of both mind and the world in knowledge, and maintain that, as separate, they can neither do nor be anything. Knowledge proceeds neither from minds nor from objects. It is the selfrevelation of the whole which comprises both, and is both in their interaction. However true it may be that experience is subjective, personal, private to every individual finite spirit, it is still a consciousness which has contents, which exists only by reference to it, and which cannot make it. To account for knowledge we must assume a reality which is wider than either subjects or objects, because it comprehends both, and neither is except in relation to its opposite. To begin with, either is comparable to the process of a mathematician who looks for a product by beginning with one of the factors, starting from either 6 or 7 in order to arrive at 42. Knowledge is the result of the interaction of the two aspects of reality which we not only distinguish but separate and then strive to bring together. We endeavor to find a way out of consciousness and into a relation with facts, whereas we are at all, and are conscious, only in virtue of our relation to the reality which comprehends both our minds and the facts.

But if this is true we shall cease to speak of the self and the not-self, of subjects and objects, of mind and matter, of soul and body, of spirit and nature, of God and man as first existing apart, and then brought together through the interaction which reveals itself in knowledge, in the fine arts, in morality and in religion; for that interaction is, as we have seen, impossible unless they are together. Our distinctions must remain and the differences must be real, and the individuality and even the personal privacy of the human spirit be maintained, but they must be maintained within the unity of the real which comprises both the opposites.

That the thought of the present day is making toward this

genuine universal standpoint is not to be doubted. There is evidence of it especially in such doctrines as that of the "natural-supernaturalism of Carlyle," in the spiritual realism of Goethe, of Wordsworth; in the indefinite view of the immanence or indwelling of the divine in nature; in the repudiation of materialism by natural science and its clearing consciousness of the abstract character of its hypotheses and task; in the growing conviction of the intrinsic interaction of man and society; in the growing suspicion of both individualistic and socialistic theories, and in the thinning down of the partition between the secular and the sacred, so that man finds his duty, which is his spiritual opportunity and privilege, in every station, and believes that every service of man may be the service of God. The sense of man's affinity with the universe is deepening in every way, and the universe itself seems to acquire a spiritual significance because man is an element in it.

The justification of this new attitude which philosophy must furnish is difficult. But psychology on the one side, and logic on the other, are preparing the way for the new metaphysic. The former finds no evidence that mind, however spontaneous, can create its own content. Even imagination. when it is more free, only selects and rearranges. creates its heaven as it pleases, it must borrow its material, as Hume has shown, from the present world, making its streets of gold and gates of pearl, etc. All knowledge is both relative and anthropomorphic, just because both man and his world are necessary factors in the function of knowing. If man is and must be spontaneous in his cognitive and conative activities, it is not because he is separate from the world. In isolation he is helpless. As he cannot lift a hand or move a foot except by means of the resistance which is also the help of the physical cosmos, so he can neither know

nor will, and is in fact only a name or nothingness in his isolation. The world is not a hindrance to man's "spontaneous" spiritual activities, but their indispensable condition. In truth, his knowledge is the activity of the real in and by him; but it is his knowledge none the less, for by it he comprises the real.

On the other hand, his affinity to and dependence upon his cosmos is also its dependence upon him. The cosmos of the materialist is as inconceivable as the knowing subject or detached self of the abstract idealist. If mind is not except in its relation to the object, neither is the object except in relation to the subject. The dependence is interdependence, and the real is never only one of its aspects. It is neither natural nor spiritual if these are considered apart.

Nor does the dependence of the world of objects on mind mean that mind, as we know it, makes them, and in making them infects them with its own subjectivity. The objects do not turn out on examination to be nothing but experience, if by experience is meant—as it ought to mean—thoughts, feelings and volitions, which somehow become substantiated into these ambiguous realities, hovering between being and nonbeing, which we call phenomena. There is no such thing as a "world of truth" which stands over against things in themselves, and mediates between them and minds, being, as Lotze called them, "a replica" of the real. The problem of discovering the connection between ideas and their objects, and all the attempted solutions of the problem by making the former images or symbols or representatives of the latter, or the latter reifications of the former, are as unmeaning and futile as the problem of the relation of the world of fairies to the world of every-day life. There are minds and there are things, and because they are elements of one reality they interact. During their interaction there is knowing,

and the result of the activity of knowing is to modify the subject which knows so that it can repeat the process, even when the objects which first contributed to it are not present. But there is no such result as a concatenated system of ideas, nor even a single idea that has any permanence or being of its own. The relation of minds and of things is *direct* in the last resort, and the relation between them is constitutive of both.

But this, it will be said, makes reality depend for its existence upon being known, and at the least derive a new stage of existence and a higher manifestation of itself from and through man's mind. In that case must not the act of knowing defeat itself? It is the object of knowing to apprehend facts as they are; but that is surely not possible if the act of knowing changes them. Knowing them changes them, I should answer, and defeats thereby its own purpose, only if we continue to assume the dualistic point of view which, at present, we are endeavoring to repudiate, and continue to treat them as separate existences brought together. But the difficulty does not arise if knowing is neither the function of mind nor of objects as apart, but of the reality which comprises them both as elements and aspects. From this latter point of view reality may be shown to enrich itself, to allow fuller being, to set free and to realize new potentialities through the cognitive activities we have been attributing to the self, but which belong to it as comprising the self.

An illustration may indicate the possibility of the truth of the view I am trying to express.

The physicist is supposed to give an account of sound. He tells us that it is wave movement. But the least analysis will show that he professes no such thing. He explains only one of the conditions of sound. Apart from the psychological structure of the human organism, and also apart from the

presumably non-physical but psychological structure of his consciousness, there is no sound. Delete any one of these three distinguishable elements—the physical, physiological, or psychical—there would be no sound and the universe would be silent. Sound is not analyzable into any one of these factors, nor attributed to any one of them rather than to the others; and when all the elements of a unity are necessary there is a true sense in which it is not possible to give priority to any one of them. On the other hand, it is true that the physical conditions of sound—the wave movements produced by the pressing down of the keys of the organ and the filling of the pipes with wind-gain new significance and value when the organ is played by a great artist and the physical conditions are subordinated to the musical purposes of a great composer. The coming in upon the scene of the musician's soul reveals a new range of meaning and beauty which before were dormant in the physical structure of the natural world; and reality as a whole, which has produced and contributed to the instruction and which comprises the musician, assumes through him a new way of being. And yet, though without him there can be no music, we cannot attribute the musical effect to him alone, as we do knowledge, an experience, to the activities of the subject. Without his context he also is helpless. The distinction of meum and tuum does not hold. The musician's spontaneous—or, as we say, creative—power is conditioned by the real world as a whole in which he lives and moves and has his being, and at the same time the real world needs him in order to realize the significance even of its natural elements.

This illustration suggests the possibility of maintaining that finite minds by their cognitive and conative activities have a more significant function in relation to the world of reality than that of "manifesting" or "expressing" its mean-

ing in the way of truth; and that their relation to it is more intimate than can be accounted for by any theory which attributes their activities to themselves alone, and which makes consciousness contain an idle, epistemological replica of reality. If in order that there may be music, or any other of the productions of fine art, reality as a whole comprising the artist must be effectively present, so reality as a whole must be that which thinks and wills. Not that minds in willing and knowing are mere instruments upon which the world of reality plays, or by which it gains better and fuller expression. The idea of "instrument" is inadequate to the occasion, and we obscure the truth and lapse back into dualism when we represent minds as operated upon. It is the mind which introduces the purpose. In the case of both the musician and the scientific man or philosopher the natural elements of the cosmos are in a sense subordinated to their purpose; and yet the purpose is not alien to the natural cosmos, or superimposed upon it from without. For nature's own potencies are realized in and by them, and in him they acquire themselves a better and fuller way of existence.

But in that case we must start from a new hypothesis as to the nature of reality. We must no longer speak of it as either natural or spiritual, nor, in order to account for it, endeavor to make the natural disappear in the spiritual. Nature as merely natural is now discovered to be only a fragment of reality, even of reality as finite minds know it. It is and remains "natural," for it is the condition of the spiritual activities, which condition is fulfilled in the finite minds into which it breaks. The facts which we speak of as given in actual experience are real as manifesting themselves in finite minds. Reality has this dual character. It functions in the thinking and volition of men as truly as in the form and the color of plants. Reality has a dual character, or

rather it is natural-spiritual. We may distinguish but we cannot separate its elements. Hence mind and reality do not need to be brought together, and thought has not the impossible task before it of going out of itself to reality. It is by comprising the real: and the real exhibits its full and true nature only in the activities by which truth and goodness are attained. When mind appears on the scene the real breaks into knowledge as well as into music, and into moral lives as well as statutes and stately edifices. It remains natural, but it is a nature with spiritual potencies that break out into actuality in man. He is nothing apart from it. He is continuous with it. He is effective as mind and will in the degree in which as subject he is saturated with its truth and purpose. For his purpose is a revelation and liberation of Nature's purpose. He is no external addendum, but her product. But when he appears, being her highest product, he recoils upon her, sublates her lower forms of being, assimilates them with and incorporates them into activities which are his activities without ceasing to be Nature's own.

There is a psychological problem for which, so far, no solution has been found. It is that of the relation of soul and body. Psychologists at present propose one of two theories. They suggest a panpsychism which converts all bodies into souls, or a parallelism between them and their phenomena. The former theory introduces more difficulties than it solves, and, so far, has not shown itself worthy of serious discussion; the latter confesses its failure in that it only states the problem and, in fact, offers no solution of it. If our criticisms have any validity, no solution of this problem is possible; and it is impossible because it contains a surd. It is like the problem of proving that  $2 \times 2 = 91$ , which would baffle all mathematicians; or of inventing a perpetually moving machine, which must baffle the physical inventor; or

like saying, "Why should we be moral?", which must baffle the moralist. The mathematician, physicist and moralist who know what they are about will not ask these questions. Nor will the psychologist endeavor to relate—that is, to bring together in thought—what he assumes to be separate in existence. He will rather take to heart what Aristotle has said of such a dualism. He will regard the soul as the highest expression, the full reality, the évêpyeia of the body -not deleting it, nor supplanting it, nor yet subordinated to it as a mere consequence or effect, but rather as that in which the body exhibits and realizes its full being, and in doing so proves its intrinsic spiritual potentialities. In man also we find exemplified always, not a soul plus a body, not merely natural or physical and superadded spiritual powers, but one being whose spiritual activities are at once conditioned by, and sublate, or take up, the so-called natural elements. The problem of the psychologist as at present stated is insoluble, because he is unjust to his body and ignores its function in all volition and thought, attributing cognition and volition to a mind in isolation, mind as merely subjective, of the existence of which there is no least item of evidence in any experience.

Man, like the cosmos, is nature at its highest and best, and nature is not a dead mechanism and mere opposite of spirit, any more than it is spiritual apart from mind. The beauty and truth and goodness which appear when man is upon the scene are not only his, but nature's also. And spirit does not dwell in it as in a dead husk, but is its own intrinsic power. This, it seems to me, is the view toward which recent thought is gradually moving. It is the theme and the inspiration of the greatest poetry of our time, from Goethe and Wordsworth to Robert Browning, and it is the aspiration of the highest morality and of the most elevated and reflective religious consciousness of the present age. It

is the special mission of philosophy to demonstrate the validity of this view, and make good the truth of the one radiant ideal.

There are evidences that philosophy has entered upon this task. But the task is great and very difficult. It implies not merely revulsion from the consequences of the abstractions which have hitherto obstructed its path, but the most fundamental revolution of all the revolutions of the world of mind. It implies a change of method. It must start from a different hypothesis and must therefore reinterpret every fact in the light of this hypothesis. I must content myself at present by mercly indicating the main obstacles which obstruct its path as it enters upon its problem, all of them due to the abstractions which we have substantiated into contradictory opposites.

The first of these are logical, and therefore metaphysical also, or ontological. I acknowledge that it is precisely in its logical doctrine that modern philosophy has made its greatest advance toward the adoption of this point of view, which, in fact, is that of spiritual realism or concrete absolutism. Nevertheless, even at its best, it is not free from the entanglements which issue from the use of the external categories, which Kant called the Categories of the Understanding. That it is not content with their use and that it aspires to a better is illustrated by its appeal to intuition. Intuition is found to achieve what lies beyond the power of the understanding. It grasps things in their veritable unity: it does not obliterate differences, but it makes them harmonious or transparent-to employ its metaphors. It bridges the gulf between knowledge and reality, and brings mind into immediate illuminating contact with that which is. But it does this at the expense of all method. Its operations are mystical and miraculous. It explains by means of the unintelligible.

It has no value except in so far as it expresses discontent with the external methods of "the mere understanding," which, after all, it cannot supplant and must merely supplement.

The method of intuitionism is too easy. It is like the optimism which finds that all is right with the world by denving or ignoring its unhappiness and wickedness. It cannot help until it turns back upon the topics of the understanding, and reveals the unity within its opposites, and shows it to be intuitive in the double sense that it always grasps unity and is always in actual touch with the real. But owing to the domination of these external categories the judgment is still treated as if it were the result either of a purely analytic or of a purely synthetic process, and reasoning as if it were either deductive or inductive. The predicate is either attached to the subject as a new thing, or it is a mere repetition of a part of the subject. In the first case the judgment is a mere accretion of elements; in the second, a mere tautology. In the first case it cannot be true; in the second it can have no meaning. Moreover, both of these processes rest upon a false supposition as to the nature of the relation of the part of the judgment, as well as of the parts themselves. Their agreement is assumed to mean their identical and indistinguishable sameness—bare unity: their disagreement or negation, to be contradiction and repulsion. In no way, therefore, can either of these theories represent the judgment—that is, any rational opinion—as concrete; and the process of judgment as beginning in the subject with what is already a system, and exposing the nature of the system in the course of judging and reasoning, distinguishing its elements and deepening its unity by the same movement.

Again, on the epistemological side, the "that," or real

being, of the object of knowledge is held to be distinct from its "what," or its qualities; and judgment is made to consist in bringing these together. And, further, as I have already indicated, knowledge itself is separated into forced abstractions, and the content is assumed to come from the data, while the form is supplied by the activity of the subject. The consequence is that knowledge and reality themselves remain in inexplicable opposition, and truth is in fundamental contradiction with itself. For it is assumed that to agree with or to represent the real as it is, it must cease to be as truth, and be merged in the real, or else be transmuted in an unknowable way by an unknowable Absolute.

But such results indicate the need, not of escaping from methodical thought by means of mystic and methodless intuition, but of recognizing that thought is always systematic and its object always a *One* in the Many, and therefore of ceasing to set the dualistic problems which baffle all attempts at solution.

The second main obstacle, and possibly the more serious in practice, may be called ethical. It is assumed, to put the matter as directly and concisely as I can, that the ethical world will disappear if man is not the genuine creator of his own actions, or absolutely spontaneous; and, further, that his creative power or spontaneity must mean that he stands apart and absolutely isolated from the so-called outer world. He is a pure subject, as represented by Kant, ontologically separate from all objects, and even from himself when he is the object of his own knowledge—his knowing self falling into the noumenal, and his known self into the phenomenal world. We are jealous, and rightly jealous, of our own intrinsic individuality, and assume that in order to maintain it we must hold the world, so to speak, at arm's length and extrude it. Let the outer conditions, and even our own past history, be what they will, we must at any moment have the

power of acting upon it and from it in a manner that, for all computation, must be contingent. "Contingency," as we have seen, must be "let into the heart of things." The inner life shuts out the outer world; or if it shuts it in or comprises it, it is only in the form of "experience"—that is, of thoughts, feelings and volitions—and its realities become "phenomena." We have "gone back to Kant," and we still dwell among his contradictions, for we have not gone forward from Kant.

Now I have no desire to minimize or to obscure in the least degree the privacy of personality, or the subjective and intensely individual character of all experience. On the contrary, there is no apparent excuse into which I would not follow the solipsist in this direction. All experience is in the fullest sense individual, and there is no such thing as universal experience in the sense that one finite man can think the thoughts or will the volitions of another. Every man's thoughts and every man's volitions are exclusively his own, and no other's; they remain his own even if it be true, as it is, that other men may know the same truth and will to bring about the same change.

When the idealist, in endeavoring to meet the evident objections to solipsism, affirms that a man's mind is not a particular thing, like his pocket-knife, but has a universal nature, which makes his mind one in intrinsic structure, subject to the same laws, active in the same manner as all minds, or as mind "as such," I have no concern in contradicting him. But such an argument does not obviate the difficulties of solipsism. However universal in nature a man's mind may be, it does not lose its intensely private and personal character, and all his experiences are his own in a sense that is exclusive. In other words, the subjective, personal, private character of experience remains, and every mind looks at the world with its own eyes. Were all men, like the gods,

possessors of the real truth of all reality, their thoughts would still be their own; and were all human wills one with the will of God, they would still be personal wills and the moral perfection would be their own.

The reputation of the solipsist is implied in his own premises. There is no solipsist who in making an affirmative does not consider that his affirmation refers to, and is an ideal construction or representation of, reality. He is expressing his own thoughts of the real, and his thoughts are his own. But, unless he confuses the results of his thinking with that about which he is thinking, and the object which he strives to comprehend with the products of his effort, he will not maintain that the real about which he thinks is also subjective. He cannot at the same time profess that he is expressing the truth and maintain that he is not dealing with the real. His thoughts, however subjective, have an objective reference, and however personal and private, they are his personal and private conception of that which is. Truth, affirmation, negation, judgment have in every instance this reference to the real. The reference is direct in every experience, and the reference is always to the real—that is, for each mind, to only one real.

Hence every solipsist considers that he knows the truth; and it is not possible to affirm or deny except on this presupposition. The question of agreement or disagreement is subsequent and secondary. What concerns us now is the universal and necessary character of every experience, however personal. The reference of a judgment is not to a private real; not even when he says, "This is only my opinion." Even that statement is a statement of a fact. And it is alleged that the result of the dealing of different minds is a different experience, or as many opinions as there are minds. Still, each mind in every affirmation refers to what is real, or to what his thought represents or misrepresents.

Nor can it be affirmed that each subject refers to a different reality, a reality infected with the illusions of his own thought. Once more it is the result that may be illusory, or merely phenomenal. And, as we have seen, the results of knowing cannot be the *data* of knowledge; nor have they any existence except as ways of the activity of the cognizing subject. Phenomena do not constitute a class of existing things, over and above the subjects which know, and the reality which the subjects endeavor to know.

Thus every experience is bipolar. It is the living relation or interaction of two elements of a reality which is at once spiritual and natural. Knowing and willing is the act of the self by means of this world and of the real world. For no existence can refer to any other.

The question of the agreement or disagreement of the different experiences, or of any community between them, is subsequent and secondary to the reference of each experience to the real, which every judgment is. And it also concerns reality, which is capable once more of being rightly or wrongly interpreted. And the real is in this and every other case the criterion of what is held to be true or false. So that the reality also is assumed in every experience, in every act of cognition, to be bipolar. It is, and it is capable of expressing itself subjectively to the knowing mind. Reality, we may perhaps be allowed to say, expresses itself in many self-conscious foci and in many degrees of accuracy and fullness. But the presupposition of the real—that is, of one single reality—is as inevitable to every subject as the presupposition of his own existence.

When the solipsist, therefore, affirms that every subject has his own experience, which is true, he overlooks the fact that the object with which each experience deals or which it endeavors to represent is that which is. No subject can assume that there are as many systems of reality as there are

interpretations of it; he denies to the experience of others that which is essential to his own and to the very possibility of experience.

It follows from this that there is one criterion for all experience, and one ideal. It is reality. It is by constant reference to it that he corrects and extends his own, and affirms or denics the truth of the experience of others: for their expressions of it are also objects for him, and parts of the reality which he endeavors to know. And the reference to the real is a reference to the Absolute—that is, to that which is all in all and exists in its own right. It is by their seeming congruence or incongruence with the presupposed whole of reality that particular opinions are called true or false. But this is as much as to say that reality is held to be a systematic whole, within which each particular fact has its own place and function. If we work to correct another person of error in any judgment, we do so by compelling him to choose between that opinion and his interpretation of that which is The admission of a new truth may compel us real. to revise our conception of the system of reality. new hypothesis may carry with it a revolution in our view of reality; but the reality which is the aim of our intellectual attempt, and the criterion of the value of its results, is no new reality. It is not true, therefore, that there are as many realities as there are opinions of reality; although there may be as many interpretations of it as there are cognitive subjects. On the contrary, each subject is necessarily assumed to be from his own standpoint endeavoring to interpret the world of reality. Experience, false or true, has otherwise no meaning.

It is this truth that Spinoza expressed when he said that knowledge is adequate in the degree in which the subject of knowledge contemplates objects sub specie aternitatis. And the moral life of man—that is, his practical life when con-

sidered in the light of its ultimate issues—gives an interesting illustration of this truth. For morality also carries within at all times this immediate reference to the Absolute. The action may be, and always is, particular in one of its aspects. But it is also a particularized universal. The right action is a specific affirmation, and the wrong action is a specific violation, of a Universal Good. The right action may be in itself insignificant—the mere giving of a cup of cold water; but being right, it is what is required in that particular context, and neither gods nor men can improve upon it. It is the particular reification or incarnation of the best. It is doing the work of God, in the language of religion. It is accord with the nature of things. And thereby it acquires inexhaustible worth and power.

Hence issues the dignity of an act which we call good, and the splendor which cannot be obscured. Hence also flows the sense of unconquerable strength which the moral agent always feels when he is in his duty. The nature of things is at his back. God is with him. His will is one with the divine. It must prevail. Its language always is, "If God be with us, who can be against us?"

Both in cognition and volition, therefore, both in knowledge and in morality, once we have freed ourselves of the fixed abstractions of the understanding, we find that immediate continuity with reality which is our own life; and the service of the true and the good, being the service of what is real, is the service of freedom so perfect that it finds nowhere aught that can limit or obstruct it. The service is fuller, the closer and the wider our communion with what is real; and the natural cosmos, in all its wealth, is not a limit but a condition of the life of our own spirit, and the living partner in all our spiritual enterprises.

HENRY JONES.

